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THE SYSTEM OF β LYRAE.[1]

By G. W. Myers.

Of the various theories which have been proposed to explain the light changes of variables, three have been most widely accepted :

First. A body whose surface possesses different degrees of brightness at different places, rotates upon an axis, bringing the differently illuminated portions into view at regular intervals.

Second. A secondary meteoric swarm circles about a primary swarm, so as to pass at regular intervals between the observer and the central swarm. The outlying meteors of the two swarms, colliding with each other at the times of periastron passage, produce also a periodic increase of brightness. A suitable combination of these two causes of light variation will explain a large variety of the light fluctuations observed in variables.

Third. The so-called satellite theory, in which two bodies whose luminous intensities may be either equal or unequal, revolve about each other in orbits whose planes pass near the solar system, and, by the mutual eclipses of the bodies, light changes of such character are produced as to explain some sort of variability.

[1] Read at the conferences held in connection with the dedication of the Yerkes Observatory, Oct. 20, 1897.

The arbitrariness involved in the fundamental assumptions of theories one and two, appears to me to be an element of weakness of so serious a nature as to compel them to yield to the third theory in all cases where the latter is applicable. By proper assumptions, indeed, regarding the extent, mode of distribution, and periodicity of the spots, or respecting the periodicity and distribution of the meteors within the swarms, as also the relative motions of the swarms themselves, any sort of light change is capable of explanation by the first two theories; but as has been pointed out, their well-nigh universal applicability is in itself an element of weakness, inasmuch as the theories in themselves are in no essential particular an advance on their underlying hypotheses.

It is my purpose to give, in a few words, an account of a recent attempt to represent the light changes of β Lyrae, on the basis of the so-called satellite theory. Argelander's third curve of this star was based upon 1500 careful photometric estimates, extending over a period of nineteen years. An earlier curve by Oudemans and a later by Schoenfeld, show no discrepancies from this curve of a magnitude great enough to affect the discussion appreciably, and, consequently, Argelander's third curve, published in a pamphlet entitled, "De Stella β Lyrae Variabili Commentatio Altera" in 1859, was taken as the basis for the discussion. This well-known curve is represented in the upper portion of the accompanying figure.

INTRODUCTORY.

The variability of this star was discovered by Goodericke in 1784, but aside from the recognition of the fact of its variability and the confirmation of the existence of two unequal minima by this same scientist, nothing further was done in its study until Argelander laid the foundation for a thoroughly scientific and, at the same time, an extremely practical method of studying the light fluctuations of variables. Applying his own method to β Lyrae, Argelander showed the star to have two unequal minima, separated by two practically equal maxima, the entire

cycle of changes being completed in about $12^d\ 22^h$ ($= 12^d.91$). At the time of the I Maximum, he found the brightness of the star to rise to the 3.4 magnitude, and to fall after about three

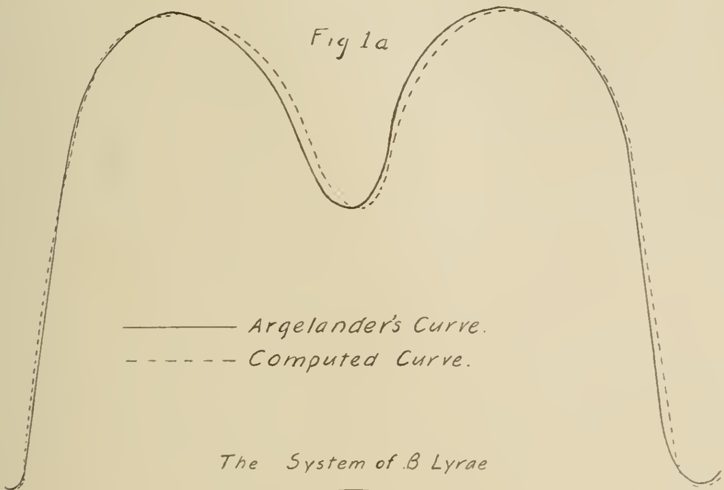


Fig 1a

The System of β Lyrae

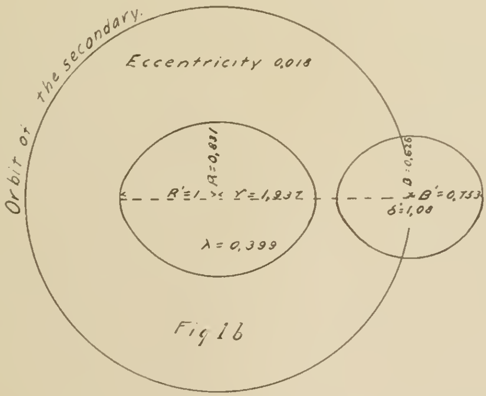


Fig 1b

days to a secondary minimum of the 3.9 magnitude, to rise again after three days to the former brightness of the 3.4 magnitude and then, after nearly the same interval, to fall to a primary minimum of the 4.5 magnitude. Or, in other words, the brightness at the maximum corresponds to 12.33 of Argelander's

grades (0.127 mag.); at the primary minimum, to 3.34 grades and at the secondary minimum, to 8.53 grades.

Assuming the light changes to be due to the mutual eclipses of two revolving bodies of unequal brightness, we should have the maxima occurring when the components stand beside each other, without either being eclipsed by the other, and consequently both disks shining full-phase. At the primary minimum, the darker component lies in front of the brighter and cuts off a portion of the latter's light, while at the secondary minimum the bright companion passes before the darker and obscures a part of its inferior brilliance. The relative positions of the components at the chief epochs are shown in the drawing (Fig. 2).

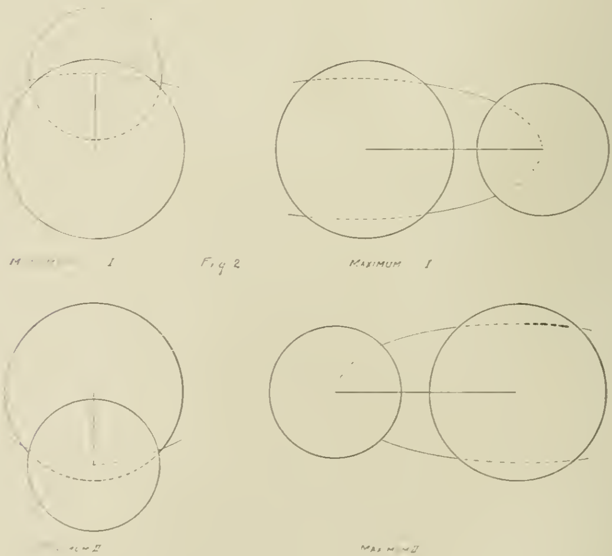


Fig 2

Since the brightness of the "system" is reduced at Minimum I (primary minimum) by 66 per cent. of its greatest brightness and at Minimum II (secondary minimum) by only 36 per cent., it is evident (1), that the disks must be assumed unequally bright or (2), that the orbital eccentricity must be assumed great or finally, that both these circumstances concur.

ECCENTRICITY.

An approximate idea of the magnitude of the eccentricity may be obtained in the following two ways:

First. If we assume the motions of the components to be in conformity to Kepler's laws we have, for the relation of the true and mean anomalies, the well-known equation:

$$v = M + 2e. \sin M + \tfrac{5}{4} e^2 \sin 2M + \quad . \quad . \quad . \quad .$$

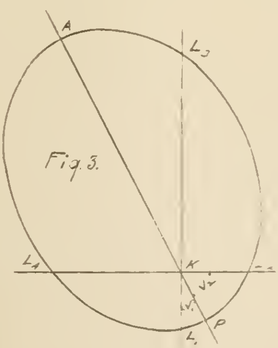
The lengths of the chief intervals of light change, from Argelander's curve are:

Min. I — Max. I	= 3.125	days.
Max. I — Min. II	= 3.250	"
Min. II — Max. II	= 3.167	"
Max. II — Min. I	= 3.368	"

The approximate equality of these intervals points unmistakably to a small orbital eccentricity. Assuming now, the eccentricity to be so small as to render its higher powers negligible, we may shorten the above equation into:

$$(a) \qquad v = M + 2e \sin M.$$

Designating the respective values of v and M at Min. I, Max. I, Min. II and Max. II by v_1, M_1; v_2, M_2; v_3, M_3 and v_4, M_4, and substituting in (a), there result the following four equations:



(1) $v_1 = M_1 + 2e. \sin M_1$

(2) $v_2 = M_2 + 2e. \sin M_2$

(3) $v_3 = M_3 + 2e. \sin M_3$

(4) $v_4 = M_4 + 2e. \sin M_4$

Subtracting (1) from (2), (3), and (4) in succession, and noting that $v_2 - v_1 = \frac{\pi}{2}$; $v_3 - v_1 = \pi$; and $v_4 - v_1 = \frac{3\pi}{2}$; we obtain

$$(5) \qquad \frac{\pi}{2} = 2\,m_1 + 4\,e.\cos(M_1 + m_1)\sin m_1$$

$$(6) \qquad \pi = 2m_2 + 4e.\cos(M_1 + m_2)\sin m_2$$

$$(7) \qquad 3\frac{\pi}{2} = 2\,m_3 + 4\,e.\cos(M_1 + m_3)\sin m_3,$$

where $2m_1 = M_2 - M_1 = 3.125\mu = 37^\circ\ 8'.4$

$2m_2 = M_3 - M_1 = 6.375\mu = 177^\circ\ 46'.2$

$2m_3 = M_4 - M_1 = 9.542\mu = 266^\circ\ 4'.8$

and $\mu = 2\pi/P, \qquad P = 12.91$ days.

We shall then have but two unknowns in (5), (6) and (7), viz., $e.$ and M_1, and any pair will suffice to determine these magnitudes.

From (5) and (6) we find,

$$(b) \quad \begin{cases} M_1 = -30^\circ 28' \\ e = 0.0186, \end{cases}$$

and these values substituted in (1), give

$$v_1 = -31^\circ\ 33'.$$

This satisfies the intervals Max. I—Min. I, and Min. II—Min. I, and requires Min. I to lie $31^\circ.5$ before periastron.

From (5) and (7), there result, similarly:

$$(c) \quad \begin{cases} M_1 = 209^\circ 32'4 \\ e = 0.0196 \\ v_1 = 209^\circ 24'. \end{cases}$$

Computing v_1 from Argelander's curve, it will be found that a rigorous satisfaction of the intervals Min. I to Max. I and Min. I to Min. II, requires a backward displacement of Argelander's Max. II by about 4 hours, while the intervals Min. I—Max. I and Min. I—Max. II, used in getting (c), necessitate a backward displacement of Max. I by about 4 hours. Since now, so small a shift in these two chief epochs, corresponds to so large a

change (nearly 180°) in the position of periastron, the eccentricity of the orbit cannot be large. The mean of the two nearly equal values of the eccentricity found above, is 0.0191 ± 0.0033. This value is small enough to justify neglecting its second and higher powers as was done above and thereby vindicates the method of treatment.

Second. It will develop later that the hypothesis of a flattening of one or both bodies must be made. Assuming the bodies to be deformed by reciprocal tidal influence, or by whatever cause, into similar ellipsoids of revolution — a permissible assumption, since such forms are figures of equilibrium — and denoting the ratio of the major and minor axes by "q," so soon as an approximate value of q is known, a superior limit of the ratio of distance of centers, to the larger semi-major axis may be derived. Thus, assuming the bodies to be spheres, with radii equal to the respective semi-minor axes of the ellipsoids, a light curve due to two revolving spheres may be computed. A lower limit for the duration of the eclipse at Min. I, for example, may be read off from this curve. A little reflection will show that the larger q be taken, the shorter the duration of the eclipse will be. Taking then, a value of q greater than that found later to be the approximate value and assuming that the eclipse has not begun until the light curve has fallen considerably, a value for the eclipse-duration will be obtained which is, at all events, small enough, perhaps much too small. q is later found to be 1.2, and assuming it to be 1.3, I find for inferior limit of eclipse-duration, 3 days and 4 hours, which must be at all events, small enough. But the smaller this inferior limit, the larger must be the ratio of distance of centers to the larger semi-major axis. Using the above value $3^d\ 4^h$ for eclipse-duration, a superior limit for this ratio may then be obtained, which will at any rate, be large enough. Taking now, the larger radius as unity and denoting the smaller by κ, the radius vector of the true orbit by r (for this, e may be put $= 0$), one-half the distance between the nearest points of the satellite at the beginning and end of the eclipse, by x, we see from the figure that,

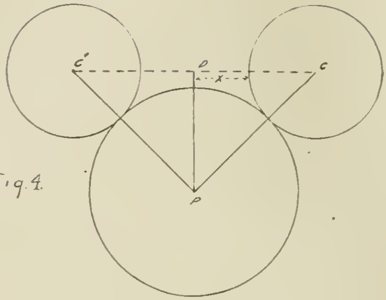


Fig. 4.

$$CPC' \geqq 3.167 \times 27^\circ.887 = 88^\circ\ 20'$$

and hence $\quad CPD \geqq 44^\circ\ 10'$.

Also $\quad r \leqq (x + \kappa) \csc 44^\circ\ 10'$

$$\leqq (x+\kappa)\ 1.438.$$

But since $x \leqq 1$, $\kappa \leqq 1$, we have $r \leqq 2.87$ times the smaller semi-axis of the larger ellipsoid or $= 2.4$ times its larger semi-axis. The distance between centers then, being so small compared with the dimensions of the larger body, the eccentricity could not be large or the masses would necessarily interpenetrate during revolution.

These considerations are sufficient to warrant the assumption of a small orbital eccentricity and to justify the hypothesis, that a first approximation to the orbit may be obtained by making $e = 0$.

FLATTENING.

The necessity of the assumption either that the disks are flattened, or that the bodies are not yet separated, is apparent from the fact that the brightness at the maxima, does not remain constant for any considerable length of time. It has been assumed in what follows that :

(1) The two bodies are distinct and separate.

(2) Both are deformed into ellipsoids of revolution.

(3) The periods of rotation and of revolution are equal to $12^d.91$.

(Possible librations being disregarded.)

Taking the origin of coördinates at the center of the larger ellipsoid and assuming the brightness of the disks uniform, we may readily find from the principles of analytical geometry of space, the equation of the light curve, which may be used while the bodies stand wholly off each other. It was found to be

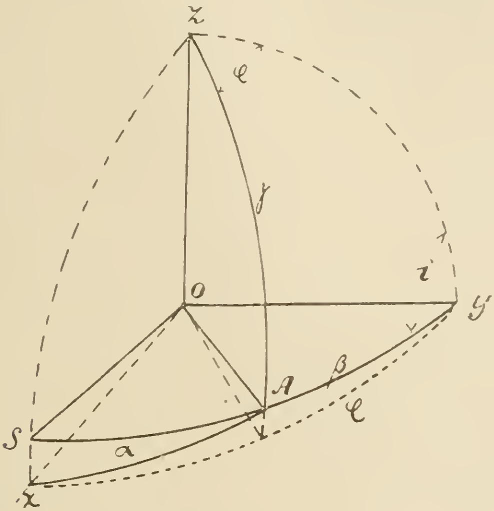


Fig. 5

$$L = \frac{1}{1 + \kappa^2 \lambda} \left\{ \frac{1}{\sqrt{\sin^2 \phi + (R / R')^2 \cos^2 \phi}} + \frac{\kappa_2 \lambda}{\sqrt{\sin_2 \phi + (B / B')^2 \cos^2 \phi}} \right\}$$

Where L denotes the ratio of the combined luminosities of both simultaneous elliptical disks to the combined brightness when the areas of these disks are greatest, *i. e.*, at the instant of the maxima :

λ is the ratio of the intensities of the disks,

κ is the ratio of the corresponding semi-axes of the two ellipsoids.

B', R' and B, R denote respectively the semi-major and semi-minor axes of the smaller and larger ellipsoids. Ox is the sight line and ZOy the tangent plane to the celestial sphere at the system.

ϕ is the longitude of the secondary in its orbit.

As a first approximation q was put equal to $R'/R = B'/B$. From the unsymmetrical character and small magnitude of the final residuals furnished by a comparison of Argelander's curve with the computed curve, it appears later, that nothing can be gained by an attempt to improve this hypothesis by ascribing different degrees of flattening to the two ellipsoids, so long as both bodies be regarded symmetrical and their disks uniformly bright. On this hypothesis the above equation becomes

$$L = \frac{1}{\sqrt{\sin^2 \phi + 1/q^2 \cos^2 \phi}}, \quad \text{where } \phi = \frac{1}{12 P} t \quad (t \text{ being in hours}).$$

An approximate value of q was found by computing light curves for various values of q, viz., 1.1; 1.2; 1.25; and 1.3, through a number of symmetrically chosen points before and after the maxima, and deducting the computed values of the ordinates from those of Argelander's curve. The correct value of q should, of course, give a curve whose ordinates, deducted from the corresponding ordinates of Argelander's curve, would be those of a curve due to two revolving spheres, and for a time on either side of the maxima, *i. e.*, while the two components are wholly uncovered, such a curve must run horizontally. The value 1.2 of q gave the following nearly equal ordinates:

10.33; 10.55; 10.76; 10.78; 10.78; 10.79; 10.78; 10.78; 10.76; 10.55; and 10.33.

This value of q was then assumed as a first approximation.

An equation of a light curve applicable during the eclipses was then computed by the process suggested in the drawing, and for Min. I, the necessary equations were found to be:

$$J_I = 1 - \frac{1}{\pi (1 + \kappa^2 \lambda)} \left\{ \phi + \kappa^2 (\pi - \phi') - \rho' \sin \phi \right\}$$

$$\rho' = \frac{\rho}{R_t'} = \frac{\sin (\phi \pm \phi')}{\sin \phi'} \text{ according as } \phi'' < 90^\circ \text{ or } \phi'' > 90^\circ,$$

$$\cos \phi = \frac{1 + \rho'^2 - \kappa^2}{2 \rho'}$$

$$\sin \phi = \kappa \sin \phi''.$$

Putting $M_I = \pi(1+\kappa^2\lambda)(1-J_I')$.

Then $M_I = \phi + \kappa^2\phi'' - \rho'\sin\phi$ or $M_I = \phi + \kappa^2\phi'' - \kappa\sin(\phi'' + \phi)$.

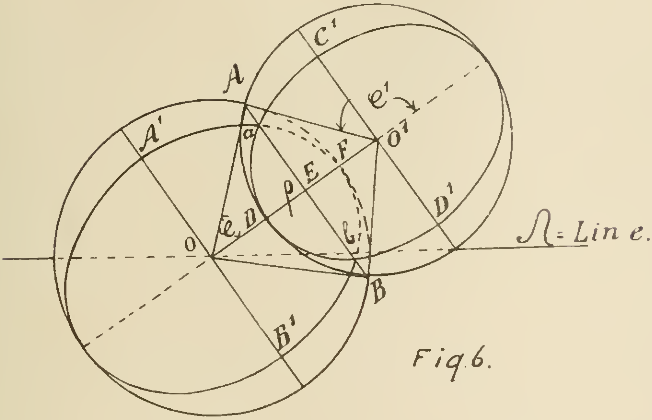


Fig. 6.

The work of computation was somewhat shortened by introducing an additional auxiliary H_I, when $\phi'' > \frac{\pi}{2}$ and by using the foregoing equations only when $\phi'' < \frac{\pi}{2}$. H_I was defined by

$$H_I = \kappa^2\pi - \pi(1+\kappa^2\lambda)(1-J_I').$$

Whereupon

$$H_I = \kappa^2\phi' - \phi - \rho'\sin\phi, \text{ or } H^I = \kappa^2\phi' - \phi + \kappa\sin(\phi' - \phi)$$

and similarly, the equations for Min. II were found to be:

$$J_{II}' = 1 - \frac{\lambda}{\pi(1+\kappa^2\lambda)}\left\{\phi + \kappa^2(\pi - \phi') - \rho'\sin\phi\right\}$$

$$M_{II} = \pi\frac{1+\kappa^2\lambda}{\lambda}\left\{1 - J_{II}'\right\}, \text{ or } M_{II} = \phi + \kappa_2\phi'' - \rho'\sin\phi$$

$$H_{II} = \kappa^2\pi - \pi\frac{1+\lambda\kappa^2}{\lambda}(1 - J_{II}'), \text{ or } \kappa^2\phi'' - \phi + \rho'\sin\phi.$$

The remaining formulæ hold without modification for both minima. J_I' and J_{II}' are the ratios of the combined instantaneous brightness to the combined brightness due to the sum of the two instantaneous disks shining full-phase during the eclipses at Min. I and Min. II respectively.

J_I and J_{II} are the ratios of the same brightness to that due to the sum of the full disks, when these disks are of maximum area.

The relation between J_I' and J_I on the one hand, and J_{II}' and J_{II} on the other, were then derived and found to be

$$J_I = fJ_I' \text{ and } J_{II} = fJ_{II}', \text{ where } f = R_i'/R = B_i'/B$$
$$= \frac{1}{\sqrt{\sin^2\beta + q^2\cos^2\beta}},$$

and $\beta = \phi - \frac{\pi}{2}$ denotes the longitude in the orbit, counted from a point 90° ahead of the origin of ϕ. (For undefined symbols see Fig. 6.)

Designating by $\beta + \frac{\pi}{2}$, the true anomaly in the real orbit, and by a that in the apparent orbit, both counted from the node, and calling ρ and r the radii vectores in the apparent and true orbits respectively, we have

$$r.\sin\beta = \rho\cos a \text{ and } \tan a = \cos i \cot\beta,$$

wherefore $$\rho^2 = r^2.\sin^2\beta + r^2.\cos^2 i.\cos\beta.$$

Differentiating the formulæ for Min. I and reducing, we find

$$\delta\phi = \frac{\delta M_I}{2\kappa tg\phi''\sin(\phi''+\phi)} \text{ and } \delta\phi = \frac{\delta H_I}{2\kappa tg\phi'\sin(\phi'-\phi)},$$

and when $\kappa > 1$, as will be found to be the case later, the preceding equations become

$$M_I = \phi + \kappa^2\phi' - \kappa\sin(\phi + \phi')$$
$$H_I = \phi - \kappa^2\phi' + \kappa\sin(\phi - \phi')$$
$$\rho' = \frac{\kappa\sin(\phi - \phi')}{\sin\phi}$$
$$\delta\phi = \frac{\delta H_I}{2\kappa tg\phi'\sin(\phi - \phi')}.$$

J was obtained by subtracting the ordinate of Argelander's curve for the selected instants from the mean of the maximum ordinates, calling this difference ΔG, and computing J from the equation $\log J = 0.051\,\Delta G$, which is readily derived from Pogson's scale, together with the value of Argelander's grade. The

values of M_I, M_{II}, H_I, H_{II}, are computed directly from values of J obtained from the above curve, and then an approximate value of ϕ, interpolated from tables computed from the M's and H's with ϕ as an argument, are then corrected by the above differential formulæ. Thus it is possible by these formulæ to compute a light curve from known elements, and also to solve the converse problem. To compute these tables the values of κ were required. κ being unknown, an approximation of its value was obtained thus: calling b_m the brightness of a star of magnitude m, and b_M that of a star of magnitude M, by Pogson's scale

$$\log\left(\frac{b_m}{b_M}\right) = \log J = 0.4 \triangle M,$$

where $\triangle M$ is the difference of the brightness in stellar magnitudes. From this we have

$$\frac{\text{Brightness at Min. II}}{\text{Brightness at Min. I}} = e = 1.8536$$

$$\frac{\text{Brightness at Max.}}{\text{Brightness at Min. I}} = m = 2.0123.$$

The first of these gives

$$(d) \qquad \frac{1 + \kappa^2\lambda - \alpha\kappa^2\lambda}{1 - \alpha\kappa^2 + \kappa^2\lambda} = c$$

and the second

$$(e) \qquad q\,\frac{1 + \alpha\kappa^2 + \kappa^2\lambda}{1 - \alpha\kappa^2 + \kappa^2\lambda} = m,$$

where α denotes the portion of the disks common to both bodies at the middle of the eclipses.

By a few simple transformations of these equations, it may be readily seen that

$q \geqq 1.0205$, *i. e.* the disks must be flattened, and $0.8323 \geqq \kappa \geqq 1.5241$.

The values of κ selected, were then

0.8323, 0.9049, 1.0000, 1.2883 and 1.5241

and a table such as was mentioned above was computed with the argument ϕ or ϕ' according as $\kappa < 1$, or $\kappa > 1$.

The values of r, for various points before and after the minima, with a correct hypothesis for κ and i, the inclination, must all be approximately equal, since they are computed on the hypothesis of a circular orbit. $\kappa = 1.5241$ gave a fair approach to an agreement in the various values, while the other values of κ gave widely discordant results for r. For some reasons it was more convenient to have $\kappa < 1$, and hence, the larger radius was assumed unity and the necessary alterations in the formulæ were made to suit this hypothesis. The only possible assumption which could be made for i was shown to be $\frac{\pi}{2}$, since this gave the various values of r more nearly equal than any other assumption for it. λ was found to be 0.3353 from (d) and (e). Recapitulating then, the following values were taken as first approximations:

$$e = 0,\ i = \frac{\pi}{2},\ r = 1.8344.\ \kappa = 0.6561,\ \lambda = 0.3353 \text{ and } q = 1.2.$$

By differentiating the above formulæ and combining the results, the following differential equations for correcting the approximate circular elements were derived:

$$\text{(I)}\quad \pi(\lambda + \kappa^2)\, dJ_I = 2\,\kappa K_I\, d\kappa + \frac{2\rho \sin\phi'}{r}\, dr + r^2 \frac{\cos^2\beta \sin\phi'}{\rho}\, d(i'^2)$$

$$\left(i' = \frac{\pi}{2} - i\right),$$

where $\quad K_I = A J_I f^2 \cos^2\beta + B(f - J_I) - f\phi - C\rho f^2 \cos^2\beta \sin\phi'$

and $\qquad A = \pi q \lambda,\quad B = \pi\left(1 + \frac{\lambda(1-\lambda)}{\kappa^2}\right),\quad C = \frac{2q3}{m}$

q must not be included in this equation since it depends on κ by equation (e). The equation applies only during Min. I. For Min. II,

$$\text{(II)}\quad \frac{\pi(\lambda + \kappa^2)}{\lambda}\, dJ_{II} = 2\,\kappa K_{II}\, d\kappa + \frac{2\rho \sin\phi'}{r}\, dr + \frac{r^2 \cos^2\beta \sin\phi'}{\rho}\, d(i')^2$$

$$K_{II} = \frac{A J_{II} f^2 \cos^2\beta}{\lambda} + \pi(f - J_{II}) - C\rho f^2 \cos^2\beta \sin\phi'.$$

The coefficients of (I) and (II) were computed for 14 points selected symmetrically along the curve, with the approximate values given above. dJ_I and dJ_{II} were obtained by comparing corresponding ordinates of Argelander's curve with those of a light curve computed from the preceding approximate values. 14 observation equations lead to the following values:

$$d\kappa = +0.3586, \quad dr = +0.0999, \quad \text{and } d(i'^2) = -0.0434$$

i' being here imaginary, but numerically small, it was called 0, and $d\kappa$ being quite large, dq was introduced in its stead, since small changes in the fundamental data do not affect q so greatly as r, and the equations again solved, gave the corrections, $dq = -0.0007$, and $dr = -0.0889$, and the corrected values were then $q = 1.1993$ and $r = 1.8955$, and from (d) and (e), $\kappa = 0.7580$ and $\lambda = 0.4023$. The probable error was, of course, somewhat increased by dropping i', being in the latter case ± 0.1 of a "brightness." These values were regarded as the most probable on the assumption of a circular orbit.

By means of these values and differential equations, which were derived for correcting a circular into an elliptical orbit of small eccentricity, a set of 48 observation equations was obtained, for as many points of the light curve, and from them the following elliptical elements were obtained:

$T = 1855$ Jan $13^d\ 6^h.35$	$P = 12.91$ days
$a = 1.937$	$\mu = 27^\circ.887$
$i = 90^\circ$	$\kappa = 0.7528$
$\eta = 94^\circ\ 53'$ from node	$\lambda = 0.399$
$\Omega = 0^\circ$ (assumed)	$q = 1.203$
$\tau = 6^d\ 15^h.4$ after Min. I.	

Both e and i' were included in the equation and i' was again imaginary but very small.

Using the equations derived for computing the light curve for elliptical motion, the results of computation are comprised in the columns of the table given below. The columns headed J_R and J_B contain the computed and observed ordinates respectively, and those headed ΔJ_{B-R}, the corresponding residuals

from Argelander's curve. The computed curve is shown in (Fig. 1_a) drawn in a dotted line beside Argelander's curve drawn in a full line. The agreement is quite close.

TABLE OF COMPUTED AND OBSERVED BRIGHTNESS.

t	Minimum I			Minimum II		
	J_B	J_R	δJ_{B-R}	J_B	J_R	δJ_{B-R}
—72	0.9963	0.9956	—0.0007	0.9918	0.9970	—0.0052
—66	0.9870	0.9852	+0.0018	0.9836	0.9945	—0.0109
—60	0.9683	0.9701	—0.0018	0.9696	0.9833	—0.0137
—54	0.9524	0.9513	+0.0011	0.9490	0.9672	—0.0182
—48	0.9147	0.9270	—0.0123	0.9258	0.9482	—0.0224
—42	0.8732	0.8849	—0.0117	0.8995	0.9217	—0.0222
—36	0.8209	0.8268	—0.0059	0.8680	0.8886	—0.0206
—30	0.7296	0.7525	—0.0229	0.8306	0.8494	—0.0188
—24	0.5836	0.6019	—0.0183	0.7836	0.8048	—0.0212
—18	0.4336	0.4993	—0.0657	0.7246	0.7558	—0.0312
—12	0.3661	0.4275	—0.0614	0.6674	0.7044	—0.0370
— 6	0.3484	0.3487	—0.0003	0.6433	0.6542	—0.0109
0	0.3433	0.3433	+0	0.6365	0.6368	—0.0003
+ 6	0.3499	0.3488	+0.0011	0.6472	0.6372	+0.0100
+12	0.3988	0.4275	—0.0287	0.6762	0.6689	+0.0073
+18	0.5306	0.5591	—0.0285	0.7340	0.7203	+0.0137
+24	0.6572	0.6624	+0.0052	0.7978	0.7716	+0.0262
+30	0.7644	0.7528	+0.0116	0.8498	0.8289	+0.0209
+36	0.8416	0.8266	+0.0150	0.8931	0.8622	+0.0309
+42	0.8857	0.8845	+0.0012	0.9255	0.8997	+0.0258
+48	0.9234	0.9268	—0.0034	0.9524	0.9307	+0.0217
+54	0.9537	0.9508	+0.0029	0.9627	0.9545	+0.0082
+60	0.9732	0.9694	+0.0038	0.9881	0.9732	+0.0149
+66	0.9870	0.9847	+0.0023	0.9977	0.9877	+0.0100
+72	0.9940	0.9952	—0.0012	1.0049	0.9970	+0.0079

With the help of an eccentricity, therefore, the residuals are somewhat reduced, and, considering the errors necessarily attaching to photometric estimates, the curve of Argelander may be regarded as sufficiently well represented.

The eccentricity results again almost the same as before, so that at the epoch 1855, the eccentricity did not differ materially from 0.02.

Since the periastron lies near Min II, the systematic deviation of the computed from Argelander's curve, the former lying above the latter, before, and below it, after Min II, it is highly probable, that while the secondary rounds periastron, very con-

siderable augmentation of brightness due to deformations of the disks, to internal friction, etc., occurs. Because of inertia, these effects could not immediately show themselves, so that the real curve would lie below the computed mean before Min II and above it after Min II, as is represented in the figure.

Feeling somewhat suspicious that any one of several sets of elements lying near those just given might represent observations equally well, it seemed worth while to test whether this same set of elements would result, if one of the circular elements upon which the elliptical ones were based, were given an arbitrary change and then, by repeated applications of the method of least squares, the elements were again corrected by the observations. An arbitrary change of —0.01 was given to κ and of —0.07 to r and after four adjustments giving ever smaller probable errors, the former values were again obtained.

It may therefore be inferred that the above elements are the most probable. An interesting fact arising during the latter process of adjustment, was that one set of elements giving a probable error of nearly the same magnitude as the final set, gave the distance between the centers = 1.80 and the sum of the radii equal to 1.82, *i. e.*, the components are not yet separated. This fact in connection with the low mean density of the system points to the nebulous condition of the star. The indications are then, either that the companions are not yet separate, but in the act of separation, or that if separate, their separation has taken place comparatively recently. In either case we seem to have here the first concrete example of a world in the act of being born.

SPECTROSCOPIC CONSIDERATIONS.

Treating by the method of Rambaut, published in *Mon. Not.* **51**, 316, the observations of Bélopolsky made between Sept. 23 and Nov. 26, 1892, and published in *Melanges math. et astronomiques*, *t.* VII, *l.* 3, I find,

$V = -0.8$ kilometers per second
$T = 26.93$ September 1892

$$e = 0.108$$
$$\omega = 79^\circ\ 17'\ \text{from node}$$
$$a \sin i = 15836000\ \text{kilometers}$$
$$P = 12^d.91\ \text{from light period.}$$

Lockyer observes the relative displacements of three lines and finds velocities as follows:

$$H\gamma = 155.0\ \text{miles}$$
$$H\delta = 154.0\ \text{miles}$$
$$\lambda_{4025} = 158.0\ \text{miles.}$$

The mean of these values is 155.7 miles and from a private letter of Professor Lockyer, I find it belongs to the epoch, August 24.46, 1893. Correcting for the motion of the Earth and reducing to kilometers, there results for the relative velocity in the line of sight 259.8 kilometers.

Bélopolsky's measures give for the diameter of the absolute orbit of one of the components 31672000 kilometers. From the absolute orbit furnished by Bélopolsky's observations, the velocity for the above date was found to be $\frac{1}{3.168}$ of the above relative velocity. The semi-major axis of the relative orbit is then $A = 3.168 / 2 \times 31672000 = 50175000$ kilometers (calling $i = \frac{\pi}{2}$) and for the ratio of the masses

$$a / A = m / (m + M),\ \text{or}\ m / M = \frac{1}{2.168}\ .$$

Assuming that the F-line observed by Bélopolsky was produced by the smaller component, we have

$$A / a = (M + m) / m = 1 + \delta' \kappa^3,$$
$$(\delta' = \text{ratio of densities of components}),$$

and assuming his line to be produced by the larger, there results,

$$A / a = (M + m) / m = 1 + 1 / \delta' \kappa^3.$$

Using the values of a / A and κ given above, we find

$$\delta' = 5.083,\ \text{or} = 1.081.$$

This furnishes a means of deciding which of the components produced Bélopolsky's F-line. Since it is quite improbable that one of the two bodies, so near to each other as is the case here, should have a density 5.083 times that of the other, the latter

value is assumed to be the correct one, and hence, that Bélopolsky's F-line was due to the larger component.

Using now the well known relation

$$M + m = A^3 / P^2$$

and substituting the values of A in astronomical units and of P in years, there result,

$$M + m = 30.56 \text{ solar masses}$$

and since $$m / M = 1 / 2.168,$$

$$M = 20.91 \text{ solar masses}$$

and $$m = 9.65 \text{ solar masses.}$$

Calling S the solar mass, H the solar radius, R' and R, the semi-major and semi-minor axes of the minor ellipsoid, δ_1 the density of the larger companion in terms of the solar density and δ_2 of the smaller,

$$\delta_1 = (M / S)\, q^2\, (H / R')^3, \qquad (q = R' / R \text{ as before})$$

and substituting former values,

$$\delta_1 = 0.00058.$$

But

$$\delta_2 = 1.08\, \delta_1 \;(\delta' = 1.08)$$

hence

$$\delta_2 = 0.00063.$$

The mean density of the system is then somewhat less than the density of air, *i. e.*, comparable to nebular density. It appears then that β Lyrae furnishes us a concrete illustration of the actual existence in space of a Poincaré figure of equilibrium.

Using the chief epochs of Lindemann's curve constructed from Plassmann's observations, the following equations result:

$$\cos(M_1 + 49^\circ\ 57') = -0.0563 / e = -3.204 \text{ (with Argelander's } e)$$
$$\cos(M_1 + 93^\circ\ 52') = -0.0338 / e = -1.920$$
$$\cos(M_1 + 136^\circ\ 23'.5) = -0.01771 / e = -1.010.$$

The impossibility of these relations, on the hypothesis that e is not greater than it was in 1855, requires us to infer that since Argelander's time the eccentricity of the system has grown larger. Comparing the spectroscopic with the photometric

observations, it is also seen that a large motion of the line of apsides has occurred since 1855, though its precise amount can scarcely be ascertained with any considerable degree of certainty.

The following results, therefore, seem to be quite clearly indicated by the preceding discussion :

1. The photometric estimates of β Lyrae's variability may be explained easily within the limits of the errors of these estimates by the aid of the satellite theory.

2. The bodies may be regarded as similar ellipsoids of revolution.

3. The orbit of the secondary body is nearly circular and its plane passes almost exactly through the Sun.

4. The common flattening of the ellipsoids differs but little from 0.17 and, aside from librations, the periods of rotation and revolution are equal.

5. The larger body is about 0.4 as bright as the smaller.

6. The distance of centers is extremely small, about 1⅞ of the semi-major axis of the larger ellipsoid.

7. From Lindemann's chief epochs for 1892 the orbital eccentricity of the system must have increased from 1855. Bélopolsky's spectroscopic observations indicate the same.

8. The motion of the center of gravity of the system with respect to the Sun is very small.

9. The semi-major axis of the orbit of the companion is about 50,000,000 kilos.

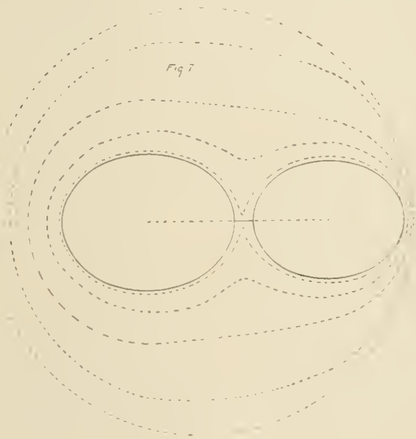
10. The mass of the larger body is 21 times, and of the smaller 9.5 times the solar mass.

11. The densities of the companions are nearly the same.

12. The mean density of the system is comparable with atmospheric density, or the "system" (for such, I think, it must now be called), is in a nebulous condition.

13. In conclusion, it may be said that the spectroscopic and photometric observations, which were available to me for the foregoing discussion, so far from being widely discordant, as some have thought, agree with each other remarkably closely.

The strong absorption lines in the spectrum of this star point to the presence of a powerfully absorbing atmospheric envelope about the nuclei of the masses. From the dynamical theory of gases we know that such an atmospheric layer would arrange itself about the combined mass of the system so that portions of equal density would be in equipotential surfaces. These surfaces would, in the immediate vicinity of the surfaces of the masses, conform somewhat closely to the surface of the bodies, but they would rapidly lose the abrupt curvatures at the surface and become more and more nearly spherical. A rough attempt to represent this is shown in the subjoined figure.



The equipotential surfaces, shown by dotted lines, would dispose themselves symmetrically about the center of gravity of the system as a center, and if the density of the larger companion were very materially greater than that of the smaller, the center of gravity would lie far within the larger companion. It might even happen that the center of gravity of the two bodies should lie beyond the geometrical center of the larger ellipsoid. Where it would lie would depend wholly upon the density of the various parts of the mass of the two bodies. It is then readily seen that the atmosphere might be, and probably is, so arranged as to permit the remote end of the smaller

to shine through a shallow layer of it and thereby to permit the smaller to appear brighter, even though it might be intrinsically darker than the larger body. No violence is done to theory, at all events, by such an assumption. This distribution of the atmosphere would also explain the absorption bands, which are seen in the spectrum of this star more distinctly at Min I than at Min II. The continuous spectrum, which would be produced most distinctly by the smaller body, must at the same time appear fainter. This accords with spectroscopic observations also.

Although some of the ideas given above may seem a little venturesome, let it be remembered that the peculiar character of the observations of this star leads one to expect an explanation of a somewhat unusual nature.

That the ellipsoids are similar is, of course, an arbitrary assumption.

In conclusion, let it be observed that an attempt at a formal representation of the condition of things prevailing in the system of β Lyrae, leads to the assumption of a single body (such as Poincaré's or Darwin's figures of equilibrium). The above has, of course, only a formal significance, but on account of the poverty of observational material at my disposal an attempt to push the discussion farther on a mathematical basis could not have proved profitable. It is believed, however, that the discussion may help us to orient our views with regard to this wonderfully interesting star. Fig. 1*b* represents the most probable relative dimensions of the bodies and orbit of the system, as based on Argelander's photometric estimates up to 1859. Professor Pickering has kindly offered to place all the earlier estimates of β Lyrae's brightness at the writer's disposal, and it is the latter's intention to make a full investigation and discussion of them at the earliest possible date.

THE ALGOL VARIABLE $+17^{\circ}$ 4367. W DELPHINI.

By Edward C. Pickering.

An ephemeris of the times of heliocentric minima of the Algol variable, W Delphini, for the years 1896 and 1897 will be found in the Astrophysical Journal 3, 200; 4, 320. The formula employed is J. D. $2412002.500 + 4.8064$ E. A continuation of this ephemeris for the year 1898 is given below in Table I. The first column gives the number of the epoch, the second the Julian Day and fraction after subtracting the constant number 2410000. The observations of the last year indicate that the period of this variable, like those of some, if not all, other variables of the Algol type, is not constant. The deviation from the ephemeris has now become so large, nearly one hour at the beginning of 1898, that corrected times of minima expressed in calendar days, and in hours and minutes of Greenwich Mean Time are given in the third and fourth columns. The amount of the correction is $0^{m}.45$ (E–357) which closely represents the observations for the last two years, and can readily be applied to the preceding ephemerides. While, therefore, the second column should be used in comparing the results of observation with those of preceding years, the fourth column in connection with the light curve (this Journal, 4, 323) will indicate more closely the times at which observations should be made. Before the next annual ephemeris is published I hope to discuss the observations so far made of this star, and to indicate the nature of the variations in period.

TABLE I.

EPHEMERIS FOR 1898.

E.	J. D.	Date	Corrected G. M. T.	E	J. D.	Date	Corrected G. M. T.
477	4295.1528	Jan. 5	2^h 46^m	515	4477.7960	July 6	17^h 55^m
478	4299.9592	" 9	22 07	516	4482.6024	" 11	13 16
479	4304.7656	" 14	17 28	517	4487.4088	" 16	8 37
480	4309.5720	" 19	12 48	518	4492.2152	" 21	3 57
481	4314.3784	" 24	8 09	519	4497.0216	" 25	23 18
482	4319.1848	" 29	3 30	520	4501.8280	" 30	18 39
483	4323.9912	Feb. 2	22 50	521	4506.6344	Aug. 4	13 59
484	4328.7976	" 7	18 11	522	4511.4408	" 9	9 20
485	4333.6040	" 12	13 32	523	4516.2472	" 14	4 41
486	4338.4104	" 17	8 53	524	4521.0536	" 19	0 02
487	4343.2168	" 22	4 14	525	4525.8600	" 23	19 22
488	4348.0232	" 26	23 34	526	4530.6664	" 28	14 43
489	4352.8296	Mar. 3	18 55	527	4535.4728	Sept. 2	10 04
490	4357.6360	" 8	14 16	528	4540.2792	" 7	5 25
491	4362.4424	" 13	9 36	529	4545.0856	" 12	0 45
492	4367.2488	" 18	4 57	530	4549.8920	" 16	20 06
493	4372.0552	" 23	0 18	531	4554.6984	" 21	15 27
494	4376.8616	" 27	19 39	532	4559.5048	" 26	10 48
495	4381.6680	Apr. 1	15 00	533	4564.3112	Oct. 1	6 09
496	4386.4744	" 6	10 20	534	4569.1176	" 6	1 30
497	4391.2808	" 11	5 41	535	4573.9240	" 10	20 50
498	4396.0872	" 16	1 02	536	4578.7304	" 15	16 11
499	4400.8936	" 20	20 23	537	4583.5368	" 20	11 32
500	4405.7000	" 25	15 44	538	4588.3432	" 25	6 53
501	4410.5064	" 30	11 04	539	4593.1496	" 30	2 14
502	4415.3128	May 5	6 25	540	4597.9560	Nov. 3	21 34
503	4420.1192	" 10	1 46	541	4602.7624	" 8	16 55
504	4424.9256	" 14	21 07	542	4607.5688	" 13	12 16
505	4429.7320	" 19	16 27	543	4612.3752	" 18	7 37
506	4434.5384	" 24	11 48	544	4617.1816	" 23	2 58
507	4439.3448	" 29	7 09	545	4621.9880	" 27	22 18
508	4444.1512	June 3	2 30	546	4626.7944	Dec. 2	17 39
509	4448.9576	" 7	21 51	547	4631.6008	" 7	13 00
510	4453.7640	" 12	17 11	548	4636.4072	" 12	8 20
511	4458.5704	" 17	12 32	549	4641.2136	" 17	3 41
512	4463.3768	" 22	7 53	550	4646.0200	" 21	23 02
513	4468.1832	" 27	3 13	551	4650.8264	" 26	18 23
514	4472.9896	July 1	22 34	552	4655.6328	" 31	13 43

HARVARD COLLEGE OBSERVATORY,
CAMBRIDGE, MASS., November 27, 1897.

OF ATMOSPHERES UPON PLANETS AND SATELLITES.[1]

By G. JOHNSTONE STONEY.

INTRODUCTION.

THE present writer began early in the sixties to investigate the phenomena of atmospheres by the kinetic theory of gas, and in 1867 communicated to the Royal Society a memoir,[2] which pointed out the conditions which limit the height to which an atmosphere will extend, and in which it was inferred that the gases of which an atmosphere consists attain elevations depending on the masses of their molecules, the lighter constituents overlapping the others. This was disputed at the time, on account of its supposed conflict with Dalton's law of the equal diffusion of gases;[3] but physical astronomers now recognize its truth.

On December 19, 1870, the author delivered a discourse before the Royal Dublin Society, which was the first of the series of communications, of which an account is given in the following pages. One of the topics of that discourse was the absence of atmosphere from the Moon. This was accounted for by the kinetic theory of gas; inasmuch as the potential of gravitation on the Moon is such that a free molecule moving in any outward direction with a velocity of 2.38 kilometers[4] per second

[1] Reprinted from an advance copy of a paper in the *Transactions of the Royal Dublin Society*, Vol. VI, Part 13. Communicated by the author.

[2] See an extract from this Memoir on p. 28, below.

[3] According to the Kinetic theory Dalton's Law will be true of mixtures of gases if the free paths of the molecules between their encounters are straight. This is the case, to an excessively close approximation, in all laboratory experiments; but the law ceases to hold at elevations in the atmosphere where the longer and more slowly pursued free paths are sensibly bent by gravity.

[4] It is very desirable that the names of metric measures should be made English words, and pronounced as such. Thus kilometer, hektometer, and dekameter should be pronounced with the accent on the second syllable, as in thermometer, barometer,

would escape, and, accordingly, the Moon is unable to retain any gas, the molecules of which can occasionally reach this speed at the highest temperature that prevails on the surface of the Moon.

Shortly after, a second communication was made to the Royal Dublin Society, at one of its evening scientific meetings, based on the supposition that the Moon would have had an atmosphere consisting of the same gases as those of the Earth's atmosphere, were it not for the drifting away of the molecules. It was shown that if the molecules of these gases can escape from the Moon, it necessarily follows that the Earth is incompetent to imprison free hydrogen ; and this was offered as explaining the fact that, though hydrogen is being supplied in small quantities to the Earth's atmosphere by submarine volcanoes and in other ways, it has not, even after the lapse of geological ages, accumulated in the atmosphere to any sensible extent. This communication was followed at intervals by others, in which the investigation was extended to other bodies in the solar system, in which an endeavor was made to trace what becomes of the molecules that filter away from these several bodies, and in which it was suggested that the gap in the series of terrestrial elements between hydrogen and lithium may be accounted for by the intermediate elements (except helium) having escaped from the Earth at a remote time, when the Earth was hot.

In one of the earlier of these communications, it was pointed out that it is probable that no water can remain on Mars — a probability which is now raised to a certainty by the recent discovery, that helium (with a molecular mass twice that of hydrogen) is being constantly supplied in small quantities to the Earth's atmosphere by hot springs, and probably in other ways, and that nevertheless there is no sensible accumulation of it in the Earth's atmosphere after the infiltration has been going on for cosmical ages of time. In the absence of water, carbon dioxide was sug-

etc. This would have the further useful effect of better distinguishing these names from decimeter, centimeter, and millimeter, which have accents on the first and third syllables.

gested as, with some probability, the substance that produces the polar snows upon Mars. Moreover, on the Earth, snow, rain, and cloud are produced by the lightest constituent of our atmosphere ; but if the atmosphere of Mars consist of nitrogen and carbon dioxide, snow, frost, and fog on that planet are being produced by the heaviest constituent. An attempt was made to follow out the consequences of this state of things, and to refer to it those recurring appearances upon Mars which, though very imperfectly seen owing to the great distance from which we observe them, have been (perhaps too definitely) mapped and described under the name of canals.

Of this series of communications, though known to many, only imperfect printed accounts have appeared ; and it is the object of the present communication to present the subject in a more complete form. The opportunity will be taken of substituting better numerical results for those originally given, by basing them on the fact which has recently come to our knowledge, that not only hydrogen, but helium also, with a density twice that of hydrogen, can escape from the Earth. The most notable change that this makes is, that what was before probable is now certain — that water cannot in any of its forms, be present upon Mars.

CHAPTER I.

Of the fundamental facts.—In order to see why neither hydrogen nor helium remains in the Earth's atmosphere, and why there is neither air nor water on the Moon, it is necessary to understand the conditions which determine the limit of an atmosphere. These were investigated under the kinetic theory of gas by the present writer in a memoir communicated to the Royal Society in May, 1867 : see his paper "On the Physical Constitution of the Sun and Stars," in the *Proceedings of the Royal Society*, No. 105, 1868, from page 13 of which it will be convenient to make the following extract : [1]

[1] Further information on this subject will be found in sections 22, 24, 25, 26, and in the footnote to section 93, of the paper here quoted.

"23. Let us consider what it is that puts a limit to the atmosphere. Let us first suppose that it consists of but one gas, and let us conceive a layer of this gas between two horizontal surfaces of indefinite extent, so close that the interval between them is small compared with the mean distance to which molecules dart between their collisions, but yet thick enough to have, at any moment, several molecules within it. Molecules are constantly flying in all directions across this thin stratum. Some of them come within the sphere of one another's influence while within the layer, and therefore pass out of it with altered direction and speed. Let us call them the molecules emitted by the layer. If the same density and pressure prevail above and below the layer, the molecules which strike down into it will, on account of gravity, arrive with somewhat more speed on the average than those which rise into it. Hence those molecules which suffer collision within the stratum will not scatter equally in all directions, but will have a preponderating downward motion, so that of the molecules emitted by the stratum more will pass downwards than upwards. This state of things is unstable, and will not arrive at an equilibrium until either the density or the temperature is greater on the underside of the layer. If the density be greater, more molecules will fly into the stratum from beneath than from above ; and if the temperature be greater the molecules will strike up into it, both more frequently and with greater speed. In the Earth's atmosphere it is by a combination of both these that the equilibrium is maintained ; both the temperature and the density decrease from the surface of the Earth upwards."

"24. We have hitherto taken into account only those molecules which, after a collision, have arrived at the stratum from the side on which the collision took place. But besides these there will be a certain number of molecules which, having passed through the stratum from beneath, fall back into it without having met with other molecules, either by reason of the nearly horizontal direction of their motion, or because of their low speed. The number of molecules that will thus fall back into the stratum will be a very inconsiderable proportion of the whole number passing through the stratum, so long as the temperature and density are at all like what they are at the surface of the Earth. In the lower strata of the atmosphere, therefore, the law by which the temperature and density decrease will not be appreciably affected by molecules thus falling back. But in those regions where the atmosphere is both cold and very attenuated, where accordingly the distance between the molecules is great and the speed with which they move feeble, the number of cases in which ascending molecules become descending without having encountered others will begin to be sensible. From this point upwards the density of the atmosphere will decrease by a much more rapid law, which will, within a short space, bring the atmosphere to an end."

It appears, then, that the atmosphere round any planet or

satellite will, *cæteris paribus*, range to a greater height the less gravity upon that body is; and that if the potential of gravitation be sufficiently low, and the speed with which the molecules dart about sufficiently great, individual molecules will stream away from that body, and become independent wanderers throughout space.

Thus, we shall presently see that, in the case of the Earth, a velocity of about eleven kilometers per second (nearly seven miles) would be enough to carry a molecule at the boundary of our atmosphere off into space, if the Earth were alone and at rest; and a somewhat less velocity of projection (about 10.5^{km} per second) is sufficient, on account of the rotation of the Earth, and because westerly winds sometimes blow in the upper regions of the atmosphere. The modification introduced by these subsidiary causes will be examined in Chapter IV, and the amount of their effect will be determined. The behavior of molecules is also slightly affected by the Moon, which is near enough sensibly to alter the orbits of molecules if shot up in some directions.

Let us now consider what would happen if free hydrogen could remain in our atmosphere. Hydrogen is, in modern times, being supplied in small quantities to the Earth's atmosphere by submarine[1] volcanoes and in other ways. Even if there were no tendency in hydrogen to leak away, it could not, in the free state become a *large* constituent of our atmosphere, because, when it came to be a certain proportion of the atmosphere, it would, on the occasion of the first thunderstorm or on account of fires, enter into combination with the oxygen, which is, in modern times, a large constituent of the atmosphere; but after each such explosion it would accumulate until it became a minor constituent like carbon dioxide were it not for the events described in this paper; and in former times, before there was vegetation to evolve free oxygen, it might have been a large constituent but for those events. The free hydrogen which con-

[1] The hydrogen evolved by terrestrial volcanoes burns into water on reaching the air, and ceases to be free hydrogen.

tinues in modern times to be supplied in small quantities to the atmosphere is used up in some way. A little may be occluded, some may suffer surface condensation, and the rest is escaping.

The evidence that there is an escape of gas from the Earth's atmosphere is still more conspicuous in the case of helium. Small quantities of this gas are constantly being dribbled into the atmosphere by hot springs and probably in other ways, and it was probably supplied more copiously in former times. Now helium is so little disposed to enter into combination with other elements, that the efforts of chemists to effect any such union have been unavailing. We must conclude, therefore, that this gas remains unchanged within the atmosphere, where it would therefore, in the lapse of time, have accumulated so as to be now a sensible and perhaps a large constituent of the Earth's atmosphere were it not that it is escaping from the atmosphere's outer boundary as rapidly as it enters it below—indeed so promptly escaping, that the amount *in transitu* is too small for the appliances of the chemist to detect it.

On the other hand, water is not sensibly leaving the Earth. From which we learn that the potential of the Earth and the temperature at the boundary of its atmosphere are such as enable our planet effectually to imprison the vapor of water with molecules whose mass compared with molecules of hydrogen is 9 (and probably ammonia with a density of 8.5.). The other constituents of the Earth's atmosphere, such as nitrogen, oxygen, and carbon dioxide, have still heavier molecules. Accordingly, none of these escape in sufficient numbers to produce any perceptible diminution of the quantity of gas upon the Earth.[1] We may infer from this that *the boundary between those gases that can effectually escape from the Earth and those which cannot, lies somewhere between gas consisting of molecules with twice the*

[1] We need not suppose that there is absolutely no escape of the molecules of the denser gases, but only that the event is an excessively rare one. Thus, if the molecules of a gas escape so very seldom that only a million succeed in leaving the entire atmosphere of the Earth in each second, then a simple computation will show that it would take rather more than 30 millions of years for a uno-twenty-one (the number represented by 1 with 21 ciphers after it) of these molecules to have escaped. Now a

mass of molecules of hydrogen and gas with molecules whose mass is nine times[1] *the mass of molecules of hydrogen.*

This we may take to be one fact which we can ascertain by observing what occurs upon the Earth, and the telescope has been able to reveal to us another fact of a like kind, viz., that there is either no atmosphere upon the Moon, or excessively little—a fact which has been made certain by the application of very delicate tests.

CHAPTER II.

Interpretation by the kinetic theory.—In order to make these facts the starting-point for fresh advances, we must study their precise physical meaning when interpreted by the kinetic theory of gas.

The velocity whose square is the mean of the squares of the velocities of the individual molecules of a gas—"the velocity of mean square" as it has been called—was determined[2] by Clausius to be

$$w = 485\sqrt{\frac{T}{273\,\sigma}} \text{ meters per second,}$$

where w is the velocity of mean square, T the absolute temperature of the gas measured in Centigrade degrees, and σ its specific gravity compared with air. We shall find it convenient to use ρ instead of σ, where ρ is the density of the gas compared with hydrogen. Accordingly $\sigma = \rho / 14.4$, whereby Clausius' formula becomes

uno-twenty-one is about the number of molecules which are present within every cubic centimeter of the gas at such temperatures and pressures as prevail at the bottom of our atmosphere. An escape of molecules of the denser constituents of the atmosphere on this excessively small scale, or even on a scale considerably larger, may be and probably is going on. See a paper on the "Internal Motions of Gases" in the *Philosophical Magazine* for August 1868, where the number of molecules in a gas is estimated. Readers of that paper are requested to correct a mistake at the end of the third paragraph, where 16^2 was by an oversight inserted instead of $\sqrt{16}$.

[1] We shall find in the chapter on Venus that the presence of water on that planet enables us to somewhat lower the upper of these two limits.

[2] *Phil. Mag.*, **14**, 124, 1857.

$$w = (111.4)\sqrt{\frac{T}{\rho}}, \tag{1}$$

in meters per second. This formula gives a velocity of 1603 meters, nearly a mile a second as the "velocity of mean square" in hydrogen at an absolute temperature of 207°, *i. e.*, at a temperature which is 66° C. below freezing point. This is the "velocity of mean square" of the molecules of hydrogen in an atmosphere consisting either wholly or partly of hydrogen, at any situation in which the gas is at that low temperature. Similarly by putting $\rho = 2$ and $T = 207$, we find the velocity of mean square for helium at the same low temperature. It is about 1133 meters per second. The actual velocities of the molecules are, of course, some of them considerably more and others considerably less than this mean, even if the hydrogen or helium be unmixed with other gases; and the divergences of some of the individual velocities from the mean will become exaggerated when the encounters to which the molecules of these lighter gases are subjected are sometimes with molecules many times more massive, and which may, when the encounter takes place, be moving with more than their average speed, as must often happen in our atmosphere. Under these circumstances we should be prepared to find that a velocity several times the foregoing mean is not unfrequently reached; and the evidence (see Chapter IV) goes to show that *a velocity which is between nine and ten times the velocity of mean square*, a velocity which is able to carry molecules of either hydrogen or helium away from the Earth, *is sufficiently often attained to make the escape of gas effectual.*

We are now in a position to aim at making our results so definite that they may be extended to other bodies in the solar system.

CHAPTER III.

Dynamical equations.—In making our calculations with reference to the planets and satellites of the solar system, it will simplify the work, and be sufficient for our purpose, to treat

them as spherical bodies, consisting of layers each of which is a spherical shell of uniform density. In that case, if B be one of these bodies

a (the acceleration of the surface of B, due to attraction)

$$=\frac{M}{R^2}, \tag{2}$$

and

$$K \text{ (the potential of gravitation at its surface) } =\frac{M}{R}, \tag{3}$$

where M is the mass of B, and R the radius of its spherical surface.

Now K, the potential, as we learn in the science of dynamics, expresses the kinetic energy stored up per unit of mass by a small[1] body in falling upon the surface of B from infinity. Hence,

$$K=\frac{v^2}{2}, \tag{4}$$

where v is the velocity which would be acquired by a small mass

[1] By a small body is to be understood one whose mass bears to the mass of B, a ratio so small that, from the physical standpoint, it may legitimately be regarded as a small quantity of at least the first order. For this purpose, a ratio of a tenthet, that is, of a unit in the tenth place of decimals, is sufficiently small in almost every branch of physical inquiry. If M be the mass of B, and m the mass of the body falling upon it, then the energy changed from potential into kinetic energy, by allowing them to fall together from infinity,

$$=\frac{m\,v^2}{2}+\frac{M\,V^2}{2},$$

if we suppose them to have started from rest, and if on coming together they have acquired the velocities V and $-v$. Now, by the principle of the center of mass, $MV+mv=0$. Therefore the acquired kinetic energy may be written

$$=\frac{m\,v^2}{2}\left[1+\left(\frac{m}{M}\right)\right],$$

which differs from being

$$=\frac{m\,v^2}{2}$$

by an insensible quantity if the ratio m/M is sufficiently small. And it is much more than sufficiently small from the physical standpoint, in the cases we are concerned with, where m is the mass of a gaseous molecule, and M the mass of a planet or satellite. In fact m is here of about the fifth order of small quantities compared with M, if we take a tenthet (10^{-10}) as about the ratio between quantities of two consecutive orders.

in falling from infinity. If a missile were projected from B with this speed, it would just be able to reach infinity, *i. e.*, this speed is the least which would enable a molecule to get completely away from B. We may, therefore, call it *the minimum speed of escape* from B when B is at rest. If B rotates, a less velocity relatively to the surface of B will suffice, provided that the missile is shot off in the direction towards which the station from which it starts was being carried by the rotation at the instant of projection.

CHAPTER IV.

Of the Earth.—Let us apply these elementary dynamical considerations to the Earth. In doing this, we may assume—

R (the Earth's equatorial radius), - -	$= 6378$ kilometers
h (the height of the atmosphere), - -	$= 200$ "
g (gravity at E, a station on the equator, at the bottom of the atmosphere), - -	$= 978.1^{\text{cm}}$./sec./sec.
u (the velocity at the equator due to the Earth's rotation), - - - -	$= 464^{\text{m}}$./sec.

We shall need one other datum, viz., the highest temperature which can be reached by the air at station E', where E' is a station at the top of the atmosphere, over the equator. To enable us to arrive at definite results, we shall regard this temperature as $-66°$ C. Our numerical results would be affected, but would only suffer a slight alteration, by substituting for this particular temperature any other which is admissible. It is, accordingly, legitimate to make our computation on this assumption, viz., that the temperature at station E' is 66° C. below freezing point. At this temperature Clausius' formula, equation (1) gives for the velocity of least square in a gas

$$w = (111\cdot4)\sqrt{\frac{207}{\rho}},$$

$$= 1603 / \sqrt{\rho}, \qquad (5)$$

if we here use w to signify the velocity of least square at this particular temperature.

Let us next calculate a, the acceleration due to the attraction of the Earth at station E (on the equator, and at the bottom of the atmosphere). Here

$$a = g + \gamma, \tag{6}$$

where g is gravity at the equator, and γ the acceleration due to the Earth's rotation, *i. e.*,

$$\gamma = \frac{u^2}{R} = \frac{(464\ \text{II})^2}{6378\ \text{V}},$$
$$= 3.4^{\text{cm}}./\text{sec.}/\text{sec.}, \tag{7}$$

where we use II for the two additional ciphers, and V for the five additional ciphers, which are necessary to express u and R in C. G. S. measure.[1]

Introducing this value of γ into equation (6), we find

$$a = g + \gamma = 978.1 + 3.4$$
$$= 981.5\ \text{cm.}/\text{sec.}/\text{sec.} \tag{8}$$

Again, $a = M/R^2$, where M, the mass of the earth, is expressed in gravitation units, and K' (the potential at station E', which is at the top of the atmosphere) $= M/(R+h)$. Taking the ratio of these we get rid of M, so that it is immaterial in what units it has been expressed. We thus find

$$K' = a\frac{R^2}{R+h} = (981{\cdot}5)\frac{(6378\ \text{V})^2}{6578\ \text{V}},$$
$$= l^{-1}\ 11.7830, \tag{9}$$

where l^{-1} means "the number whose logarithm is." This result is expressed in C. G. S. measure.

Now $K' = v^2/2$ (see equation 4), where v' is the minimum speed of projection which would carry a molecule clear away from the Earth, if the Earth were stationary. We thus find

$$v' = 1101500\ \text{cm.}/\text{sec.},$$

[1] The author has found it very convenient, especially in investigations touching on molecular physics, to use Roman figures to represent factors consisting of 1 followed by the number of ciphers indicated by the Roman figure. In this way VI means a million, XII means a billion; similarly, XXI means a uno-twenty-one, which is about the number of gaseous molecules in each cubic centimeter of air at the bottom of our atmosphere.

which is the same as

$$v' = 11.015 \text{ km.} / \text{sec.} \tag{10}$$

Now the rotation of the Earth carries station E' along at the rate of 0.478 km. / sec. Hence a velocity

$$\begin{aligned} v' - u' &= 11.015 - 0.478, \\ &= 10.537 \text{ km.} / \text{sec.}, \end{aligned}$$

will suffice, if the molecule be shot off in the direction in which it is already traveling in consequence of rotation. And, finally, if a strong west wind is blowing at station E', which must sometimes happen, a speed of

$$v' - u' - a = 10.5 \text{ km.} / \text{sec.}, \tag{11}$$

may be enough. This, then, we may take to be the least velocity which enables molecules to escape from the Earth.

Let us now turn to what happens in gas. By Clausius' formula, p. 31,

$$w \text{ (the velocity of mean square in a gas)} = (111{\cdot}4)\sqrt{\frac{T}{\rho}} \text{ m.} / \text{sec.},$$

which, at 66° below zero (which we regard as the temperature at station E') gives

$$\begin{aligned} w &= (111{\cdot}4)\sqrt{\frac{207}{\rho}}, \\ &= 1603 / \sqrt{\rho} \text{ m.} / \text{sec.}, \end{aligned} \tag{12}$$

where w means the velocity of mean square in a gas at the temperature — 66° C.

If in this we put $\rho = 1$, we find

$$w = 1603 \text{ m.} / \text{sec. in hydrogen.}$$

This is nearly a mile a second. Similarly putting $\rho = 2$, we find

$$w = 1133 \text{ m.} / \text{sec. in helium.}$$

which is somewhat more than a kilometer per second. And, finally, if we put $\rho = 9$, we find

$$w = 534 \text{ m} / \text{sec., in the vapor of water,}$$

which is somewhat more than half a kilometer per second.

Now, we found above that, in order that any gas may cease to be imprisoned by the Earth, its molecules must now and then

be able to attain at least a speed of 10.5 kilometers per second; see equation (11). Whenever this happens to a molecule favorably circumstanced it escapes. Hence, since hydrogen succeeds in leaking away from the Earth, its molecules must in sufficient numbers attain this speed, which is 6.55 times the velocity of mean square in that gas at a temperature 66° below zero; and since helium can escape, its molecules must sufficiently often reach a speed equal to or exceeding 9.27 times what we have found to be the velocity of mean square in helium at a temperature of — 66° C.

On the other hand, in order that a molecule of water may escape from the Earth, it has to get up a speed of 19.66, nearly twenty times the velocity of mean square in that vapor at the above temperature: and the fact that water does not drain away from the Earth in sensible quantities shows that this seldom happens.

We are now in a position to make a very important deduction in molecular physics from these facts, which is that *in a gas a molecular speed of 9.27 times the velocity of mean square is reached sufficiently often to have a marked effect upon the progress of events in nature;* while, on the other hand, a molecular speed of twenty times the velocity of mean square is an event which occurs so seldom that it exercises no appreciable influence over the cosmical phenomena which we have been considering. We must remember, however, that there are other events in nature—in chemistry, and especially in biology—which may be, and probably are, determined by conditions that occur far more rarely.

The separation of the swiftest moving molecules from the boundary of our atmosphere is, of necessity, accompanied by a lowering *pro tanto* of the temperature of the atmosphere left behind. It is one of the many operations carried on by nature to which the second law of thermodynamics does not apply. We must remember that this law is only a law of molecular averages, and therefore is not a law of nature where, as in this case, nature separates one class of molecules (those moving fastest) from the rest.

CHAPTER V.

Extension of the inquiry to other bodies.—In order to extend our inquiry to the atmospheres upon other bodies of the solar system, we have to determine the potential of gravitation upon them. We can do this where r, the radius of the new body B, and m / M, the ratio of its mass to the mass of the Earth, are known. For then

$$k \text{ (the potential at the surface of } B) = \frac{m}{r}$$

$$= \frac{m}{M} \cdot \frac{R+h}{r} \cdot \frac{M}{R+h}, \tag{13}$$

of which the last factor is the K' which is given in a numerical form in equation (9).

Combining this with the dynamical equation (see p. 33)

$$k = v^2 / 2 \tag{14}$$

we can calculate v, which would be the minimum velocity of escape from B, if B were at rest. In general B rotates, and then the minimum velocity of escape is

$$v' = v - u, \tag{15}$$

where u, the velocity at the equator of B due to its rotation, is easily found, if we know from observation the period of rotation.

Having calculated v', we can determine what density a gas must have to escape from B with the same facility with which helium leaves the Earth. For this purpose, let w_1 be its velocity of mean square. Then, in accordance with what is stated on p. 37, w_1 may be as large as

$$w_1 = \frac{v'}{9{\cdot}27}, \tag{16}$$

where w_1 and v' are to be expressed in meters per second: and then Clausius's equation, viz.

$$w_1 = (111{\cdot}4)\sqrt{\frac{T}{\rho_1}} \text{ m. / sec.}, \tag{17}$$

enables us to calculate ρ_1 / T, *i.e.*, the density of that gas which, at a specified temperature T, can escape from B as freely as

helium does from the Earth at a temperature of $-66°$ C. This and all lighter gases will escape.

To determine what density of gas will be imprisoned by B as firmly as water is by the Earth, we proceed in a similar way. Here

$$w_2 = \frac{v'}{19{\cdot}66}, \tag{18}$$

and the rest of the work is the same as before, giving as its result the value of ρ_2 / T, where ρ_2 is the density of a gas which will find it as difficult to escape from B as water does from the Earth. It and all denser gases will be retained.

The investigation leaves uncertain the fate of gases whose density lies between ρ_1 and ρ_2.

CHAPTER VI.

Of the Moon.—When we turn to the Moon, we find the conditions to be such that it can rid itself of an atmosphere with much ease. Upon the Moon

r (its radius), ...	$= 1738$ km.
$\frac{m}{M}$ (the ratio of its mass to that of the Earth), .	$= 0{\cdot}01235$
P (its period of rotation)	$= 2{,}360{,}591$ sec.

Calculating v', the least velocity which would enable a missile to quit the Moon by the equations in the last chapter, we find it to be about 2.38 km./sec., while on the Earth it is 11.015 km./sec., which, by the help of the rotation of the Earth and possible storm, may be, under favorable circumstances, furnished by a relative projectile velocity of 10.5 km./sec. Accordingly, more massive molecules can disengage themselves from the Moon with the same facility with which helium can leave the Earth, if ρ, their molecular mass, is greater than that of helium, in the ratio of the square of 10.5 to the square of 2.38, *i. e.*, if the molecules are 19.5 times heavier than those of helium, or, which is the same thing, 39 times heavier than those of hydrogen. Accordingly, hydrogen sulphide, with a molec-

ular mass 17 times that of hydrogen, oxygen with a molecular mass of 16, nitrogen with a molecular mass of 14, and the vapor of water with a molecular mass of 9, will hurry away. They will all escape with greater facility than hydrogen does from the Earth. A like fate will befall argon with a molecular mass of 20, carbon dioxide with its molecular mass of 22, carbon disulphide with its molecular mass of 38, and all others of the gases emitted by volcanoes, or from fissures, of which the vapor density is less than 39. These will escape with greater promptness than does helium from the Earth.

This is what would happen if the Moon were by itself, and if portions of its surface could rise even to a temperature of — 66°C. But the conditions are more favorable. Lord Rosse infers from his observations that the temperature of the Moon's surface rises something like 280°C. when exposed to the fierce glare of the Sun's rays. Even if it shall turn out that this is an overestimate, it at all events makes it probable that the maximum temperature is very much higher than 207° above the absolute zero, which is the same as 66° below the freezing point. Moreover, the proximity of the Earth would somewhat assist the process at its present distance; and its greater proximity in former ages must have more assisted it. In fact, on this account, any of the gases or vapors in question which had been developed upon the Moon while the Moon was close to the Earth must have been for the most part transferred over to the Earth, if the Earth was then cool enough to retain them. Those molecules that have escaped from the Moon since its distance from the Earth became considerable have for the most part become independent planets traveling in a ring round the Sun, of which ring (roughly speaking) the Earth's path is the central line. There they are accompanied by most of the molecules of hydrogen and helium that have leaked away from the Earth. A very few of the latter which happened to be shot off at unusually high speed, and in the direction towards which the Earth was at the time traveling in its orbit, may have been able to disengage themselves altogether from the solar system; but

this can have happened to but few of those thrown off from the Earth, and not to almost any of those ejected from the Moon.

CHAPTER VII.

Of Mercury.—The radius of Mercury may be obtained by assuming the equatorial radius of the Earth to be 6378 kilometers, and applying to it the data given in the preface to the *Nautical Almanac* for 1899. We thus find the planet's radius

$$r = \frac{3''{\cdot}34}{8''{\cdot}848}\, 6378 = 2406 \text{ km.}$$

The mass of Mercury is less satisfactorily known. We shall use the value

$$\frac{m}{M} = 0{\cdot}065.$$

Mercury's rotation period is also in doubt. The difficult observations that have hitherto been made seem to be about equally consistent with a rotation period of nearly a day, and a rotation period of 88 days (the period of Mercury's revolution round the Sun). Possibly observations could be made in the daytime which would determine between these. Meanwhile

$u = 2$ m. / sec., if the rotation period is 88 days.

$u = 175$ m. / sec., if the rotation period is 1 day.

By using the above values for r and m / M in equations (13) and (14), we find

v (the minimum velocity of escape, if Mercury were at rest) = 4643 m. / sec.,

which is a little more than $4\frac{1}{2}$ km. / sec. Hence

$v' = v - u = 4641$ m. / sec., if the rotation period is 88 days,

and

$= 4468$ m. / sec., if the rotation period is 1 day.

By employing these values in equations (16) and (17), we find that

ρ (the density of the gas that will escape from Mercury, as freely as helium does from the Earth) . . = 10.25, on the 88-day hypothesis

and = 11, on the 1-day hypothesis,

and on the further supposition that the absolute temperature of the gas where it escapes is 207°, that is 66° C. below zero.

If the highest temperature at the upper surface of Mercury's atmosphere over his equator is higher than this, and it is probably much higher, the foregoing values for ρ will have to be increased in the ratio of $T / 207$, where T is the highest temperature reached. It must also be remembered that helium is so prompt in escaping from the Earth that it is probable that gases somewhat denser could escape; and, as a consequence, that the limiting density of the gases that can escape from Mercury has to be increased in the same proportion.

The general conclusion then is:

1. That water with a density of 9 certainly cannot exist upon Mercury. Its molecules would very promptly fly away.

2. That it is in some degree probable that both nitrogen and oxygen, with densities of 14 and 16, would more gradually escape.

It is, therefore, not likely that there are, in whatever atmosphere Mercury may be able to retain, any of the constituents of the Earth's atmosphere except perhaps argon and carbon dioxide.

CHAPTER VIII.

Of Venus.—The state of Venus' atmosphere need not detain us long. The potential of gravitation is so nearly the same on this planet as on the Earth that its atmosphere almost certainly retains and dismisses the same gases as does the atmosphere of the Earth. The only element of uncertainty arises from its period of rotation being imperfectly known, but the nearly globular form of the planet assures us that its rotation cannot be swift enough seriously to affect the problem.

The similarity of the two atmospheres is confirmed by the appearance of the planet. Venus is presumably a much younger planet than the Earth, and its temperature is consequently what the Earth's was many ages ago, when through excessive evaporation water was the largest constituent of our atmos-

phere, and when clouds were present everywhere and without intermission.

The conditions upon Venus are so nearly akin to those on the Earth that we cannot be mistaken in regarding the vapor which forms the abundant cloud we see on that planet as none other than the vapor of water. If we may assume this, we can advance a step farther than the statements made in Chapter IV.

The detailed computations in the case of Venus give

$$r = \frac{8''.40}{8''.848} 6378 = 6053 \text{ kilometers},$$

$$\frac{m}{M} = 0.769;$$

and as such observations as are practicable seem to indicate that on that planet

$$P = 83779 \text{ seconds},$$

we find that

$$v = 10000 \text{ m. / sec.},$$

$$u = 454 \text{ m. / sec.};$$

whence we infer that

$$v' = v - u = 9546 \text{ m. / sec.},$$

is the least speed which will carry a projectile away from Venus.

Now, in water, $\rho = 9$. Whence, in accordance with Clausius' formula, p. 31, the velocity of mean square in water, at the temperature of $-66°$ C., is

$$w = \frac{1603}{\sqrt{\rho}} = 534 \text{ m. / sec.}$$

Now v' is almost exactly 18 times this value of w; so that the circumstance that Venus is able to retain its hold upon water means that the molecules of a gas do not attain a velocity 18 times that of mean square sufficiently often to enable the gas to escape from an atmosphere in appreciable quantities.

We are accordingly now in a position to go beyond the statement made on p. 314. We may now say:

1. A velocity of 9.27 times that of mean square is attained by the molecules of a gas sufficiently often to enable helium to escape from the Earth.

2. A velocity 18 times that of mean square is so seldom attained that Venus has been able to retain its stock of water.

3. Since Venus can prevent the escape of water, the Earth, with its larger potential, is competent to retain its hold upon a gas of somewhat less density, viz., one whose density is $\rho = 7.43$.

Accordingly, as regards the Earth, we may come to the following conclusions: (1) Gases with a density of 2 or less than 2 can certainly escape from the Earth; (2) a gas with a density of 7.43, and all denser gases,[1] are effectually imprisoned by the Earth; (3) the information supplied by Venus, supplemented by our present chemical knowledge, does not determine what would be the fate of a gas, if there be such, whose density lies between 2 and 7.43.

CHAPTER IX.

Of Mars.—The case of Mars is one of exceptional interest. Using the data furnished by the *Nautical Almanac*, we find its radius to be

$$r = 3372^{\mathrm{km}}.$$

As in the case of Mercury, its mass is not yet known with exactness. It has become better known since observations have been made on the elongations of its satellites, which seem to furnish the value:

$$\frac{m}{M} = 0{\cdot}1074.$$

Its period of rotation is known, viz., 88,643 seconds; whence, and from its radius, we find

u (the velocity at the equator due to rotation) $= 239^{\mathrm{m}}/\mathrm{sec}.$

By following the same steps as in the case of Mercury, we find successively

$$v = 5042^{\mathrm{m}}/\mathrm{sec.},$$

[1] Ammonia NH_3, and Methane CH_4, are a little above this limit, and therefore can neither of them escape. Ammonia is no doubt washed out of the Earth's atmosphere by rain; but it is not easy to see what becomes of the methane. It seems unlikely on chemical grounds that it directly combines with oxygen, furnishing water and carbon dioxide. Possibly it meets with a trace of chlorine, and furnishes methyl chloride and hydrogen in the presence of sunshine; or possibly it is nitro-methane that is formed.

for the least velocity which would carry a missile away from Mars, if Mars were not rotating, and

$$v' = v - u = 4803^{m}/\text{sec.},$$

for the relative velocity which is sufficient in consequence of the rotation.

From this, and equations (16) and (17), we find

$$\rho = 9.57,$$

as the density of a gas which would escape from Mars at a temperature of $-66°$ C., with the same facility as helium from the Earth. Hence, and since $9.57 : 9 = 207 : 194.7$, it follows that water would quit Mars at the absolute temperature of 194.7°, that is at $-78.3°$ C., as freely as helium can escape from the Earth at the temperature of $-66°$ C.

We must make some allowance for the probability that the highest temperature at which a gas has an opportunity of escaping from Mars may be lower than the corresponding temperature on the Earth. And we must, on the other hand, remember that the molecules of helium are almost certainly not quite the heaviest molecules that can rid themselves of the Earth. Taking both considerations into account, *it is legitimate to infer that water, in which $\rho = 9$, cannot remain on Mars.*

As to what happens to gases with densities of 14 and 16, we cannot speak with confidence. They may perhaps be imprisoned. And the conspicuous polar snows of Mars make it in a considerable degree probable that carbon dioxide, of which $\rho = 22$, is abundantly present.

It appears here to be worth reviewing the state of things that must prevail if the atmosphere of Mars consists mainly of nitrogen and carbon dioxide. Without water, there can be no vegetation upon Mars, at least not such vegetation as we know; and, in the absence of vegetation, it is not likely that there is much free oxygen. Under these circumstances, the analogy of the Earth suggests that the atmosphere of Mars consists mainly of nitrogen, argon, and carbon dioxide.

Carbon dioxide, the most condensible gas of such an atmos-

phere, would behave very differently from the way in which water behaves on the Earth. Water in the state of vapor is so much lighter than the other constituents of our atmosphere that it hastens upward through the atmosphere; and, accordingly, its condensation into cloud, whether of droplets of water or spicules of ice, takes place usually at very sensible elevations. There would be no such hurry to rise on the part of carbon dioxide, it would, on the contrary, show great sluggishness in diffusing upward through an atmosphere of nitrogen. When brought to the ground in the form of snow or frost (for there would probably be no rain), and when subsequently evaporated, the carbon dioxide gas would crawl along the surface, descending into valleys, occupying plains and pushing its way under the nitrogen, mixing only slowly with the nitrogen; and, as a result, only a very small proportion of the whole stock would be at any one time found elsewhere in the atmosphere than near the ground. It is suggested that the fogs, the snows, the frosts, and the evaporation of such a constituent of the atmosphere may account for the peculiar and varying appearances upon Mars, which, though recorded in our maps as if they were definite, are in reality very imperfectly seen from our distant Earth. In fact, Mars, when nearest the Earth, which unfortunately seldom happens, is still 140 times farther off than the Moon. Fogs over the low-lying plains which on Mars correspond to the bed of our ocean, with mountain chains projecting through the fog, and a border of frost along either flank of these ranges, would perhaps account for some of the appearances which have been glimpsed; and extensive displacements of the vapor, consequent upon its distillation towards the two poles alternately, would perhaps account for the rest.

CHAPTER X.

Of Jupiter.— In the case of the planet Jupiter, we have the following data:

$$r \text{ (Jupiter's equatorial radius)} = \frac{97''\cdot 36}{8''.848} 6378 = 70170 \text{ km},$$

P (the periodic time of his rotation) $= 35.728$ seconds,

$\frac{m}{M}$ (m being Jupiter's mass, and M the mass of the Earth) $= 311 \cdot 9$.

Using these data we find—

u (the velocity at his equator, owing to the rotation) $= 12.337^{km}/sec.$,

v (the least velocity which would carry a missile away, if Jupiter were not rotating) $= 59.570^{km}/sec.$,

$v' = v - u$ (the least velocity which enables a missile to escape when helped by the rotation) $= 47.233^{km}/sec.$,

ρ_1 (the density of gas which would escape from Jupiter, at a temperature of $-66°$ C., with as much ease as helium does from the Earth) $= 0.099$ of the density of hydrogen,

ρ_2 (the density of a gas which would be imprisoned by Jupiter as effectually as water is by Venus) $= 0.373$ of the density of hydrogen.

Hence gases with a density less than $\frac{1}{10}$ of that of hydrogen (if any such exist) could escape from Jupiter. But Jupiter can prevent the escape of a gas which has a density a little more than a third of the density of hydrogen, and of all denser gases.

Jupiter is accordingly able to imprison all gases known to chemists. His atmosphere may therefore, so far as can be determined by the present inquiry, have in it all the constituents of the Earth's atmosphere, with the addition of helium and hydrogen, and any elements between hydrogen and lithium which the Earth may have lost; except that, if the hydrogen is sufficiently abundant, there can be no free oxygen. Owing to the chemical reaction that would then take place, the oxygen will have been used up in adding to the stock of water.

CHAPTER XI.

Of Saturn, Uranus, and Neptune.—Our information with reference to these three planets is less satisfactory. Computing their radii from the data given in the *Nautical Almanac*, we find

$$r = 61060 \text{ km. on Saturn,}$$
$$= 24700 \text{ km. on Uranus,}$$
$$= 26340 \text{ km. on Neptune.}$$

Their masses compared with the masses of the Earth are also sufficiently known, viz.:

$$m / M = 93.328, \text{ for Saturn,}$$
$$= 14.460, \text{ for Uranus,}$$
$$= 16.863, \text{ for Neptune;}$$

but their rotation periods are very imperfectly known. We shall take them to be about

$$P = 36864 \text{ seconds, of Saturn,}$$
$$= 36000 \text{ seconds, of Uranus,}$$
$$= 36000 \text{ seconds, of Neptune.}$$

If we may use these values, we find

$$u = 10.412 \text{ km./sec., on Saturn,}$$
$$= 4.311 \text{ km./sec., on Uranus,}$$
$$= 4.598 \text{ km./sec., on Neptune,}$$

for the velocity at the equator due to the planet's rotation. Further, by equations (13) and (14), we find for the minimum velocity of escape from each of these planets, if not rotating,

$$v = 34.92 \text{ km./sec., on Saturn,}$$
$$= 21.61 \text{ km./sec., on Uranus,}$$
$$= 22.60 \text{ km./sec., on Neptune;}$$

whence

$$v' = v - u = 24.508, \text{ on Saturn,}$$
$$= 17.299, \text{ on Uranus,}$$
$$= 18.002, \text{ on Neptune,}$$

is the least velocity which enables a missile to escape when helped by the rotation.

By dividing these last numbers by 9.27, we find the velocity of mean square of the gas which can escape as freely as does helium from the Earth, and then by Clausius' formula, we can calculate ρ_1, its density, which is

$$\rho_1 = 0.37 \text{ of the density of hydrogen on Saturn,}$$
$$= 0.74 \text{ of the density of hydrogen on Uranus,}$$
$$= 0.68 \text{ of the density of hydrogen on Neptune.}$$

On the other hand, by dividing the values for v' by 18, we

learn what is the velocity of mean square of the gas which would be detained as firmly as water is held by Venus; and then, if we calculate ρ_2 by Clausius' formula, we find

$\rho_2 =$ 1.39 times the density of hydrogen on Saturn.
$=$ 2.78 times the density of hydrogen on Uranus.
$=$ 2.57 times the density of hydrogen on Neptune.

Now hydrogen, with a density of 1, stands in each case between ρ_1 and ρ_2, and we are, therefore, left uninformed whether hydrogen is or is not allowed to escape. There is, perhaps, some ground for conjecturing that it cannot escape from Saturn, and that it can escape from Uranus and Neptune. But this must remain doubtful. Helium, with its density of 2, being more than the value of ρ_2 upon Saturn, is certainly imprisoned by that planet, but we have no satisfactory information as to what is its fate upon Uranus or Neptune.

Thus the information we gain with reference to these three planets amounts to this—that we have no definite information as regards hydrogen; that Saturn is able to detain helium, but that we do not know whether the other two planets can or cannot; that all other gases known to chemists would be more firmly imprisoned by any one of these planets than they are by the Earth; and that, if there be gases lighter than hydrogen, it is certain that Saturn cannot detain any of which the density falls as low as one-third of that of hydrogen, Neptune cannot hold any as light as two-thirds, nor Uranus any lighter than three-quarters of the density of hydrogen. On the whole, the probability seems to be that the atmosphere of Saturn is nearly the same as that of Jupiter; while the atmospheres of Uranus and Neptune more nearly approximate to that of the Earth, with perhaps the addition of any gases with densities less than 7.43 that may possibly have left the Earth when the Earth was hotter, and whose withdrawal from the Earth is perhaps what has left the gaps in the series of terrestrial elements which appear to exist between hydrogen and helium, and between helium and lithium.

CHAPTER XII.

Of the satellites and minor planets.—We have no sufficient information as to the densities of any these bodies. But the asteroids, or minor planets, which lie between the orbits of Mars and Jupiter, are all of them bodies so small that, even if they were as dense as osmium, iridium, or platinum, they could not retain their hold upon an atmosphere. The same may be said of the two satellites of Mars, of the two satellites of Jupiter, of most of the satellites of Saturn, and of the small bodies that make up the rings of Saturn. None of these can condense any atmosphere upon them. If there are molecules of gases traveling in their neighborhood, they also are, each of them, an independent satellite.

One satellite of Saturn and three of Jupiter are larger than our Moon ; and one other of Saturn and one of Jupiter, though smaller than the Moon, are not much smaller. We should need to know the densities of these bodies before we could speak with confidence about them. The presumption, however, is that, as their primaries are very much less dense than the Earth, so these satellites are probably less dense than the Moon. If so, they also, as well as the smaller satellites, must be devoid of atmosphere.

We know too little about the satellites of Uranus and Neptune to venture upon any conclusion about them. The satellite of Neptune appears to be a body of considerable size, and, with some probability, it may have an atmosphere.

CHAPTER XIII.

What becomes of the molecules that escape.—The speed of the Earth in its orbit is about 30 km./sec. Now it follows, from the dynamics of potential, that the potential of the Sun at the distance of the Earth is represented by the square of this number if the Sun's mass be measured in gravitational units. That is

$$k = \frac{m}{r} = 900,$$

where m is the mass of the Sun, and r the radius of the Earth's orbit.

We have already found, on p. 35, the potential of the Earth at the boundary of our atmosphere to be

$$K' = \frac{M}{R+h} = \frac{v'^2}{2} = \frac{121}{2} = 60{\cdot}5.$$

Therefore the joint potential of the Sun and Earth at that station is

$$\frac{m}{r} + \frac{M}{R+h} = 960{\cdot}5.$$

This, then, is equal to $v^2/2$, when v is the least velocity which would enable a missile to escape from both these bodies if stationary. Therefore

$$v = \sqrt{(2 \times 960{\cdot}5)} = 43{\cdot}83 \text{ km.} / \text{sec.}$$

If the missile be shot off in the direction towards which the Earth is traveling, it has already got, in common with the rest of the Earth, 30 km./sec. of this velocity; and therefore, if fired off in that direction, the speed with which it would need to part from the Earth is 13.83 km./sec. Accordingly, this is a velocity which would suffice to set the molecule completely free, if the Earth were arrested in its orbit immediately after the molecule left it. But since, on the contrary, the Earth persists on its course, a slightly greater speed of projection is actually needed. Now, as 11 km./sec. is enough to enable a molecule to leave our atmosphere, it can be but very seldom that a molecule quits it with a velocity somewhat exceeding 13.83 km./sec.; and, accordingly, nearly all the molecules that have left the Earth have remained in the solar system, and are in fact now traveling as independent planets round the Sun.

We have taken the special case of a molecule leaving the Earth's atmosphere. A similar treatment applies to molecules leaving the atmospheres of other planets and satellities. In every case the velocity required to enable a molecule to quit the solar system is markedly in excess of that which enables it to escape

from its own atmosphere. Accordingly, almost all such wandering molecules are still denizens of the solar system.

CHAPTER XIV.

Former size of the Sun.—The Sun is contracting, and therefore in past time was larger than it now is. The question then arises, how much larger may it have been while it was still globular? We can place a limit on its possible size *if we assume that it was then, as now, able to prevent the escape of free hydrogen*, and if we assign a temperature below which its outer boundary did not fall.

In order to arrive at definite results, let us suppose this temperature to be 0°C. Here we might take into consideration the probability that, at a sufficiently remote period, the planets formed part of the Sun. But it is needless to do this, as the addition to be then made to its present mass would be only about $\frac{1}{750}$ part, which is too slight an increase sensibly to affect our present computation.

We have first to ascertain what the "velocity of mean square" of hydroden is at the freezing temperature. It is got by putting $T = 273$ and $\rho = 1$ into Clausius' formula, page 310. We thus find $w = 1.841$ km./sec. This multiplied by 9.27 (see page 314) gives us a velocity v_1 which the molecules of hydrogen could, at this temperature, get up sufficiently frequently, for the purpose of escape. And if multiplied by 18 (see page 320), it furnishes a velocity v_2 which hydrogen is unable to get up sufficiently frequently for effective escape. We thus find

$$v_1 = 17^{\text{km}}/\text{sec.} \qquad , \qquad v_2 = 33.14\ ^{\text{km}}/\text{sec.}$$

We have next to find how large the Sun should be in order that one or other of these velocities should be that which is just sufficient for the escape of a molecule. For that, r_1 and r_2 being the corresponding radii, the potentials must amount to

$$\frac{m}{r_1} = \frac{17^2}{2} = 144.5, \qquad , \qquad \frac{m}{r_2} = \frac{(33.14)^2}{2} = 549.$$

But at the distance of the Earth we found $m/r = 900$. Therefore

$$\frac{r_1}{r} = \frac{900}{144.5} = 6.227, \qquad \frac{r_2}{r} = \frac{900}{549} = 1.64.$$

That is, the surface of the Sun would need to have been about $6\frac{1}{4}$ times farther from the Sun than the Earth now is, in order that hydrogen at 0°C. should escape from it as freely as helium does from the Earth at —66° C. And it would need to have been 1.64 times farther than the Earth to imprison the hydrogen as firmly as water is held by Venus.

Hence, the *greatest* size which the Sun can have had since it became a sphere, consistently with its not allowing hydrogen at 0° C. to escape, is an immense globe extending to some situation intermediate between the orbits of Mars and Jupiter. From some such vast size it may have been ever since slowly contracting.

CHAPTER XV.

Of motions in a gas.—In carrying on an inquiry such as that of the present Memoir, we should keep in mind that the encounters between molecules have not the same effect on their subsequent motions as mere collisions between elastic or partially elastic solids would have. Let us, for simplicity, picture to ourselves two molecules which approach one another along a straight line, and after an encounter, which is in fact a complex struggle, recede from one another along the same line.

If they were solid particles with elasticity e, the equations of their motion would be

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2,$$
$$u_1 - u_2 + e(v_1 - v_2) = 0,$$

where $v_1 v_2$ are the velocities before, and $u_1 u_2$ the velocities after, the collision, and where e, the coefficient of elasticity, depends on the amount of the kinetic energy which is expended on internal events during the collision. It is therefore necessarily a proper fraction; so that e, in the case of solid particles, cannot exceed 1, whereas, in the encounters between molecules, it may have any value whether above or below 1. This is because, dur-

ing an encounter between molecules, energy is in some cases imparted to, and in other cases withdrawn from, the motions of the molecules along their free paths, whereas, in a mere collision, energy is always withdrawn. In fact, the *internal* events of individual molecules are in communication with heat motions in the ether, and interchange energy with it. A molecule may thus absorb energy from the ether during the whole of the long flights which it makes, when near the top of an atmosphere, between its encounters; and any excess of energy thus acquired will be shared with the motions of translation of the molecules when the next encounter takes place. Accordingly, the value of e will vary from one encounter to another, and, near the boundary of an atmosphere, there may be changes in the velocities of the molecules which are more abrupt than in situations where the gas is denser.

The effect here spoken of would be more marked in the case of helium, water, nitrogen, or oxygen, than in that of hydrogen, inasmuch as solar rays of the kind that hydrogen can absorb reach the Earth in a feebler state than those which the other gases absorb, owing to the partial absorption by hydrogen which has already taken place in the hot outer atmosphere of the Sun. On this account the rays that can affect hydrogen are the relatively feeble radiations from Fraunhofer lines, whereas the molecules of the other gases are exposed on the confines of our atmosphere to the glare of full sunshine. This is evidenced by the Earth-lines of the solar spectrum, especially those due to oxygen and aqueous vapor.

These considerations were taken into account in fixing on -66° C. as the maximum temperature to be attributed to the outer layer of our atmosphere. No doubt it would, in some slight degree, improve the investigation to use a rather lower temperature in the solitary case of hydrogen; but it was not thought necessary to make a distinction of this kind in an investigation which, from the nature of the case, could only be approximate. The only effect of introducing the refinement would have been to show that the facility with which hydrogen escapes

from an atmosphere is not quite so much in excess of the facility with which helium escapes as the numbers in Chapter IV indicate. This is almost certainly true to some small extent; but it leaves our main conclusions undisturbed. Accordingly, the simpler mode of inquiry, in which these and other small differences are ignored, has been an adequate investigation for our purpose.

HELIOGRAPHIC POSITIONS. III.

By Frank W. Very.

The last step in the passage from plane polar to heliographic coördinates shall now be explained. Fig. 4, Plate XXIV,[1] is a diagram representing the more important points required in the determination of heliographic positions, as they would be seen from the Earth at about the time of the autumnal equinox. P is the Sun's north pole. K and N' are the intersections of the Sun's surface with the axes of the ecliptic and of our celestial sphere, passing through the Sun's center. $ABMN$ is the solar equator, AB its invisible and BMN its visible branch, N being its ascending node. A line from the Sun's center to the Earth pierces the Sun's surface at C, the center of the visible solar disk, and the arc CM (symbol $= D$) measures the heliographic latitude of the Earth's position, or of the point on the Sun which has the Earth at its zenith. $N'C$ is a celestial hour-circle, and PD is a solar meridian through the spot (S) whose heliographic latitude is SD (symbol $= d$). The heliographic longitude of the Earth from the Sun's node is $NABM$ (symbol $= L$), and that of S is $NABD$ (symbol $= l$). Once in each solar rotation the assumed prime meridian coincides with the node. It will simplify matters to assume that the drawing is made at one of these recurrences of the conditions obtaining at the primary epoch, in which case the *nodal* heliographic longitude, $NABD$, will also be the heliographic longitude of the spot from the prime meridian, PN, or the condition $l' = l$ will be fulfilled.

To avoid complicating the figure, the ecliptic axis KC, and a line at right angles to it, through C, representing the ecliptic trace, also a line at right angles to $N'C$, through C, terminating in the points (E and W) of a celestial parallel, are omitted.

In the triangle PSC, the angle PCS is the algebraic difference of the position-angle of the Sun-spot ($P = N'CS$), reckoned

[1] *Ap. J.*, 6, Dec., 1897.

from north towards east as a positive angle, and the position-angle of the Sun's north pole. The latter angle is $N'CP$, or in our symbols, $-(G+H)$, taking G and H with the signs already given. Hence

$$PCS = N'CS - N'CP = P + G + H = \chi.$$

The arc SC (symbol $= \rho$) is the angular distance of the spot from the center, and since $PC = 90° - D$, the solution of the triangle PSC gives us:

$$\sin(90° - PS) = \cos SC \sin(90° - PC) + \sin SC \cos(90° - PC) \cos PCS,$$

$$\sin SPC = \frac{\sin PCS \sin SC}{\sin PS},$$

or substituting the more general symbols:

$$\sin d = \cos\rho \sin D + \sin\rho \cos D \cos\chi.$$
$$\sin(L - l) = \sin\chi \sin\rho \sec d$$

which are the fundamental equations enabling us to state the heliographic latitude of a spot and its heliographic longitude from the Earth's position.

The spot's longitude from the solar prime meridian is determined as follows: Since there are no fixed reference points on the Sun's visible surface, we must assume an imaginary prime meridian, or, if this be possible, one determined by some recurring phenomenon connected with the solar rotation. If Carrington's prime meridian has been chosen, and if the time of the Sun-picture is stated in astronomical reckoning, which at present is taken to begin at mean noon, we must transform the interval from noon into the decimal of a day and look up the day of the year in the almanac, taking the beginning of the year, or $0^d.00$ at mean noon of December 31, of the preceding civil year. We then have the interval in days from Carrington's epoch

$$t = 365(y - 1854) + (d - \tfrac{1}{2}) + b,$$

where y is the year, d is the day of the year and fraction of a day counting from mean noon (or in present astronomical reckoning), and b is the number of leap years which have intervened between 1854 and the beginning of the current year. The latter

year, if it be a leap year, and the date after February 28, will have its extra day included in the day of the year as given in the almanac. *If civil time is used, d must be diminished by one, instead of by one-half.*

We must now divide by the sidereal period of the Sun's rotation, getting:

$$\frac{t}{25.38} = m + T,$$

where T is the remainder, or the interval of time from the previous conjunction of the solar prime meridian with the Sun's node. Converting this interval into degrees, we have for the nodal heliographical longitude of the prime meridian:

$$l' = 360^\circ \times \frac{T}{25.38} = 14^\circ.1844 \times T.$$

If the nodal heliographic longitude of the spot be l its true heliographic longitude will be $l - l'$, reckoned always from the prime meridian toward heliographical east, or in the same direction in which right ascensions and celestial longitude are reckoned.

Some observers, while using Carrington's system, have departed from his epoch, and employ the solar meridian which coincided with the Sun's node at Greenwich mean *noon*, Jan. 1, 1854, their epoch being exactly twelve hours later than Carrington's.[1]

[1] In the *Greenwich Spectroscopic Observations* for 1884, we read (p. 106): "The rotations adopted in the following table correspond to the synodic rotation of the Sun, and the commencement of each is defined by the coincidence of the assumed prime meridian with the central meridian, the assumed prime meridian being that meridian which passed through the ascending node at mean noon on January 1, 1854, and the assumed period of the Sun's sidereal rotation being 25.38 days. The rotations adopted in the volumes of *Greenwich Observations*, 1877 to 1883, correspond on the other hand to the sidereal rotation of the Sun, the commencement of each being defined by the coincidence of the assumed prime meridian with the ascending node. The numeration of the rotations is in continuation of Carrington's series (*Observations of solar spots made at Redhill* by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853, Nov. 9. The dates of commencement of the rotations are given in Greenwich civil time reckoning from midnight." The statement that the "numeration of the rotations is in continuation of Carrington's series" seems sufficiently explicit, but the execution of the precept is discordant. Apparently someone has erred in carrying the times forward, by failing to notice that the dates, or intervals, were given originally in civil time, a practice which is most convenient for daylight observations.

Consequently their heliographic longitudes will be $7° 5'.5$ greater than those expressed in the original system. Carrington's statement is as follows: "As the fractions of the day [intervals] are throughout counted in civil time from the preceding midnight, $1854 - 0^d.000$ here signifies mean midnight on December 31, 1853." He says further that he has given, "(1) the day and fraction from midnight of the observation [expressed usually not as a date, but as an interval]; (2) the difference from the preceding epoch; (3) this difference converted into rotation angle in the proportion of $360° : 25^d.38$,[1] or the angle through which the prime meridian had rotated since its last coincidence with the node. The deduction of this amount for each day manifestly leaves us a heliographical longitude reckoned in all cases from a prime meridian, which if our period be correct, is constant, if incorrect, varies slowly with the time." (*Observations of the Solar Spots*, p. 16.)

The solar prime meridian which coincided with the node, 1854, Jan. 1.000 (civil date), arrived at the center of the solar disk, 1854, Jan. 3.0057, a date determined by the condition

$$L - l' = 0.$$

The mean synodic period of the Sun's rotation is given by the equation

$$S = \frac{365.25636}{\frac{365.25636}{25.38} - 1} = 27^d.27523,$$

or the diurnal synodic rotation is on the average $13°.1988$, or about $13° 12'$; but since the Sun's longitude changes $4'$ faster on January 1 than on July 1, the synodic rotation varies in the course of the year, and owing to the day and year being incommensurable, we cannot draw up a table of the exact values of $L - l'$ according to the dates which will be true for

[1] The sidereal rotation $25^d.38$ was adopted because of "its admitting conveniently of much subdivision without remainders;" but it happens to be a close, as well as a convenient approximation for the phenomena of spot rotation.

all years.[1] The simplest method of computing the dates of conjunction of the solar prime meridian with the center is to assume an approximate date, compute L and l' for this date and apply to it the correction

$$\frac{L - l'}{14^{\circ}.1844 - \Delta\odot},$$

where $\Delta\odot$ is the diurnal change of the Sun's longitude on the given date, the unit of the correction being the mean solar day.

Carrington gives the date of his first published position

$$\text{Nov. } 9, 1853 = 1853, 312^{d}.489,$$

stating the time as an interval from the beginning of the civil year; that is to say, the observation was made at 0.489 of the 313th day of the year 1853, civil reckoning. Central conjunction of the prime meridian occurred on this same day, 1853, Nov. 9.3625, or $3^{h}.036$ before Carrington's observation. This constitutes the beginning of "Rotation No. 1," and the central conjunction of Jan. 3.0057, 1854, was the beginning of "No. 3." As an example of the computation, let us carry the last forward twenty years. We must add

$$268 \times 27.27523 = 7309.7616$$

to 3.0057, and subtract

$$(20 \times 365) + 5 = 7305. \text{ days},$$

there having been five leap years during the interval, giving Jan. 7.7673, 1874, as a first approximation to the date of the beginning of the 271st rotation. The correction to the first approximation is in this case $+0^{d}.0494$, and the date of the beginning of rotation No. 271 is January 7.8167, 1874.

A little care in regard to the epoch of the rotations will be necessary if the student of this subject would avoid vexatious

[1] SIR ROBERT S. BALL, in his *Atlas of Astronomy* (p. 21), has adopted a plan which will give approximate values. He tabulates a mean correction to $L - l'$, from which interpolated angles can be taken and subtracted from the value of $L - l'$ accurately computed for noon on the *previous* first day of January. It may be noted that Professor Ball repeats the error that "the moment of noon on January 1, 1854," "was the epoch selected by Carrington."

discrepancies. I will take an example from the *Companion to the Observatory* for 1889 (No. 144, p. 37). We find there the following data for January 1, 1889: $P = +1° 34'$, $D = -3° 19'$, $L = 133° 49'$; and this explanation is given:

> The position-angle of the Sun's axis, P, is the position-angle of the north end of the axis from the north point of the Sun, read in the direction N., E., S., W. In computing D (the heliographic latitude of the center of the Sun's disk), the inclination of the Sun's axis to the ecliptic has been assumed to be $82° 45'$, and the longitude of the ascending node to be $74°$. In computing L (the heliographic longitude of the center of the disk)[1] the Sun's period of rotation has been assumed to be 25.38 days, and the meridian which passed through the ascending node at the epoch 1854.0 has been taken as the zero meridian.

This description of the epoch is not sufficiently explicit. We might question whether the year is the common year beginning January 0 in astronomical parlance, that is noon of December 31, 1853, or whether the civil year beginning at midnight between December 31, 1853, and January 1, 1854, is meant. Since Carrington is careful to state that the latter is the epoch chosen by him, the incautious student might be pardoned for supposing that it is the one adopted here, but a computation[2]

[1] Called $L - l'$ in this article.

[2] Illustration, using the nomenclature of this article, but epoch of *Companion*:

1889. January 1.5 (civil) $\odot = 281° 20'.6$ $\omega = 23° 27'.2$

$N = 74$

" " " " $\odot - N = 207° 20'.6$ $I = 7° 15'$

$l \tan \omega$	9.63733	$l \tan I$	9.10454	$l \cos I$	9.99651
$l \cos \odot$	9.29378	$l \cos (\odot - N)$	9.94854n	$l \tan (\odot - N)$	9.71358
$l \tan G$	8.93111	$l \tan H$	9.05308n	$l \tan L$	9.71009
		$l \sin I$	9.10106		
		$l \sin (\odot - N)$	9.66211n		
		$l \sin D$	8.76317n		

$G = +4° 52'.6$ $H = -6° 26'.8$ $L = 27° 09'.4$ $D = -3° 19'.4$

Position-angle of Sun's north pole (referred to center of disk, and reckoned from N. towards E.) $= -(G + H) = +1° 34'.2$.

Interval from *mean noon*, January 1, 1854 = 12784 days.

$503 \times 25.38 = 12766.14$ "

$T = 17.86$ "

$17.86 \times 14°.1844 = l = 253° 20'.0$

$L = 387° 9'.4$

Heliographic longitude of center $= L - l = 133° 49'.4$

shows that neither the first mentioned nor the second year is used, but one beginning at Greenwich mean noon on January 1, 1854.

Carrington appears to have considered his determination of N trustworthy to certainly less than a degree. Taking his exact value (73° 40′ for 1850) and allowing for precession, the value of N for 1889 is 74° 12′.6, which, with Carrington's epoch, gives for midnight 1889, January 1. 0 (civil), the interval being the same in both cases,

$$\odot = 280^\circ\ 50'.0 \qquad\qquad L = 26^\circ\ 26'.4$$
$$\odot - N = 206^\circ\ 37'.4 \qquad\qquad D = -3^\circ\ 14'.5$$
$$L - l' = 133^\circ\ 6'.4.$$

The change of epoch chiefly affects the longitudes, and these may be readily transformed from one system to the other. For example, since the prime meridian revolves with the Sun's sidereal rotation, while the center of the disk revolves according to the Sun's synodic rotation,

Companion's value,	$L - l' = 133^\circ 49'.4$, January 1, 1889, noon,
minus ½ true daily ⊙ rotation,	$-\ 7^\circ\ \ 5'.5$,
minus difference in assumed N,	$-\ \ \ \ 12'.6$,
gives Carrington's value,	$L - l' = 126^\circ\ 31'.3$, January 1, 1889, noon.
This *plus* ½ synodic daily ⊙ rotation,	$+\ 6^\circ\ 35'.1$,
gives Carrington's value	$L - l' = 133^\circ\ 6'.4$, for previous midnight.

An example of the complete reduction of a Sun-spot position is appended. The spot in question was a large one in rapid change, and the leading one of a great group. The position of the apparent center of the particular umbra chosen, altered rapidly from day to day, and the original observation, being made on a small paper drawing is possibly inaccurate to a considerable fraction of a degree. Nevertheless, it will serve to illustrate the method. The drawing, which is reproduced in Plate I, was made by Mr. F. Slocum, a student at this Observatory, by the method of projection, using the 12-inch equatorial with its aper-

ture reduced to six inches.[1] Meridians and parallels 30° apart, have also been drawn, giving a very effective idea of the solar

[1] Spot *A*. September 18, 1896. See Mr. Slocum's note, p. 92 of this number.

Date = 1896, September 18.33 Greenwich M. T. (astronomical),
= 1896, " 18.83 " " " (civil),
= 0.83 of the 262d Greenwich day of the year 1896,
= $15601^d.83$ from 1854.0 (civil).

Radius of spot, corrected for distortion = $\frac{r'}{R'} = \frac{1.85}{3.00} + 0.016 = 0.633$.

$P = 302° 35'$. $R'' = 958''.2$.

Reduction from tabular ⊙ = + 1161″.

⊙ = 176° 2′ 59″ + 19′ 21″ = 176° 22′ 20″.

N_{1850} = 73° 40′ }
Precession to 1896.717 = + 39′ 9″ } 74 19 9

⊙ − N = 102° 3′ 11″
ω = 23° 27′ 17″
I = 7 15′ 0″

l tan ω	9.63736	l tan I	9.10454	l cos I	9.99651
l cos ⊙	9.99913n	l cos (⊙ − N)	9.31977n	l tan (⊙ − N)	0.67055n
l tan G	9.63649n	l tan H	8.42431n	l tan L	0.66706n

l sin I 9.10106
l sin (⊙ − N) 9.09032

l sin D 9.09138

G = − 23° 24′ 46″, D = + 7° 5′ 22″, H = − 1° 31′ 18″
L = − 77° 51′ 9″ = 282° 8′ 51″

$\rho' = 0.633 \times 958.2 = 607''$
$\rho = \sin^{-1}(0.633) - 607''$
= 39° 16′ − 10′ = 39° 6′

P = 302° 35′
$(G + H)$ = − 24° 56′
χ = 277° 39′

l cos χ	9.12425			l sin χ	9.99612n
l cos D	9.99667	l sin D	9.09138	l sin ρ	9.79981
l sin ρ	9.79981	l cos ρ	9.88989	l sec d	0.00708
	8.92073		8.98127	l sin $(L - l)$	9.80301n

sin d = 0.08332 + 0.09578 = 0.17910
= sin 10° 19′.0

Interval from 1854.0 = $15601^d.83$
614 × 25.38 = 15583 .32
Remainder = T = $18^d.51$

$L - l$ = − 39° 26′.7
L = 282 8 .9
l = 321 35 .6
18.51 × 14°.1844 = l' = 262 33 .2
$L - l'$ = 59° 2 .4

Spot *A*, heliographic latitude = + 10° 19′, heliographic longitude = 59° 2′, referred to the initial epoch of Carrington, or if the epoch be taken twelve hours later, heliographic longitude = 66° 8′.

presentation. Since the computation of heliographic positions has to be repeated many times, it is desirable to employ a printed form; and since the values of I and N are not known with quite the degree of accuracy implied in the accompanying example, the labor may be further abridged by tabulating the values of G, H, L, and D, for single degrees of the arguments, $\odot$ and $\odot - N$.

Heliographic latitudes can be determined somewhat more accurately than longitudes, because less affected by foreshortening near the limb. They are also subject to less actual variation, because the large vertical movements in solar spots are combined with the normal axial rotation of the Sun's mass, which is sufficient to give a drift in the direction of the latitude parallels. The longitudinal elongation of Sun-spot groups bears witness to this tendency.

Spots close sometimes by horizontal inrush, but more frequently by alterations which are probably to be interpreted as a physical change of state, occurring over a wide area, rather than as extensive transportation of materials. But however produced, the result is a sudden shifting of apparent spot-positions.

As a favorable example of spot-recurrence, and an illustration of the amount of variation which may be expected in spot-positions, I select the following groups observed by Carrington (see table on opposite page). The columns headed Δl and Δd exhibit the apparent diurnal changes in heliographic longitude and latitude. A positive drift in latitude means one towards the pole.

It will be seen by a comparison of positions that the groups are really but a single one which, on further comparison, proves to be nearly identical with one of the previous March, a recrudescence which is not unusual. A new number, however, has been given to the group at each reappearance by the Sun's rotation. The positions bracketed together are those of individual spots belonging to the group, which fluctuate and disappear; but the initial spot was exceptionally long-lived. Its mean

No.	Date and interval	Heliographic longitude	Δl	Heliographic south latitude	Δd
	1860				
710	May 4	22 13'	?	11 27'	?
	124.496	12 23	?	11 53	?
710	May 5	21 56	— 17	11 43	— 16'
	125.492	12 26	+ 3	11 30	— 23
710	May 6	21 40	15	11 17	— 25
	126.553	——			
710	May 7	21 27	— 14	11 35	+ 19
	127.485	15 48	?	12 58	?
710	May 9	21 28	+ 1	11 2	— 15
	129.644	13 23	?	11 55	?
710	May 13	21 2	— 7	11 32	+ 8
	133.627	——			
730	May 30	——			
	150.382	21 23	+ 1	12 36	+ 4
730	June 5	30 31	?	16 14	?
	156.358	21 16	?	18 50	?
		20 10	— 12	12 29	— 1
730	June 6	29 15	— 64	15 58	— 13
	157.546	19 39	— 26	12 47	+ 15
730	June 8	32 37	?	15 2	?
	159.528	24 31	?	17 56	?
		19 6	— 17	12 48	+ 1
753	June 25	18 5	?	21 21	?
	176.616	18 22	— 3	13 4	+ 1
753	June 26	20 38	?	20 37	?
	177.336	19 54	+ 128	12 46	— 25
753	July 1	27 14	?	16 31	?
	182.576	19 3	— 10	12 15	— 6
		14 56	?	18 45	?
		9 23	?	19 27	?
753	July 3	18 34	— 15	12 31	+ 8
	184.563	15 16	+ 10	18 58	+ 7
		9 22	— 1	19 22	— 3
753	July 4	18 32	— 2	12 26	— 5
	185.530	14 57	— 20	18 51	— 7
		8 0	— 85	18 52	— 30
753	July 6	14 29	— 13	18 47	— 2
	187.723	18 11	10	12 13	— 6
777	July 22	——			
	203.490	34 8	?	15 42	?
777	July 24	33 1	— 31	15 42	0
	205.629	22 37	?	16 51	?
		18 52	+ 2	11 41	— 2
777	July 25	33 16	+ 15	15 39	— 3
	206.641	23 3	+ 26	17 25	+ 34
		18 44	— 8	11 27	— 14
777	July 30	33 32	+ 3	14 47	— 11
	211.545	22 37	— 5	17 32	+ 1
777	Aug. 1	33 31	— 1	14 21	— 12
	213.664	22 14	— 11	17 0	— 15

positions and mean diurnal drifts, from one apparition to the next, were:[1]

	Interval from Jan 1.0	Long.	Δ l	South Lat.	Δ d
1st rotation	126ᵈ.6	21°.8		11°.5	
2d rotation	156 .0	20 .1	—3′	12 .7	+2′
3d rotation	183 .5	18 .9	—2′	12 .4	—1′
4th rotation	206 .0	18 .8	0	11 .6	—2′

These and other similar mean variations being for long intervals, and representing the determinations of four to six days in each case, we may conclude that proper motions, or variations in the mean drift, of 3′ or 4′ per day, corresponding to horizontal linear movements of 370 and 493 miles per day, are established beyond a doubt. The larger and irregular fluctuations noted from day to day are in many cases also probably real, but since the movement is not continuous in any given direction, these fluctuations are eliminated from the mean. The mean deviations from the adopted period refer in this case to a spot in heliographic latitude 12° south, not far from the latitude (about 14°) which gives the average or typical solar rotation. The residuals in longitude are much larger than these as the latitude departs from that of the mean spot-zone, becoming about —40′ in latitude 35°. This constitutes the evidence for Carrington's great discovery that the solar surface does not revolve as one piece. We must distinguish, however, between those mean residuals which result from the combination of the periods of a large number of spots by the elimination of the average period, and those departures from the mean rotation for a given latitude, which indicate a proper motion or divergence from a mean surface drift. The latter are especially liable to occur in large and vigorous spots, and are of peculiar interest. The group selected for analysis was a comparatively quiet one, with small proper motion; but individual spots show occasional movements of large amount, and certainly much greater than the probable error of a single observation. In addition to the visible movements of spots whose unbroken history gives them an appear-

[1] *Observations of the Spots on the Sun*, p. 182.

ance of individuality and continuity which is doubtless far from the reality, we see the frequent birth of new spots in the rear of old ones, testifying, in all probability, to an invisible lateral transportation of materials at a very rapid rate. The spectroscope assures us that the lighter constituents of the Sun are moving in the higher regions of the chromosphere at very great velocities, and although we do not know whether the erupted masses of hydrogen, helium, calcium, and "coronium," are sufficient to account for spot growth, the intimate association of prominences and Sun-spots warrants the hypothesis of a chain of causal connection through a hidden, complex circulation of material between neighboring spots.

The study of proper motions either of a spot as a whole, or of its component filaments, plumes, bridges, or reticulations, needs extensive series of instantaneous photographs of the Sun, taken at short intervals on a large scale, and carefully reduced. Nothing comes amiss here. Bits of information given by spectroscopic measures of velocities in the line of sight, studies of facular development by the spectroheliograph, or of local variation of radiative power by the bolometer, observations of those evanescent changes in spot-forms and spectral details, which must escape detection by occasional photographic registration, and which can only be captured by continuous visual study, and the rarer events of coronal and chromospheric displays, must be combined together and interpreted by the aid of extensions of chemical and physical theory suitable to the novel problem to be dealt with, before we can have anything like a complete life-history of a Sun-spot. Evidently coöperation is needed, and the fragmentary contributions of individual workers are not to be despised.

In finishing a Sun-drawing, there is a temptation to bring any group of spots which may be under inspection near the margin, to the center of the field, in order to secure the advantage of a sharper image. But such a procedure must be executed with great caution, lest the relative positions of details be altered because of the difference in distortion. If the leading positions

and outlines have been drawn to scale with a correctly centered image, there is no objection to a free-hand infilling of the finer detail seen under better conditions, but alterations of scale will vitiate any conclusions which may be drawn as to positions or dimensions.

The determination of spot-dimensions is intimately connected with that of positions. With a transparent plate of glass or mica, ruled in squares (*e. g.*, 0.5^{mm} on a side), apparent areas of umbras or penumbras may be easily estimated. These areas require correction for distortion and foreshortening near the limb. The area of one of the little squares at the center of the Sun's disk in terms of the area of the visible solar hemisphere, will be $\frac{1}{2\pi R^2}$, R being here the number of divisions of the counting plate in the radius of the projected solar disk; while near the limb, the measured areas must be increased in the ratio, $\sec \rho : 1$; and the general expression for the area of the Sun's spherical surface, covered by a square division, is: $\frac{\sec \rho}{2\pi R^2}$, ρ being the heliocentric angle of the position from the center of the disk. If optical distortion has increased the marginal radius from r' to r, the measured area (a) must be diminished, for this cause, in the proportion, $1 : \left(1 + \frac{r - r'}{r'}\right)^2$; whence the final value of the corrected spherical area is:

$$a' = \frac{a \sec \rho}{2\pi R^2} \left(\frac{r'}{r}\right)^2.$$

This formula of course assumes that the area measured is a relatively small part of the sphere.

The summation of spot-areas has been pursued assiduously in the belief that it furnishes a reliable indication of solar surface activity. The methods followed in obtaining spot-areas are therefore important. In his memoir on *Photographs and Drawings of the Sun*,[1] Rev. S. J. Perry endeavors to account for a persistent difference in penumbral areas measured from photo-

[1] S. J. PERRY, *Mem. R. A. S.*, 49, 286, 1889.

graphs and from drawings (the photographic areas being the larger) by suggesting that "from the state of the sky, or from the length of the exposure, or from a slight variation in the sensitiveness of the film, or from some modification in the development, the penumbra of a spot does not present so well defined an outline in the photograph as in the projected image, and that the measure is taken of what would appear to be the extreme limit of the spot."

Two other alternatives might be suggested. The first is that the drawings may have been finished with the spot removed to the center of the field in order to diminish chromatic aberration, thereby decreasing the size of the details drawn, even though the general disposition of the outline may not have been intentionally changed. Mr. Perry states, however, that "the unexplained differences for spots within 10° of the limb are almost nil, and those between 10° and 20° are exceptionally small.[1] This hypothesis, therefore, does not appear to be warranted in Mr. Perry's case.

My other suggestion is that the discrepancy is possibly sufficiently explained by the fact that the rays which affect the photographic plate are more strongly absorbed by the solar atmosphere than those which produce vision. It must always be a matter of judgment to decide how faint a shade shall be considered penumbral. The greater intensity of absorption of the photographic rays will throw the balance in favor of counting in a photograph what would be rejected in making a drawing. In favor of this hypothesis, it may be noted that very near the limb, the general absorption of the photographic rays is so great that variations of absorption, due to slight differences of penumbral elevation, may be masked, and the discrepancy between drawings and photographs diminished. The point is one which can probably be settled by comparing measurements on ordinary and on orthochromatic photographic plates.

LADD OBSERVATORY,
Providence, R. I., Aug. 1897.

[1] *Loc. cit.* p. 285.

ON THE CONDITIONS OF MAXIMUM EFFICIENCY IN ASTROPHOTOGRAPHIC WORK.

PART II. EFFECT OF ATMOSPHERIC ABERRATION ON THE INTENSITY OF TELESCOPIC IMAGES.

By F. L. O. Wadsworth.

Introductory note.—When Part I of this paper was written it was intended to follow it at once with Part II (thus completing the general theory of contrast), and then apply the general results obtained to the consideration of the special cases, which have been referred to in detail elsewhere.[1] But as stated in another note,[2] this investigation had to be laid aside for a time to complete other work of a more pressing character, and it was therefore decided to publish, without waiting for the completion of the general theory, the results that had already been obtained in considering the special cases of the photographic and visual observations of the planets. In each of these cases (as in that of the objective spectroscope previously considered[3]) the effect of atmospheric aberration had been individually investigated.[4] There was, however, one other case which was considered at the same time, in which this effect was unfortunately not taken into account, not indeed because it was overlooked (any more than in the other cases), but because its importance in this particular line of work had been at first underestimated. This was in part due to the fact that we were concerned principally with small linear and angular apertures (the aberrational effect due to a given disturbance will vary in general, at least as the square, if not as a higher power of the aperture), and in part to there hav-

[1] See *Ap. J.*, **6**, 135; also *M. N.*, **57**, 588–589.

[2] See p. 77 of this number.

[3] *Ap. J.*, **3**, 62–64, June 1896.

[4] See paper "On the Photography of Planetary Surfaces," *Obs'y* **20**, 303, 365, 404, Nov. 1897. "On the Effect of the Size of an Objective on the Visibility of Linear Markings on the Planets." *Ast. Jour.*, 413, Oct. 1897.

ing been an error in one of the results used in developing the general theory of contrast for this case, which happened to be of such a nature and magnitude as to just mask the effect of atmospheric aberration, and thus enable conclusions to be reached which explained and were completely confirmed by the actual results of observation; a test of theory which is generally regarded as one of the most conclusive and satisfactory that can be applied. As it turns out, therefore, the main conclusions reached in this case are (through the balancing of the two errors, or rather one error and one omission) correct, if the general theory of contrast and "delineating power" upon which they are based is correct.[1]

The result which is referred to as in error, is the expression for I^2_{III} which represents the intensity of illumination of the focal plane due to an infinitely extended area. It does not enter into the expression for contrast in any, save the last, of the cases, yet considered; but it has been judged of sufficient theoretical importance at least to be considered, together with another error of an exactly similar character in the expression I^2_{II}, for *the intensity in the image of a long line*, more at length in another note.[2]

Taking into account these errors, the expressions for the contrast between the image and field in cases A, B, C, of the preceding paper become

$$\text{(A)} \qquad K_A - 1 = \frac{k_a}{k_{III}} \cdot \frac{\pi b^2}{4\lambda^2} = \text{Const.}\,\frac{k_a}{k_{III}}\,b^2 \qquad \text{(36a)}$$

[1] For further remarks on this point see pp. 82, 85 of this number.

[2] See p. 77 of this number. Wholesale criticism of the results of my recent investigations have been recently published by Professor Schaeberle (*Ast. Jour.*, No. 421) and Mr. Newall (*M. N.*, Nov. 1897) on the ground that this one expression I^2_{III} was *probably* in error. The cursory and superficial manner in which these gentlemen must have examined my papers is evidenced not only by this, but by the further facts, that both entirely overlook the references to Rayleigh's work; both fail to find the real error that was committed in his work and in my own (Professor Schaeberle's only analytical criticism is directed against a part of the work that is entirely correct.), and finally, as might have been expected, both entirely overlook the other error of exactly the same nature in the expression I^2_{II}. These facts have been pointed out and some other criticisms briefly answered in notes to the *Ast. Jour.*, No. 424, and to the *M. N.* (to be published probably in the January number), the publications in which Professor Schaeberle's and Mr. Newall's articles appeared.

$$\text{(B)}\ \left\{\begin{matrix}(1)\ B_1\\(2)\ B_2\end{matrix}\right\}\quad K_B - 1 = \frac{k_b}{k_{111}}\cdot\frac{8}{3}\cdot\frac{b}{\pi\lambda}\cong\frac{k_b}{k_{111}}\cdot\frac{1}{\alpha}\qquad (37a)=(38a)=(40)$$

$$\text{(C)}\qquad K_C - 1 = \frac{k_c}{k_{111}}\qquad (39a)$$

The expression for case D is, as already given

$$K_D - 1 \cong \frac{k_d}{k_{111}}\cdot\frac{1}{\alpha}\qquad (40)$$

i. e., the same as for B.

These expressions must be regarded as representing the effective contrast under:

1. Theoretically perfect conditions; *i. e.*, entire absence of both instrumental and atmospheric disturbances; or

2. Such conditions of observation as enable the effect of a moderate amount of atmospheric disturbance to be eliminated; *i. e.*,

a. Visual observations (with instruments of moderate aperture) in which as already pointed out[1] the eye makes the most of the intervals of "good seeing" without being influenced to any great extent by the intervening intervals of "fuzziness," due to temporary instrumental or atmospheric disturbances.

b. Photographic exposures which are either practically "instantaneous" as in the case of solar photography, or of the "intermittent" nature recently proposed by the writer[2] in which the favorable conditions of visual observations are, as nearly as may be, approximated.

The expressions for the *effective photographic contrast* in the case of prolonged continuous exposures will be developed in the present paper.

§ 1. GENERAL CONSIDERATIONS.

It has long been recognized that the limits of the practical resolving and defining power of our astronomical instruments are set, not so much by instrumental limitations as to size and accuracy of workmanship, as by the difficulty of securing sufficiently

[1] *Pop. Astron.*, **5**, 205; *Obs'y* **20**, 336, Sept. 1897. [2] *Ibid.*

good atmospheric and meteorological conditions to enable the full theoretical advantages attendant upon increase of aperture to be practically realized. Long before the invention of the modern refractor, when the size of the largest instruments was counted in inches rather than feet as can be done now, this difficulty was clearly recognized by Newton,[1] and nearly every astronomer of note since his time has contributed something of either a theoretical or an experimental nature toward the determination of the most favorable meterological conditions, or the selection of the most suitable localities on the Earth's surface for various classes of astronomical and astrophysical observations. During the last half of this century, since instruments of large aperture have been rapidly coming into general use, many expeditions have been undertaken with the latter object particularly in view, and as a consequence a number of observatories of either a temporary or permanent character have been established at points which have been found to offer characteristic advantages for the prosecution of particular lines of work.

Among the more important of these expeditions may be mentioned that of Bond to the mountains of Switzerland in 1851; of Lassell to Malta in 1852; of Piazzi Smyth to Teneriffe in 1856; of the United States coast survey parties under Young and Davidson to the Sierras and the mountains of Wyoming in 1872; of Draper to the mountains of Utah and Wyoming in 1876; of Tacchini to Ætna in 1877; of Burnham to Mt. Hamilton in 1879; of Langley to Mt. Whitney in 1881; of Copeland to the Andes of Peru in 1883; of Abney to the Riffel in 1886; of Vallot and Janssen to Mt. Blanc in 1887–1890; of the Harvard College Observatory expeditions to Mt. Wilson and Peru in 1887–1890; of Hale to Pike's Peak and Ætna in 1893–4;

[1] "If the theory of making telescopes could at length be fully brought into practice, yet there would be certain bounds beyond which telescopes could not perform. For the air through which we look upon the stars is in a perpetual tremor, as may be seen by the tremulous motion of shadows cast from high towers and by the twinkling of the fixed stars. The only remedy is a most serene and quiet air, *such as may perhaps be found on the tops of the highest mountains above the grosser clouds.*"—*Optice*, 107 (2d ed. 1719). See, also, Clerke's *History of Astronomy*, 519, and Holden's *Mountain Observatories*.

and of Mr. Lowell's expedition to Flagstaff, Arizona, and Mexico in 1894–1897.[1]

The subject of the general nature of atmospheric disturbances and their effect on the "seeing" has also received much attention. Arago, Laplace, von Humboldt, Montigny, Wolf, Hann, Secchi, Respighi, Dufour, Helmholtz, Davidson, Lord Rayleigh, André and Angot, Ventosa, Douglass, Hale, and many others have made important contributions of both an experimental and theoretical nature to our knowledge of the subject.[2] Much, however, still remains to be done in this field, particularly in the nature of systematic explorations of the meteorological conditions in the upper regions of the atmosphere, not by means of mountain stations, at which the conditions are abnormal because of the presence of the mountain itself, but over extensive plateaus and table lands by mean of self-registering instruments carried up by kites or captive balloons. Happily this field of work is now beginning to receive the attention it deserves.

In the present paper it is not so much our purpose to enter into a discussion of the cause and nature of atmospheric disturbances, as it is to inquire as to their effect on the *intensity* of the images formed at the focal plane of the telescope, a part of the subject that has not, as far as I know, been quantitatively and systematically investigated. In order to do this, it is, first of all, desirable to classify the effects of atmospheric disturbances on the position and character of the focal plane images.

[1] The attitude and expressed opinions of the Flagstaff observers in reference to this matter of the study of atmospheric conditions are curiously narrow ones. They believe evidently that theirs is a case in which "the last shall be first." Thus, according to Mr. Douglass (*Pop. Astron.*, **5**, 65, June 1897), "*Professor Pickering* (W. H.) *was unquestionably the first to intelligently appreciate the importance of seeking a good atmosphere*," and according to Dr. See (*A. N.*, No. 3438, 81), "*comparatively few observers have heretofore made a critical study of the conditions essential to good seeing* . . . . and . . . . *few observatories have been located for securing good definition and steady images*." In view of the actual facts of the case comment on such remarks seems unnecessary.

[2] For a very well written (though somewhat incomplete) review of the subject reference may be made to Holden's recent pamphlet, *Mountain Observatories*, published by the Smithsonian Institution; *Smith. Miss. Coll.*, No. 1035, 1896.

The various observed effects may be divided into three general classes :

1. Movements of the image as a whole without *sensible* loss of definition. These movements may be either
   a) Rapid and irregular ("jumping" or "dancing" of the image).
   b) Slow and oscillatory.
2. A general blurring and enlargement of the images ("haziness" or "fuzziness") without lateral movement.
3. Vibration and blurring combined.

Each of these effects is due to a characteristic change in the nature of the atmospheric disturbance. The first is produced by a regular and progressive change in density from one side to the other of the column of air through which the light has to pass to reach the telescope objective. This produces a general turning of the whole beam; the same effect as would be produced by a thin *prism* of glass placed in the path of the beam. If the equivalent "air prism" is perfectly homogeneous, we would have a displacement of the image simply, without *any* loss of definition (save the very slight effect due to the chromatic aberration of the "air prism"); this, however, as might be expected, very rarely happens, particularly if the objective is of any considerable size, since the "air prism" is being continually broken up and reformed with the refracting edge in a new direction. In many cases the "air prism" is formed over only a part of the the lens, in which case we have a "ray" shot out from the image in a direction at right angles to the refracting edge of the "prism."

Effects of the second kind may in the same way be considered to be due to the momentary interposition in front of the objective of air clots of more or less globular form, varying in density from the center outward.

The effect of these is to momentarily change the focus of the telescope, just as a thin *lens* of glass would do. Here again the effect is rarely a pure one, the "air lens" being irregular in

structure. If it were regular in structure and persistent in position and form, its effect could be almost completely compensated by a change in focus. And it is interesting to note that when the atmospheric disturbances are mainly of this nature the most frequent intervals of "good seeing" may be obtained when the eyepiece is set either a little inside or a little outside the position which would be best if the air were undisturbed, *i. e.*, inside, where the "air lenses" most frequently formed are thickest (most dense) at the center; outside, where the reverse is the case.

It is easy to see that the third effect is produced either by momentary combination of "air lenses" and "air prisms," or by "air lenses" alone, when the latter are not concentric with the axis of the telescope.

In considering the effect of these various disturbances on the intensity of the focal plane image, we can see in a general way that the relation between the magnitude of the effect and the dimensions of the object glass will be decidedly different in cases 1 and 2. In 1 there will be no loss in the intensity in case of visual observations, but in prolonged and continuous photographic exposures the loss will be (for small sources) directly proportioned to the square of the focal length and independent of aperture. In case 2 there will be a loss in both visual and photographic work (except at the instants of "best seeing"), which is proportional to the square (and in some cases to a higher power) of the aperture, but which decreases in proportion as the focal length increases. In 3 again we have a combination of both effects. In visual observations 2 is the most important; in prolonged photographic exposures 1 is the one to be chiefly considered.

YERKES OBSERVATORY.
Dec. 24, 1897.

(*To be continued.*)

Minor Contributions and Notes.

A NOTE ON THE DISCOVERY OF AN ERROR IN THE PAPERS OF STRUVE AND LORD RAYLEIGH, DEALING WITH THE APPLICATION OF THE PRINCIPLES OF THE WAVE THEORY TO THE DETERMINATION OF THE INTENSITY OF THE IMAGES OF FINE LINES AND EXTENDED AREAS AT THE FOCUS OF A TELESCOPE.[1]

The general expressions for the intensity at various points in the images of point, line, and surface sources, as formed by a telescope of symmetrical aperture (rectangular, triangular, elliptical, or circular) have been developed on the principles of physical optics by Airy Stokes, Lommel, André, Struve, and most fully and comprehensively of all by Lord Rayleigh, whose memoir on the general principles of the wave theory[2] is a standard of reference for all workers in the subject. These results rendered possible for the first time the development of a satisfactory theory of the resolving power of optical instruments, which is the only logical basis upon which to discuss their comparative merits and efficiency with reference, either to ACCURACY (metrological or measuring power), or RESOLUTION (separating power); the two properties which chiefly concern us, in what may, for want of a better

[1] This error was discovered about three weeks ago when I resumed the investigation (temporarily laid aside by reason of the pressure of work attending dedication determination of color curve, and computing of photographic correcting lens for 40^{in} telescope. etc.), of the "Conditions of Maximum Efficiency in Astrophotographic Work," of which Part I, "General Theory of Telescopic Images," was published in the August number of this JOURNAL; and of which Part II, "General Effect of Atmospheric Aberration on the Intensity of Telescopic Images," appears elsewhere in the present number. The present note was not completed in time for insertion in full in the December number, and a short note simply announcing the discovery of the error was inserted in its place. Since then two articles on the same subject have appeared, one by Professor Schaeberle in the *Astronomical Journal* (No. 421), and the other by Mr. Newall in the *Monthly Notices* (November 1897). Both of these articles, however contain themselves a number of errors which have been pointed out in two communications addressed to the respective journals in which they appeared. I have therefore, allowed the present note to stand as it was originally written without reference to either Professor Schaeberle's or Mr. Newall's articles.—F. L. O. W., January 3.

[2] "Wave Theory," *Enc. Brit.*, 9th ed. **24**.

term, be called *quantitative* observations. More recently these same results, together with others deduced by similar methods, have been used in developing the theory of "contrast" and "delineating power," which, in conjunction with the older theory of resolving power, has been made the basis for comparing the efficiency of optical instruments with reference to DEFINITION (defining power): the property with which we are chiefly concerned in what we may call (in contradistinction to the term already used) *qualitative* observations.

The full discussion of "the conditions of maximum efficiency" in both classes of observations, as determined by considerations of resolving and delineating power, is of the greatest value and importance in indicating the methods of work and the forms of instruments calculated to secure the best results in any given line of research.

My attention was first directed to the investigation of the conditions of maximum efficiency in the use of the spectrobolometer, by my desire to obtain the best possible results in the investigation of the infra-red spectrum at the Smithsonian Astrophysical Observatory, and by my having been unable to find at the time this work was put into my hands any complete and satisfactory discussion of many of the various questions involved.[1] Subsequently my field of investigation was greatly extended by my having been placed in charge of the design and construction of the instruments and apparatus for the various lines of spectroscopic, spectrographic, and astrophotographic research at the Yerkes Observatory. A number of papers embodying the results

[1]At that time the only papers that had been published (as far as the writer is aware) in which both the general theory and practical design of optical instruments had been taken up and discussed satisfactorily by the methods of physical optics were those of Lord Rayleigh, whose results were derived with special reference to *laboratory* spectroscopes (see particularly "On the Manufacture and Theory of Diffraction Gratings, *Phil. Mag.*, 47. 5, 1874; and "Investigations in Optics with Special Reference to the Spectroscope," *ibid.*, 8, 9, 1879–80); and of Professor Michelson, who had investigated the theory and pointed out a number of applications of the *interferometer* to astronomical measurements ("Interference Phenomena in a New Form of Refractometer," *Phil. Mag.*, 46, 395, 1882; "Measurement by Light Waves," *Amer. Jour. Sci.*, 39, 115, 1890; "Application of Interference Methods to Astronomical Measurements," *Phil. Mag.*, 30, 1, 1890; "Application of Interference Methods to Spectroscopic Measurements," *ibid.*, 31, 338, 1891, and 34, 280, 1892, etc.). Since that time the theory of the microscope has been fully treated by Lord Rayleigh ("On the Theory of Optical Images with Special Reference to the Microscope," *Phil. Mag.*, August 1896), and the theory of the spectroheliograph by Michelson (*Ap. J.*, 1, 1, January 1895). Several papers on the theory of the telescope have also been recently published by Strehl in the *Zeit. für Instrumentenkunde*.

of these investigations as they have progressed have been published during the last three years in this and other journals, and the conclusions that have been reached have been in general, I believe, accepted by astronomers and astrophysicists.[1]

But there is one result which has recently been used by the writer in developing the theory of contrast and "delineating power" that has been (as it now appears rightly) questioned. This is the result for the illumination of the focal plane of a telescope due to an infinitely extended luminous area, which was first announced (I believe) several years ago by Lord Rayleigh in his great memoir on the wave theory. After reviewing the work of Stokes and Struve, both of whom had investigated closely related problems (exactly the *same* problem from a mathematical point of view), he concludes as follows (§ 12):

"If we integrate (30)" . . . *i. e.*,

$$d\xi \int_{-\infty}^{+\infty} I_1^2 \, d\eta \qquad (30)$$

"with respect to ξ between the limits $+\infty$ and $-\infty$, we obtain πR^2 as has already been remarked. This represents the whole illumination

[1] For convenience in future reference the following partial list of these papers is here given: "Electric Control and Governors for Astronomical Instruments," *A. and A.*, 13, 265, 1894; "An Improved Form of Littrow Spectroscope," *Phil. Mag.*, 37, 137, 1894; "Fixed Arm Spectroscopes." *ibid.* 38, 337, 1894; "General Considerations Respecting the Design of Astronomical Spectroscopes," *Ap. J.*, 1, 52, 1895; "Design of Electric Motors for Constant Speed" (for astronomical instruments), *ibid.* 1, 1895; "New Designs of Combined Grating and Prismatic Spectroscopes of the Fixed Arm Type and a New Form of Objective Prism," *ibid.* 1, 232, 1895; "A Multiple Transmission Prism of Great Resolving Power," *ibid.* 2, 264, 1895; "Fixed Arm Concave Grating Spectroscopes." *ibid.* 2, 370, 1895; "The Use of a Concave Grating as an Analyzing Spectroscope," *ibid.* 3, 47, 1896; "Further Notes on Astronomical Spectroscopes," *ibid.* 3. 176, 1896; "The Conditions of Maximum Efficiency in the Use of the Spectrograph," *ibid.* 3, 321, 1896; "The Objective Spectroscope," *ibid.* 4, 54, 1896; "A New Form of Fluid Prism and Its Use in an Objective Spectroscope," *ibid.* 4, 274, 1896; "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Mem. Spectr. Ital.*, 26, 2, 1897, *Phil. Mag.*, May, *Wied. Ann.*, June, and *Jour. d. Phys.*, August 1897; "On the Conditions which Determine the Ultimate Optical Efficiency of Methods of Observing Small Rotations," *Phil. Mag.*, 44, 83, 1897; "The Effect of the General Illumination of the Sky on the Brightness of the Field at the Focus of a Telescope," *M. N.*, June 1897; "On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," *A. N.*, No. 3439, *Knowl.*, August and September 1897; "On the Conditions of Maximum Efficiency in Astrophotographic Work. Part I. General Theory of Telescopic Images,"

over the focal plane, *or reciprocally the illumination at O*[1] (*the same as at any other point*), *due to an infinitely extended luminous area*." Earlier in the same memoir (§ 11) Rayleigh makes a similar statement with reference to the intensity of illumination in the images of *linear* sources. On this point he says (in the case of a telescope with rectangular aperture), "If the image of a line be at $\xi = 0$" (the center line in the field), "the intensity at any point ξ, η, in the diffraction pattern may be represented by

$$\int_{-\infty}^{+\infty} I_1^2 \, d\eta \tag{8}$$

and again (for circular aperture), "If we integrate (8) for I^2, with respect to η, we shall obtain a result applicable to a linear luminous source" . . . .

Before using these results in the development of the theory of contrast I had obtained a seeming verification of them along a slightly different line of analysis. I first obtained three general expressions for the intensity at any point in the image of the most general form of radiating source (one of any form or extent, and having any distribution in intensity), of which the elements vibrate independently. These expressions [(13), (16), and (17), of my paper in the Astrophysical Journal, August 1897][2] were first applied to the special cases of a short line and a small circular area of uniform intensity, and correct expressions (24) (28) obtained for the intensity at the centers of the images of such sources. They were then applied to the cases of lines of infinite length and areas of infinite extent and uniform intensity (which are also evidently special cases of the general one already stated), and two other expressions (19) (31) were obtained which appeared, and were, at first, assumed to be identical with the corresponding integrals (8) and (30) given by Rayleigh. Then, having independently carried through the analytical work of integration by the methods of both Stokes, Struve, and Rayleigh (as

Ap. J., **6**, 119, 1897; "A Comparison of the Photographic and of the Hand and Eye Methods of Delineating the Form and Surface Markings of Celestial Objects," *Pop. Astron.*, **5**, 200, 1897; "On the Photography of Planetary Surfaces," *Obs'y* **20**, 303, 365, 404, 1897; "On the Effect of the Size of an Objective on the Visibility of Linear Markings on the Surface of a Planet," *Ap. J.*, No. 413, etc. When the investigations now on hand are completed the writer hopes to publish these papers in a collected form.

[1] *O* is the center of the field; the origin of coördinates for ξ η.

[2] See paper, "General Theory of Telescopic Images," pp. 128–9.

given in Rayleigh's memoir), and not finding any error in the mathematical part of the work, I felt confirmed in the belief that the results reached were correct. And, indeed, although they seem at first sight sufficiently startling and contradictory to our established ideas on the matter (particularly the one for an extended uniform area), further consideration seemed to show that they were not altogether unreasonable. So far as the laws of geometrical optics were concerned they were at least no more in contradiction to them than the results obtained for the intensity in the physical images of point sources, or sources of very small angular magnitude, for which the ordinary geometrical laws are so greatly in error that the actual intensity at the centers of the images of such sources is in some cases less than 2 per cent. of what it would be on the laws of geometrical optics.[1] Naturally less discrepancy between the results obtained on the geometrical and on the physical theories would be expected in the case of very large areas than in the case of very small ones, but considered at least approximately (and I must confess that in general I am inclined, perhaps too much so, to look upon the results of the geometrical theory as at best but approximations) the result reached for an infinitely extended area seemed explicable under both theories. Geometrically considered, the image of an infinite area is itself infinite in extent, and since the intensity of illumination at any point of the image is geometrically proportional to the area of the objective divided by the area of the image, and since this latter quantity is constant (there being no question of infinities of different orders) the intensity would likewise be constant and proportional simply to the area of the objective.[2]

On the ground of the physical theory the difficulty of explaining the result seems even less, because we know that the illumination at the center of the field (which is the same as at any other point) is due to the effect not only of that portion of the source which would correspond to the geometrical image, but to all the outlying portions as well; even (in some degree) to those that lie at an angular distance of 90° from the optical axis. The effect of each one of these outlying

[1] In the case of many stars (whose angular magnitude must be less than 0″.001) it must be considerably less than 0.1 per cent., perhaps not more than 0.01 per cent. See table in *Ap. J.*, August 1897, p. 132.)

[2] The fallacy in this argument lies in the fact that the focal surface is not a plane, but (approximately), a portion of a sphere of radius *f*.

elements (distant more than a few minutes of arc from the edge of the geometrical image) is, it is true, extremely (mathematically, differentially) small, but on the other hand the number of the elements, contributing each its effect, is infinitely large. *A priori* there is no reason for considering that the summation of an infinite number of infinitely small effects may not be itself finite; indeed, it generally is. And if the effect of these outlying elements was finite it would (since the integration extends from $-\infty$ to $+\infty$) be independent of the distance, from the diffracting aperture, of the point at which the effects are summed up, *i. e.*, of the focal length of the telescope. When such a result was reached, then, the writer was not prepared to dispute its correctness, particularly after it had been accepted and announced by such an authority as Lord Rayleigh.

In addition to this there was another strong apparent confirmation of this mathematical result, in the complete and satisfactory explanation which it seemingly offered of the remarkable experimental results obtained by Professor Barnard with lenses of small aperture and short focal length. The striking agreement between the observed times of exposure required to obtain a given result with different lenses, and the computed times as determined by the theory of contrast (in the development of which this result had been used), was considered to be too remarkable and exact to be accidental and fortuitous.

For all these reasons, but chiefly for the last, the result for the focal plane illumination, due to an infinitely extended area, was at first regarded as well established, although doubts of its correctness were freely expressed by a number of astronomers and astrophysicists (among whom I may mention Keeler, Hale, Runge, Deslandres, Schaeberle, and Lord). No one with whom I discussed the matter was, however, able to point out any flaw in the reasoning or in the mathematical analysis by which the result was obtained.

It was only recently[1] that I was able to take up this investigation again with special reference to the general effect of atmospheric aberration on the intensity of telescopic images. When I found that the general effect of this, in prolonged photographic exposures was to diminish the *effective photographic intensity* of point and small surface sources in the ratio $\frac{1}{f^2}$ and thus, if the first result for the focal plane illumination, due to the sky, had been correct, make the *effective photographic*

[1] See preceding footnote.

contrast between field and such sources vary as $\frac{1}{f^4}$, instead of $\frac{1}{f^2}$, as found in practice (and as found at first in theory); I was led to suspect an error somewhere in Rayleigh's work, and to discover, on a reinvestigation of the whole problem, where it lay.

In Rayleigh's case the error was made in assuming (without proof) that the general principle of "reciprocality" or "reversibility" of images was applicable to the problem under consideration. The integral

$$\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} I_1^2 \, d\eta \tag{32}$$

in which the integration is extended *over the whole of the focal plane*, is, it is true, the same in form as the integral

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} I_1^2 \, dy \tag{31}$$

in which the integration is extended *over the whole of an infinitely extended uniformly luminous area;* but they are not identical; in other words, although the first correctly represents the total illumination over the whole of the infinitely extended focal plane, due to a point source, *it does not* "reciprocally represent the illumination at a point due to an infinitely extended luminous area." My own error was essentially the same, *i. e.*, the assumption of the identity of the two integrals (31), (32);[2] but in making this assumption I also committed an analytical blunder, which, under the circumstances, seems well-nigh inexcusable. The two integrals would be identical (the limits being infinity) if the variables x, y, were symmetrically involved with the variables ξ, η, in the expression for I^2. And although this may appear at first sight to be the case, an inspection of equation (14),[3] which expresses the value of r (the variable in I^2) in terms of x, y, ξ, η, shows that this is not so. The two variables x, y, are each multiplied by a factor of dissymmetry $\frac{f}{D}$. In order to obtain an expression, of which the integral part is identical with the integral (32), we must introduce in (31) two new variables

[1] See paper: "Conditions of Maximum Efficiency in Astrophotographic Work, Part II. General Effect of Atmospheric Aberration on the Intensity of Telescopic Images," p. 70, of the present number of this JOURNAL.

[2] See equation (1), *Monthly Notices*, **57**, 587, June 1897.

[3] *Ap. J.*, **6**, 132, August 1897.

$$\xi_1 = \frac{f}{D} x. \qquad \eta_2 = \frac{f}{D} y.$$

Substituting in (31) we obtain

$$\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} I_1^2 \, dy = \frac{D^2}{f^2} \int_{-\infty}^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} I_1^2 \, d\eta_1 = \frac{D^2}{f^2} (Z_{111}) \tag{31a}$$

and similarly in (19), (the expression for the intensity in the images of long lines),

$$\int_{-\infty}^{+\infty} I_1^2 \, dy = \frac{D}{f} \int_{-\infty}^{+\infty} I_1^2 \, d\eta_1. \quad = \frac{D}{f} (Z_{11}). \tag{19a}$$

The integrals (Z_{111}) and (Z_{11}) are now respectively identical with (32) and (20) of my paper, which again are the same as the ones (30_R) and (8_R) used by Lord Rayleigh.[1]

These integrals were correctly evaluated in the preceding papers. The correct expressions for the intensity in the images of an infinitely extended uniformly luminous source, and of a long uniformly luminous line (at the center of the diffraction pattern) are therefore respectively

$$i_{111} = \text{const.}\ \frac{b^2}{f^2} = \text{const.}\ \beta^2 \tag{33a}$$

$$i_{11} = \text{const.}\ b\, \beta^2 \tag{23a}$$

or the same as the intensity at the centers of the images of areas and lines of finite dimensions (28) and (24).

And the expressions for the theoretical contrast (contrast under perfect atmospheric and instrumental conditions) as given in the paper, "General Theory of Telescopic Images," Part I. are modified by the introduction of a factor (f^2) in the numerator in cases (A), (B_2), and (C), and a factor f in (B_1), as elsewhere indicated. At the same time the expressions for the *practical photographic contrast* (during prolonged exposure) remain (owing to the introduction of the factors $\frac{1}{f}$ and $\frac{1}{f^2}$, expressing the effect of atmospheric aberration during such exposures) practically identical with the expressions (36), (39), which have been used in the detailed consideration of cases (A) and (C) from the astrophotographic side. ("On the Conditions which Determine the Limiting Time of Exposure of Photographic Plates in Astronomical Photography," *A. N.*, No. 3439, and *Knowledge*, August and Septem-

[1] "Wave Theory," *Enc. Brit.*, **24**, §§ 11 and 12.

ber 1897). The main conclusions reached in the preceding paper, dealing with these particular cases are, therefore, correct (if the general theory of contrast upon which these conclusions are based is correct, and this has not, I believe, been questioned), as well as those in my preliminary note to the R. A. S. ("The Effect of the General Illumination of the Sky on the Brightness of Field at the Focus of a Telescope," *M. N.*, June 1897). One of the minor conclusions of the first paper (*A. N.*, 3439) was based directly on the erroneous result for I^2_{III} and is consequently wrong. This is the conclusion with respect to the "fogging" of photographic plates by the light from the sky. But this result was not considered at the time it was obtained as of particular importance [as may be seen from the following quotation, § 100: "The mere ability to lengthen the time of exposure (at least beyond twenty-four hours) by decreasing the size of the photographic objective would not, in itself, be of great importance because there would be too much risk and difficulty in accurately following an object for a much greater length of time than is done at present"], and it was not, therefore, included in the summary of conclusions at the end of the paper.

The only other case yet considered in detail is case *D*, for which the expression for the theoretical contrast (40) was correct, as originally derived, and in the consideration of which the effect of atmospheric aberration was fully taken into account.[1] It was very unfortunate that this effect was not also considered in detail in cases (*A*) and (*C*) at the very first, as it was such consideration that led later to the discovery of the error in Rayleigh's result. But although the effect had not by any means been lost sight of, (having already been considered in the theory of the spectroscope and spectrograph,[2] as well as in case (*D*) already referred to), its importance was at first underestimated, and its consideration therefore deferred to the second part of my general paper, "Conditions of Maximum Efficiency in Astrophotographic Work," which, as already indicated, appears elsewhere in the present number.

F. L. O. WADSWORTH.

YERKES OBSERVATORY,
December 16, 1897.

[1] See papers: "On the Photography of Planetary Surfaces," *Obs'y* **20**, 333, 365, 404; and "The Effect of the Size of the Objective on the Visibility of Linear Markings on the Planets," *Ap. J.*, 413.

[2] See particularly *Ap. J.*, 4, 59, 60, June 1896.

THE PHOTOGRAPHIC NORMAL SOLAR SPECTRUM.

CONSECUTIVE WAVE-LENGTH EDITION.

Shortly after the issue of my "Photographic Studies of the Solar Spectrum," several leading men of science on both sides of the Atlantic expressed opinions in favor of a map on the same lines in regard to the various aspects under different conditions of the atmosphere and of solar altitude, but in consecutive wave-length parts for convenience of general reference.

Selections were submitted for consideration or approval, but owing to the fact that a number contained two orders for the purpose of showing the relative wave-lengths, it was felt that the resulting want of uniformity would still be an objection.

For this and other reasons the work has been entirely remodeled, and although the production of a uniform series was the primary object, no pains have been spared to make such improvements as seemed to be necessary, not only in respect to the general appearance and correctness of delineation, but in the introduction of some new features, which it is hoped will prove of service.

The work consists of thirty-eight sections, eight of which may be regarded as supplementary. Each section contains two or more subjects, and measures $8\frac{1}{2} \times 3\frac{1}{4}$ inches. The scale and other numbers have been written with a fine point and are not obtrusive enough to mar the subject.

The degree of enlargement requiring mm. units for the wave-length scale has been found sufficient for the visible and infra-red portions. In adopting $1\frac{1}{3}^{mm}$ as units for the violet end, the ratio of the two units is the same as that existing between the 3d and 4th orders.

No feature discernible in the original negatives under microscopical examination is wanting in the finished prints. A hand magnifier on the supplementary parts may, however, be required to reveal the structure of dense groups or close doubles. With the same object in view, short exposures have been given on the margins of plates for the region between wave-lengths 3200 and 5000.

With the exception of two or three supplementary parts, the ordinary method of enlarging has been employed, and is referred to in the descriptive supplement.

INDEX.

10 SCALE DIVISIONS = 13½^{mm}.

Where the same remarks are applicable to both editions the numbers of the first edition are referred to in the third column and on the prints themselves.

Section	No.	Reference	Solar altitude	Wave-lengths		Including
A	1	21	55	2988:3149		*t*, T, *s*, S, *r*
	2	22	53	3124:3285		*r*, R
B	3	37	51	3196:3357		Q
	4	38	51	3286:3446		P, O
C	5	46	49	3392:3552		O
	6	49	41	3468:3628		N
D	7	50	45	3560:3720		N
	8	63	31	3643:3803		M
E	9	73	..	3740:3900		L
	10	74	..	3815:3975		K, H
F	11	77	..	3887:4047		K, H
	12	78	..	3958:4118		H, *h*
G	13	83	.	4052:4212		*h*
	14	..	..	4187:4347		G, *g*

10 SCALE DIVISIONS = 10^{mm}.

Section	No.	Reference	Solar altitude	Wave-lengths		Including
H	15	39	..	4263:4478		G, *g*
	16	40	50	4381:4595		
I	17	47	49	4522:4736		
	18	48	49	4625:4839		
J	19	51	45	4747:4961		F
	20	52	31	4857:5071		F
K	21	66	47	5040:5255		*b*
	22	75	48	5085:5300		*b*, E
L	23	80	..	5222:5436		E
	24	82	32	5402:5616		
M	25	85	34	5513:5727		
	26	86	6	5597:5811	Low sun	
N	27	..	20	5672:5886	Moist	
	28	9	10	5682:5896	Low temperature	

INDEX—*continued.*

10 SCALE DIVISIONS = 13½mm.

Section	No.	Reference	Solar altitude	Wave-lengths		Including
n	29	7	28	5688:5902		D
	30	10	10.	5706:5920	Doppler effects	D
O	31	15	13	5844:6058		D
	32	18	1	5844:6058	Rising sun	D
P	33	13	20	5777:5991	Moist	D
	34	25	40	5983:6197		
Q	35	27	43	6160:6374		α
	36	29	38	6310:6524		
R	37	28	7	6188:6402	Low sun	α
	38	..	7	6398:6612	Low sun	C
r	39	30	45	6390:6604		C
	40	31	12	6390:6604	Low sun	C
s	39A	..	9	6390:6604	Low sun	C
	41A	..	9	6498:6713	Low sun	C
S	41	..	45	6498:6712		C
	42	42	32	6686:6900		B
T	43	43	37	6840:7054		B
	44	44	7	6850:7064	Low sun	B
U	45	53	33	7015:7229		*a*
	46	55	50	7145:7359		*a*
V	47	57	11	7100:7314	Low sun	*a*
	48	57	11	7255:7469	Low sun	
W	49	56	44	7296:7510		
	50	67	44	7405:7619		A
X	51	69	60	7555:7762		A
	52	58	11	7520:7734	Low sun	A
Y	53	71	52	7573:7787		A
	54	69	60	7759:7973		
Z	55	70	52	7928:8142		
	56	70	52	8132:8346		Z

SUPPLEMENTARY SECTIONS.

Most of these are made up of four strips, and contain all the portions which abound with complex groups and close doubles. The first seven are under a wave-length scale of 2^{mm} units. The remainder consists of highly magnified isolated groups, together with subjects possessing points of special interest.

Section	No.		
a	1A	3051 : 3158	Shows coincidences
	2A	3130 : 3237	with red spectrum of the first order
b	3A	3196 : 3303	
	4A	3296 : 3403	
d	5, 6	3396 : 3503	3496 : 3603
	7, 8	3596 : 3703	3696 : 3803
f	9, 11	3793 : 3900	3898 : 4005
	12, 13	4003 : 4110	4106 : 4213
h	14, 15	4211 : 4318	4316 : 4423
	16, 17	4421 : 4528	4526 : 4633
j	18	4629 : 4736	4732 : 4839
	19, 20	4838 : 4945	4944 : 5051
o	31, 32	5845 : 5952	Comparison high and low sun (cylindrical lens)
	31, 32	5926 : 6033	
p	.....	Groups 3880, 5007, 5205, 5270, 5276, 5603, 5884, 6103, and 6164. Magnified 12 : 18 times. (Cylindrical lens)	
q	1	5914 : 6870	8228 Groups
	2	7130 ; 7740	Low sun ; low temperature ; dry
	3	7130 : 7740	Low sun ; high temperature ; moist
	4	7592 : 7699	A line, 2^{mm} scale. (Cylindrical lens)

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δ	7, 8	6680 : 7750	7280 : 8350

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GEORGE HIGGS.

TUEBROOK, LIVERPOOL,
December, 1896.

NOTE ON SUN DRAWINGS.

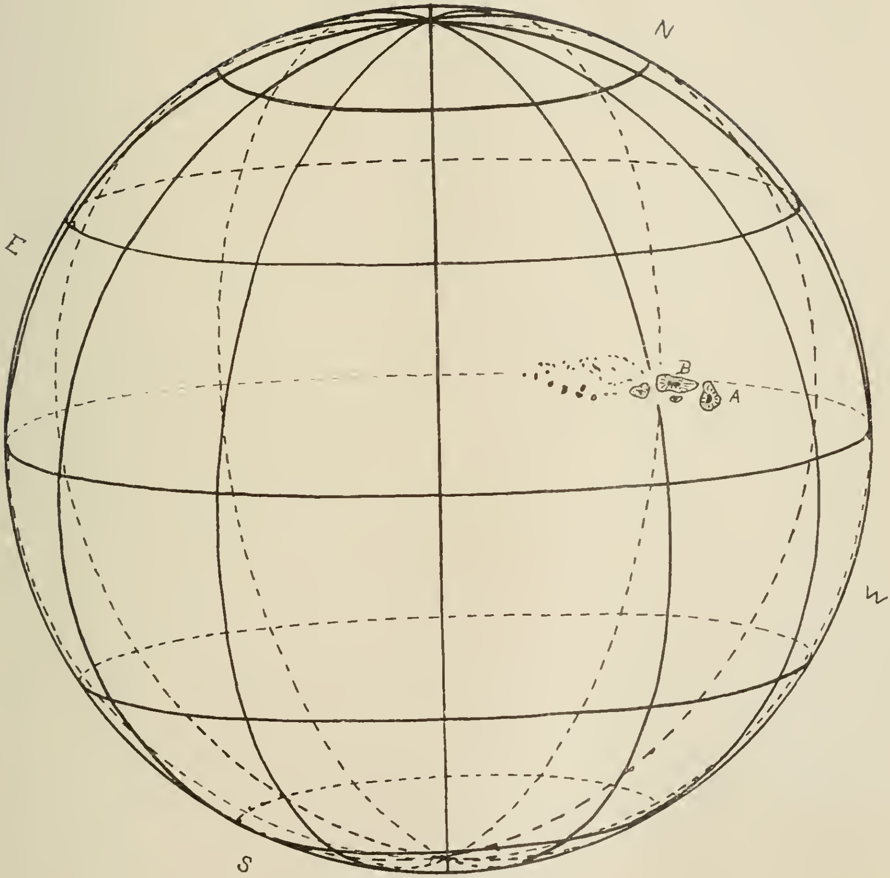
The accompanying results were obtained by the well-known method of projection-drawing. The drawing was made by attaching a board in front of the eyepiece of a twelve-inch equatorial in such a manner that an image of the Sun of some convenient size could be formed upon it. In our case, an image of six inches diameter was chosen. A piece of drawing paper, upon which was drawn a circle of six inches diameter, was tacked to the board so that the center of the circle was in the line of collimation of the telescope. The focus and the distance of the board were then so adjusted that the image of the Sun should exactly fit the circle. An east and west line was determined by allowing the image to pass across the paper and tracing the path of some small spot. Then, keeping the image fixed by means of the driving clock, the outlines of the spots were traced. From the drawing thus made are taken the distance from the spot to the center of the circle, and its position-angle from the north and south line. By means of these data. the heliographic latitude and longitude of the spot may be calculated by Carrington's method.

The accompanying observations, of course, do not include all the spots which appeared between the given dates.

Plate I represents the Sun, as seen in the direct view September 18.83, 1896, Greenwich Mean Time (civil reckoning), showing a large group of spots, the position of the poles and the parallels and meridians for every thirty degrees. The parallels and meridians were drawn according to the method given by R. A. Proctor.

Plate II, Fig. 1, represents the Sun on January 7.80, 1897, showing conspicuously the large spot P. Fig. 2, February 10.85, 1897, shows the same spot P again, after it has completed more than a revolution. Figs. 3, 4, 5 and 6 represent, respectively, the solar appearance on November 7.74, 9.83, 13.74, and 14.55, 1896. These show nicely the motion of the large spot G and the changes in the smaller spots.

PLATE I.



SUN SPOT GROUP SEPT. 18.83, 1896.

PLATE II.

N

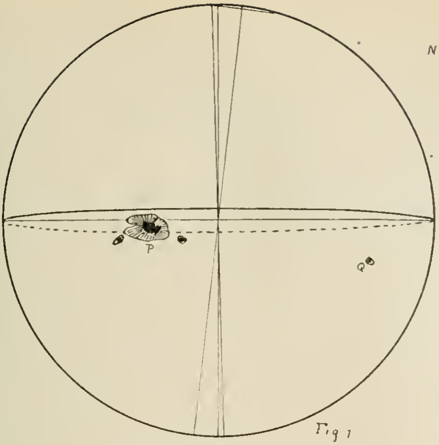


Fig 1

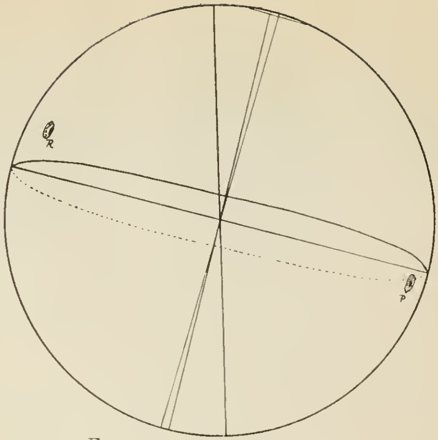


Fig 2

E

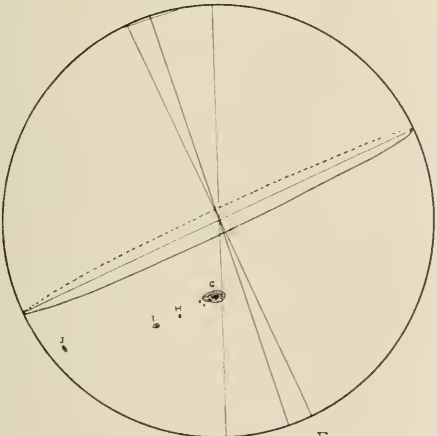


Fig 3

W

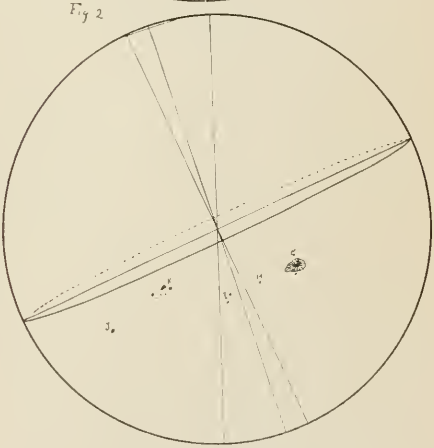


Fig 4

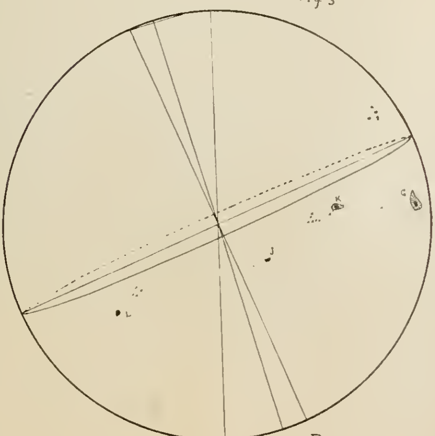


Fig 5

S

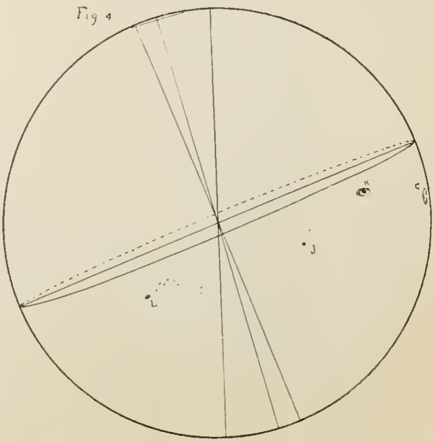


Fig 6

Gr. Mean Time (civil)	Group	Spot	Dist. from centre in inches	r/R corrected for distortion	Position angle	Sun's radius	Sun's longitude	Heliographic latitude of spot	Longitude from node	Heliographic longitude
1896										
Sept. 18.83	1	A	1.85	.633	32° 305′	15. 97	176 23	10° 19	321 38	59 5
	1	B	1.60	.549	307 0			12 31	315 23	52 50
Sept. 19.80	1	A	2.37	.802	303 30	15. 97	177 20	10 58	336 49	60 30
	1	B	2.13	.724	307 0			13 30	329 43	53 24
	2	C_1	2.00	.682	101 45			14 13	240 5	323 46
	3	D	2.84	.952	128 0			- 9 59	212 53	266 34
	3	E	2.89	.967	129 55			12 22	210 8	293 49
Sept. 21.58	1	A	2.93	.980	303 30	15. 98	179 4	9 24	4 17	62 43
	1	B	2.80	.939	306 0			12 25	355 35	54 1
	2	C_1	.99	.342	92 5			14 22	265 59	324 25
	3	D	2.33	.789	134 10			- 10 13	235 40	294 6
	3	F	2.61	.879	133 15			12 6	226 13	284 39
Sept. 23.74	2	C_2	.59	.205	327 0	15. 99	181 11	12 58	276 40	304 28
	3	D	1.27	.438	157 30			- 10 25	267 43	295 31
	3	F	1.68	.576	148 30			- 12 11	257 27	285 15
Nov. 7.74	4	G	1.06	.367	176 0	16. 19	225 52	- 15 36	321 44	71 14
	4	H	1.42	.489	155 20			- 15 53	309 40	59 10
	4	I	1.70	.583	146 10			15 29	301 19	50 49
	5	J	2.80	.939	127 50			- 12 27	263 23	12 53
Nov. 9.83	4	G	1.24	.427	243 30	16. 20	227 58	- 15 43	350 37	70 29
	4	H	.95	.329	218 50			- 15 10	339 15	59 7
	4	I	.94	.325	190 50			15 18	329 52	49 44
	5	J	2.05	.697	132 10			- 11 1	291 56	11 48
	6	K	1.10	.380	134 30			- 5 5	313 9	33 1
Nov. 13.74	4	G	2.81	.943	274 45	16. 21	231 55	14 58	44 15	68 39
	6	K	1.68	.576	277 10			- 6 0	9 20	33 44
	5	J	.88	.304	236 0			11 48	345 24	9 48
	7	L	1.88	.643	129 0			- 8 50	297 4	321 28
Nov. 14.55	4	G	2 91	.973	275 0	16. 22	232 44	- 15 21	53 53	66 48
	6	K	2.08	.707	280 0			- 6 20	22 36	35 31
	7	L	1.45	.499	133 0			- 9 2	310 38	323 33
	5	J	1.24	.427	254 30			- 12 19	358 57	11 52
Dec. 5.74	8	M	.83	.288	319 25	16. 28	254 10	9 25	13 45	86 5
	9	N	1.08	.373	32 30			20 41	352 27	64 47
	10	O	2.67	.898	123 45			- 17 35	297 31	9 51
Dec. 7.79	8	M	1.93	.658	297 30	16. 28	256 16	8 19	42 10	85 26
	9	N	1.30	.448	335 0			19 57	19 31	62 47
1897	10	O	1.90	.649	128 30			- 15 28	324 7	7 23
Jan. 7.80	11	P	1.00	.345	94 50	16. 30	287 51	- 5 56	13 13	336 37
	12	Q	2.13	.724	254 25			- 12 53	79 16	42 40
Feb. 10.85	13	R	2.67	.898	62 15	16. 24	322 27	7 12	7 38	208 3
	11	P	2.83	.948	251 0			- 4 44	139 40	340 5
Feb. 13.71	13	R	1.31	.452	45 25	16. 23	325 20	5 40	47 9	207 1
Feb. 17.87	13	R	1.50	.516	275 10	16. 22	329 31	5 49	103 19	204 10
eb. 20.71	13	R	2.75	.924	259 25	16. 21	332 23	5 26	144 18	204 52

FRED. SLOCUM.

LADD OBSERVATORY, Providence, R. I.
November 1897.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

It is intended to publish in each number a bibliography of astrophysics, in which will be found the titles of recently published astrophysical and spectroscopic papers. In order that this list may be as complete as possible, and that current work in astrophysics may receive appropriate notice in other departments of the JOURNAL, authors are requested to send copies of all papers on these and closely allied subjects to both Editors.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE
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ON THE ARC-SPECTRA OF THE ELEMENTS OF THE PLATINUM GROUP. I.

By H. Kayser.

Some years ago very little was known of the spectra of the elements of the platinum group. For Pt and Pd, Kirchhoff,[1] Huggins,[2] Thalén,[3] and Lecoq de Boisbaudran[4] had measured the strongest lines in the visible part of the spark-spectrum; for Os Huggins[2] gave eighteen lines; for Ir Kirchhoff[1] had found three lines in the spark-spectrum, and Lockyer[5] some lines of the arc-spectrum. Of Rh and Ru we knew nothing.

The first modern research with a Rowland grating was made by McClean.[6] He published photographs of parts of the spark-spectra of the six elements, together with the solar spectrum. Then Rowland[7] gave the first exact determination of the arc-spectra of these elements, excepting Ir, between the wave-lengths 3000 and 4500. Lastly Exner and Haschek[8] have published the ultra-violet spark-spectra of the six elements, photographed with a small Rowland grating, but measured with less accuracy than is desirable.

The determination of these spectra is exceptionally difficult because of the impossibility of preparing the elements suffi-

[1] Kirchhoff, *Untersuchungen ueber das Sonnenspectrum*, Berlin, 1861.

[2] Huggins, *Phil. Trans.*, 154 (1864).

[3] Thalén, *Nova Acta. R. Soc. Sc. Upsala* (3), 6 (1868).

[4] Lecoq de Boisbaudran, *Spectres Lumineux*, Paris, 1874.

[5] Lockyer, *Phil. Trans.*, 1881.

[6] McClean, *Comparative Photographic Spectra of the Sun and the Metals*, 1893 (see *M. N.*, No. 52).

[7] Rowland, *Ap. J.*, 2 (1895) and 3 (1896).

[8] Exner and Haschek, *Sitz. d. Wien. Akad.*, 104, 2 (1895), 105 (1896).

ciently pure and free from traces of one another. I had the fortune to get, through the kindness of Dr. Bettendorff, in Bonn, different salts of these elements, which he had prepared some years ago with all possible care, with the purpose of a new determination of the atomic weights. They were incomparably purer than the material to be had in commerce, and I hope that by their use I have been able to ascribe most lines to the elements to which they really belong.

The wave-lengths were determined with the aid of iron lines. For that purpose I have first measured the iron spectrum. On the basis of Rowland's[1] standards of the iron spectrum in the arc, not in the Sun, all of the stronger lines between 2300 and 4500 were determined with a mean error of, say, 0.003 A. U. For greater wave-lengths than 4500 it is impossible to interpolate with sufficient accuracy between the very few lines that Rowland has given. For this part of the spectrum I have therefore taken as standards the iron lines measured by Rowland[2] in the solar spectrum. I am well aware that this is inaccurate, but the shift between the solar lines and the arc lines will be a small one, and when it has once been determined, it will be easy to apply a correction to my numbers.

The wave-length of every line of the platinum elements is the mean of at least four determinations made on at least two different plates. The mean error of all the final values, excepting the diffuse lines, lies between 0.000 and 0.005 A. U. I think an error of 0.010 A. U. will very seldom occur.

The agreement between Rowland's wave-lengths and my own is very satisfactory. For Os, *e. g.*, we have 141 lines in common. For 73 of these lines the difference is between 0.000 and 0.005 A. U.; for 37 lines between 0.006 and 0.010; for 27 lines the difference is greater than 0.010 A. U. Rowland has measured only the stronger lines of the elements, so that my tables contain many more lines. A small number of Rowland's lines I have not found. This is partly due to the overlapping of the cyanogen bands, which seem to have been stronger on my plates than on Rowland's, so that I have not seen some lines lying in

[1] ROWLAND, *A. and A.*, 12, 1893. [2] ROWLAND, *Ap. J.*, 1 and 2 (1895).

them. The other lines not found by me I ascribe to impurities, for in the spectrum of Ru Rowland gives seven lines belonging to Zn.

In the following tables the intensities are given in a scale from 0 (weakest line) to 10 (strongest line). The self-reversed lines, denoted by r, are estimated from 3 to 10. u means diffuse (unscharf), U very diffuse (sehr unscharf), (such lines occur in great number in the spectrum of Pd), and uR or uV means diffuse on the side of longer and shorter wave-lengths, respectively.

I. PLATINUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2305.72	2	2450.527	2	2544.042	4
2308.12	3	2451.046	3	2544.807	2
2315.58	2	2460.160	1	2546.562	0
2318.371	2	2461.474	0	2546.986	0
2326.185	2	2467.504	6, r	2548.194	0
2331.047	1	2469.537	0	2549.552	3
2340.255	2	2471.092	3	2552.326	3
2343.468	0	2473.247	0	2560.438	0
2346.822	0	2477.365	0	2564.263	0
2347.239	0	2481.270	2	2572.723	0
2353.123	0	2483.312	2	2574.580	2
2356.415	0	2483.452	2	2582.415	2
2357.181	4, r	2487.261	4, r	2587.890	2
2357.656	0	2488.819	4	2596.081	4
2368.357	4, r	2490.217	2	2599.148	0
2380.035	0	2495.910	4	2599.986	2
2383.732	4	2497.197	1	2602.182	0
2386.886	0	2498.592	4	2603.223	4
2387.448	0	2500.895	0	2606.126	0
2389.615	3	2503.075	2	2608.333	0
2391.856	0	2504.128	2	2613.204	0
2396.243	2	2506.014	4	2613.337	0
2396.762	0	2508.589	3	2614.701	2
2401.089	1	2510.604	0	2616.839	0
2401.959	3	2513.999	0	2619.668	4
2403.180	4, r	2514.165	2	2619.977	0
2413.138	1	2515.119	3	2625.419	2
2418.151	3	2515.666	3	2627.484	4
2420.912	0	2517.273	1	2628.122	7, r
2424.964	2	2520.356	0	2635.372	0
2426.523	2	2522.616	0	2639.434	5
2428.206	8, r	2529.499	2	2645.453	4
2429.186	2	2536.068	2	2646.969	6, r
2434.551	0	2536.581	4	2650.938	4, r
2436.771	4, r	2538.361	0	2653.867	0
2439.533	1	2539.285	3	2656.907	0
2440.158	4, r	2541.433	2	2658.266	4

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2658.790	2	2788.728	0	2901.798	0
2659.535	10, r	2790.593	0	2903.129	0
2664.723	2	2790.987	0	2904.258	0
2668.748	0	2793.372	4	2906.001	4
2673.707	0	2793.736	2	2908.008	4
2674.649	4	2794.304	5	2808.928	0
2677.232	5, r	2796.165	1	2910.569	3
2686.990	0, u	2800.560	0	2911.888	0
2688.352	2	2803.338	6	2912.884	0
2694.314	4	2806.151	0	2913.361	2
2696.069	0	2807.396	0	2913.655	4
2698.498	6	2808.603	4	2914.443	0
2701.208	0	2810.921	0	2915.278	0
2702.484	6, r	2813.080	2	2916.505	2
2705.985	5, r	2814.121	0	2919.451	4
2713.215	4.	2818.354	4	2921.336	1
2714.613	0	2818.741	2	2921.498	3
2715.866	2	2821.179	0	2922.381	0
2717.709	0	2822.273	0	2927.040	1
2719.125	6, r	2822.602	2	2928.226	4
2725.433	2	2825.192	1	2929.903	8, r
2730.002	5	2830.402	8, r	2830.904	4
2733.725	5, r	2831.981	0	2933.837	0
2734.057	8, r	2834.815	0	2938.935	4
2734.584	2	2837.338	2	2941.219	2
2736.886	0	2837.643	0	2941.908	0
2737.656	2	2839.345	2	2942.880	4
2738.569	4	2848.406	0	2944.879	3
2744.928	2	2849.241	1	2948.844	0
2747.701	4	2853.207	4	2949.900	2
2753.850	2	2853.484	2	2950.929	0
2753.957	3	2854.781	0	2951.341	2
2754.327	0	2855.866	0	2958.650	0
2755.003	4	2868.783	0	2959.219	4
2757.799	2	2870.572	4	2959.825	1
2758.164	0	2878.823	1, u	2960.864	5, u
2758.333	2	2884.583	1	2967.596	0
2759.424	0	2885.447	0	2969.965	0
2763.299	0	2888.307	4	2974.252	0
2766.764	5	2890.495	2	2978.179	2
2769.940	4	2891.030	2	2982.414	0
2771.750	4, r	2891.170	0	2983.882	2
2772.925	4	2891.873	0	2984.565	0
2773.696	2	2893.335	4	2988.177	0
2774.095	4	2893.984	6	2988.913	0
2774.306	3	2896.245	1	2989.915	4
2774.880	2	2897.988	5	2994.916	2
2776.111	1	2899.764	1	2998.087	7, r
2776.859	0	2900.903	0	3001.304	2
2777.558	0	2901.282	2	3002.385	4

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3003.400	2	3104.170	0	3252.785	1
3004.269	2	3112.718	0	3253.319	0
3005.911	2	3118.547	0	3255.356	0
3010.051	0	3119.911	4	3256.048	6
3012.498	2	3122.192	0	3256.634	1
3014.636	0	3123.065	0	3258.551	0
3015.013	0	3132.187	0	3259.282	1
3015.510	2	3133.443	4	3259.866	4
3017.450	2	3133.785	1	3261.202	2
3018.003	4	3134.413	1	3261.818	4
3019.961	0	3136.381	0	3263.737	1
3022.957	3	3139.503	7	3268.557	4
3024.410	2	3141.767	4	3282.104	5
3025.179	2	3154.858	1	3283.336	2
3025.671	2	3156.686	5	3283.443	2
3026.446	2	3159.841	0	3285.367	0
3036.554	6	3160.314	1	3287.245	0
3039.612	0	3169.006	1	3290.363	6
3041.323	2	3174.959	2	3293.615	0
3042.752	4, r	3176.081	1, u	3293.820	0
3048.6	2, U	3177.707	1	3298.688	0, u
3054.4	2, U	3179.650	1	3300.070	1, u
3054.8	2, U	3191.604	0	3302.015	8, u
3055.402	4	3192.635	3	3311.504	1
3056.719	0	3199.076	0	3311.959	2
3059.748	4	3199.215	0	3312.614	3
3061.905	1	3200.848	4	3313.186	1
3062.3	0, U	3204.165	6	3315.186	4
3062.845	0	3207.347	0	3323.914	6
3064.825	6, r	3208.968	0	3325.861	2
3069.207	2	3212.502	2	3327.234	0
3070.369	2	3218.603	1	3338.214	2
3072.042	5	3218.972	0	3342.429	1
3074.938	1	3220.904	3	3344.031	4
3075.122	0	3221.416	0	3367.139	4
3078.948	0	3222.680	0	3368.628	2
3079.674	4	3222.930	0	3372.960	0
3081.172	0	3223.928	0	3406.733	2
3082.779	0	3227.305	2	3408.286	7
3084.217	3	3230.401	5	3414.610	2
3084.978	2	3233.550	5	3417.227	2
3087.319	0	3240.324	5	3418.311	0
3088.677	0	3241.652	1	3420.493	0
3089.780	0	3243.224	0	3426.887	2
3098.887	0, u	3243.533	2	3428.079	4
3100.146	4	3247.388	2, d ?	3431.495	0
3101.077	4	3248.623	2	3432.002	2
3102.710	0	3248.843	0	3448.523	1, u
3103.231	1	3250.481	4	3454.290	3
3103.704	2	3252.117	5	3464.097	2

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3472.080	0	4092.426	3	4831.371	0
3483.588	5	4118.854	5	4854.067	4
3485.430	6	4164.709	4	4862.577	0
3488.877	1	4192.577	4	4879.700	4
3491.155	1	4201.374	2	4980.532	1
3498.321	1	4247.838	1	4998.123	2
3505.848	1	4251.277	1 [1]	5002.762	2
3514.869	4	4263.664	2	5033.686	4
3528.700	2	4269.411	2	5037.859	0, u
3611.057	2	4274.042	2	5038.681	0
3615.443	0	4281.905	1	5044.194	4
3621.839	2	4288.215	4	5044.645	6
3628.275	5	4291.070	2	5050.006	1
3629.025	3	4327.243	4	5059.658	5
3638.956	6	4334.827	2	5095.950	0, u
3643.331	6	4343.852	0	5118.583	1
3652.411	1	4358.522	2, u	5194.050	1
3654.132	1	4364.624	4	5208.775	0
3659.571	2	4391.999	4	5227.782	6
3663.239	4	4411.580	3	5257.609	0, u
3668.564	1	4414.420	2	5260.982	3
3672.165	4	4437.470	4, u	5265.290	0
3674.207	4	4442.730	6	5275,008	0, u
3675.107	1	4445.710	4	5286.289	0, u
3681.227	0	4473.633	3, u	5295.918	0
3683.169	4	4481.808	3	5301.182	6
3687.582	4	4484.882	5, u	5306.493	0
3700.070	4	4493.350	3	5319.540	0
3706.685	3	4498.926	6	5324.799	0
3818.827	5	4511.417	3, u	5369.188	4
3898.880	4	4521.099	5, u	5388.105	2
3900.873	4	4523.192	5, u	5391.010	4
3903.864	2	4548.056	3, u	5452.984	0
3904.534	3	4552.116	2	5469.714	2
3906.433	2	4552.586	5, u	5475.996	6
3911.045	3	4554.759	4	5478.722	6
3923.105	5	4560.209	4	5514.324	4
3925.483	4	4577.584	4	5526.077	4
3948.550	4	4580.685	2	5560.245	2
3953.780	1	4580.828	2	5684.908	2
3966.507	3	4639.982	4	5699.190	1
3976.460	1	4650.192	1	5700.672	0
3980.746	1	4658.105	5	5728.369	0
3996.720	3	4684.255	4	5762.877	3
4002.649	2	4737.722	2	5763.778	3
4054.928	2	4739.924	1	5840.354	5
4066.087	2	4746.046	1	5845.050	4
4081.631	1	4772.467	1	5861.074	2

[1] Rowland.

11. PALLADIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2316.569	0	2605.157	4	3025.094	0
2319.328	0	2609.716	0	3028.031	4, r
2327.575	0	2614.270	1	3028.894	1
2336.522	0	2631.692	0	3029.600	0
2336.663	0	2641.149	0	3032.324	1
2347.611	0	2653.7	2, U	3036.220	1
2351.428	0	2655.1	4, uR	3038.830	0
2357.732	0	2658.819	2	3039.128	0
2360.614	0	2663.249	2	3039.748	0
2362.406	0	2676.050	2	3041.102	0
2368.044	0	2686.373	3	3046.614	2
2373.701	0	2734.095	0	3049.502	0
2414.850	0	2742.532	2	3062.3	0, u
2418.835	0	2751.972	2	3062.430	0
2421.	3, U	2763.199	8, r	3063.537	1
2424.564	0	2802.009	3	3065.425	4, r
2426.964	0	2806.561	1	3066.210	1
2431.051	0	2807.8	2, uR	3067.243	0
2435.408	0	2835.133	0	3068.564	0
2441.52	6, r, U	2835.385	0	3073.924	0
2446.275	0	2837.2	2, U	3075.274	4
2447.998	10, r	2839.5	4, uR	3078.356	0
2457.361	0	2846.4	2, U	3088.636	0
2461.2	1, U	2849.912	2	3089.756	0
2469.353	0	2854.694	2	3103.176	0
2470.091	0	2875.875	2	3103.909	0
2471.275	0	2922.615	7, r	3107.435	0
2473.011	2	2932.4	0, uR	3109.276	2
2476.509	10, r	2936.570	0	3114.157	5, r
2482.05	0, U	2936.901	2	3122.917	0
2486.618	1	2938.552	0	3138.417	0
2489.010	4	2950.920	1	3139.531	0
2498.873	3	2951.134	0	3139.804	2
2503.597	0	2956.811	0	3142.932	6
2505.804	2	2962.443	0	3146.075	1
2513.0	1, U	2968.356	0	3147.730	0
2521.102	0	2975.953	0	3148.532	0
2536.872	2	2995.400	0	3168.022	1, u
2539.690	0	2996.660	0	3213.018	0
2544.821	4	3002.775	4, r	3219.088	4
2546.283	0	3009.903	3	3242.824	10, r
2546.990	0	3010.980	1	3251.754	5, r
2548.165	0	3014.733	1	3258.907	6, r
2551.095	0	3015.052	0	3272.925	2
2551.971	0	3020.835	2	3284.080	0, u
2564.0	2, UR	3021.859	3	3286.337	1
2565.595	0	3022.744	0	3287.378	5

PALLADIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3299.875	1, u	4087.518	6	5110.940	6
3300.325	0	4098. bis }	, U	5114.530	2
3302.253	6, r	4101. }		5117.158	7
3310.251	1	4123.761	2	5127.849	2
3311.136	2	4170.005	5	5161.491	1
3313.093	2	4213.116	6, r	5163.970	10
3317.455	0, u	4268.4	4, U	5209.044	4
3321.100	2	4321.8	0, U	5234.992	7
3346.268	0	4328.125	0, u	5256.321	3
3373.137	6, r	4344.8	4, U	5294.267	2
3380.840	5, u	4351.1	2, U	5295.744	10
3383.025	4	4358.773	0, u	5312.752	4
3388.811	0	4360.4	0, U	5345.278	4
3389.192	2	4379.8	0, U	5346.980	0
3396.081	0	4386.614	1	5362.864	4, uR
3396.926	3	4388.776	2	5363.474	1
3404.732	10, r	4406.759	5	5377.833	1, u
3406.210	1	4421.217	1, u	5385.668	0
3419.818	4	4443.191	3	5394.958	4, uV
3421.368	8, r	4458.785	2	5395.471	8, uR
3433.582	5, r	4469.307	0	5427.425	1
3441.548	6, r	4473.771	7	5435.379	3
3442.545	2	4489.641	4, u	5497.056	4
3460.888	7, r	4497.813	2	5529.657	6
3481.308	7, r	4516.406	5, u	5542.997	10
3488.293	0	4541.314	5, u	5547.219	9
3489.930	4, r	4553.096	2	5548.514	2, u
3517.087	8, r	4590.191	3, u	5562.902	2, u
3528.881	2	4632.770	2	5601.867	3
3553.242	7, r	4677.617	4	5608.229	5, u
3566.775	2	4708.261	0	5619.667	9
3571.305	5, r	4724.204	3	5621.520	2
3574.040	2	4762.098	0	5642.898	5
3596.795	2	4776.715	1	5655.628	5, u
3609.698	9, r	4788.327	8	5664.578	1
3634.840	10, r	4791.061	0	5668.605	2
3646.116	1	4817.26	0, u	5670.263	10
3654.574	2	4817.662	9	5674.432	2
3690.491	6, r	4822.347	0	5680.993	0
3719.061	4, r	4836.654	0	5687.670	2
3799.332	5, r	4875.577	7	5690.333	4
3894.335	6, r	4919.008	3	5695.293	9
3958.777	5, r	4924.373	0	5700.978	0
3992.5	1, U	4930.145	1	5736.826	5
4007.6	0, U	4972.081	3	5737.842	3
4011.8	0, U	5063.549	4	5739.881	4
4020.3	1, U	5101.704	1	5760.122	1
4021.2	0, U				

III. RUTHENIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2335.047	2	2471.576	0	2524.952	0
2338.094	2	2472.215	0	2525.263	0
2340.767	2	2474.115	1	2525.726	0
2342.920	2	2474.506	0	2526.011	0
2351.411	2	2475.483	2	2526.914	2
2357.991	2	2476.395	0	2528.027	0
2370.251	2	2476.960	2	2528.813	0
2375.346	2	2479.010	2	2529.812	1
2392.501	2	2479.458	0	2532.128	1
2396.791	2	2479.611	0	2533.331	1
2402.802	4	2481.216	0	2535.147	0
2407.997	2	2482.628	0	2536.315	0
2408.744	1	2484.055	0	2537.776	0
2420.905	2	2490.017	2	2538.565	0
2429.672	2	2490.555	0	2539.822	1
2434.980	0	2491.847	2	2540.411	0
2437.019	0	2494.116	2	2541.381	0
2439.715	0	2494.773	1	2542.601	0
2441.051	1	2495.775	2	2543.240	0
2441.419	0	2498.512	2	2543.349	2
2443.036	0	2498.670	2	2543.778	1
2444.129	1	2499.873	2	2544.318	2
2444.497	0	2500.484	0	2545.866	0
2444.924	0	2500.940	0	2546.765	2
2445.519	0	2501.569	2	2547.600	1
2447.537	1, u	2501.990	0	2549.260	0
2448.958	0	2502.484	0	2549.576	2
2449.958	1	2502.966	0	2549.664	2
2450.464	0	2507.090	2	2550.946	1
2450.650	1	2508.377	2	2551.466	1
2454.267	0	2508.508	2	2551.822	0
2455.005	2	2509.160	1	2552.083	0
2455.614	5	2509.709	0	2552.384	0
2456.376	0	2510.238	0	2552.524	0
2456.519	4	2511.058	0	2552.965	0
2456.666	4	2511.652	1	2554.060	1
2457.050	1	2512.898	2	2554.790	1
2457.311	0	2513.417	2	2555.734	0
2458.706	2	2515 372	1	2555.955	2
2459.146	0	2516.882	0	2556.100	2
2461.506	0	2517.403	2	2556.994	0
2463.026	2	2517.728	2	2557.784	1
2464.474	0	2518.601	0	2558.359	0
2464.781	2	2520.041	0	2558.626	2
2467.674	0	2520.925	1	2559.497	0
2470.608	0	2521.700	2	2560.347	2
2470.805	0	2522.410	0	2560.920	3

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2562.252	2	2609.143	4	2650.076	0
2564.503	0	2609.573	2	2650.486	1
2564.674	1	2611.130	2	2650.693	0
2565.277	1	2612.165	2	2650.968	0
2566.666	2	2612.990	0	2651.366	2
2567.981	1	2613.143	0	2651.603	0
2568.854	4	2614.151	1	2651.936	4
2569.840	2	2614.671	2	2652.240	0
2570.180	0	2615.179	2	2653.240	1
2571.068	2	2617.882	1	2653.776	1
2572.370	2	2619.105	0	2654.563	0
2572.512	2	2619.745	2	2654.898	0
2573.654	0	2620.154	0	2655.193	0
2575.339	1	2620.713	2	2655.292	1
2577.052	0	2621.173	0	2656.328	1
2578.653	2	2623.914	1	2656.641	1
2579.071	2	2625.168	0	2656.776	1
2579.309	1	2626.290	0	2657.249	1
2579.623	2	2626.444	0	2658.482	2
2579.879	0	2627.737	1	2658.862	0
2580.316	0	2628.375	4	2660.673	0
2580.883	2	2628.621	0	2661.249	2
2581.216	2	2630.010	0	2661.690	4
2581.990	2	2630.314	1	2661.937	0
2583.131	2	2631.657	1	2664.833	4
2584.211	2	2632.210	0	2665.227	0
2585.412	1	2632.584	1	2665.542	0
2585.815	0	2633.537	0	2665.803	1
2586.157	0	2635.451	0	2667.479	1
2587.413	0	2635.927	4	2668.042	1
2589.129	0	2636.617	0	2668.421	0
2589.649	2	2636.760	2	2670.586	0
2589.886	0	2638.597	2	2670.813	0
2591.087	1	2639.205	2	2672.451	0
2591.201	2	2640.413	2	2673.089	0
2591.710	0	2641.549	0	2673.550	2
2592.093	2	2642.063	0	2673.691	2
2594.926	2	2642.607	0	2674.930	0
2595.734	0	2643.042	4	2675.273	0
2596.043	0	2643.600	0	2676.430	2
2597.417	1	2644.187	0	2677.057	0
2598.681	0	2644.711	0	2677.406	0
2600.840	0	2646.087	2	2677.967	0
2601.392	0	2646.715	0	2678.267	0
2601.553	2	2647.394	2	2678.837	4
2604.409	0	2648.019	1	2679.843	1
2605.439	2	2648.535	1	2683.756	1
2605.950	2	2648.706	0	2684.172	1
2607.440	0	2648.872	2	2684.540	0
2608.024	1	2649.608	2	2685.242	0

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2686.375	4	2730.115	0	2782.305	1
2687.214	1	2730.416	2	2784.625	0
2687.580	1	2731.028	2	2784.978	0
2688.216	1	2732.011	0	2785.746	1
2688.668	1	2733.167	0	2787.930	3
2688.969	1	2734.438	3	2789.720	0
2690.487	1	2735.806	2	2790.695	0
2690.904	0	2736.412	0	2791.164	0
2691.199	4	2736.917	0	2792.418	2
2693.392	2	2738.983	0	2792.746	2
2693.750	0	2739.311	4	2795.464	0
2696.653	0	2740.085	0	2796.652	0
2697.595	0	2740.327	1	2800.243	0
2698.161	0	2744.022	2	2800.785	1
2699.957	1	2744.541	2	2802.260	0
2700.578	1	2744.821	0	2802.907	2
2700.772	0	2745.343	0	2803.593	1
2701.434	4	2746.169	0	2806.845	0
2702.916	4	2746.991	0	2808.335	0
2703.221	0	2749.923	0	2810.131	4
2703.403	0	2750.452	0	2810.645	3
2703.891	2	2751.698	0	2810.788	0
2705.416	0	2752.548	2	2811.360	0
2708.054	2	2752.868	2	2812.925	2
2708.930	0	2753.543	2	2813.807	0
2709.157	0	2757.175	0	2815.410	0
2709.291	2	2757.912	1	2817.192	3
2709.851	0	2758.104	0	2818.460	4
2710.321	0	2760.268	0	2818.913	0
2712.169	0	2762.400	2	2819.062	2
2712.493	4	2763.232	2	2819.667	0
2712.967	0	2763.513	4	2821.279	1
2713.272	2	2764.005	1	2821.504	0
2713.824	1	2764.824	2	2822.142	2
2715.326	0	2765.530	2	2822.371	0
2715.595	2	2766.323	0	2822.659	2
2717.100	0	2768.032	0	2822.912	0
2717.510	2	2769.024	4	2824.004	0
2718.919	0	2769 993	0	2824.866	0
2719.610	5	2770.399	0	2827.627	0
2719.838	0	2770.805	2	2827.969	4
2721.653	3	2772.716	0	2829.253	2
2721.937	0	2773.068	0	2830.815	1
2722.493	0	2774.589	2	2831.280	0
2722.760	3	2775.288	1	2832.755	0
2722.903	0	2775.723	0	2834.107	3
2724.153	2	2776.009	1	2836.254	2
2725.549	4	2777.629	0	2836.684	2
2727.063	0	2779.081	0	2837.384	0
2729.540	2	2780.858	2	2838.729	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2840.657	2	2888.112	2	2934.300	2
2841.777	2	2888.739	2	2934.638	0
2842.651	1	2889.543	0	2936.131	2
2842.859	0	2891.242	2	2936.380	0
2843.277	2	2891.762	2	2936.591	0
2846.430	1	2892.654	4	2937.448	1
2846.662	0	2893.844	0	2937.679	0
2848.688	1	2895.554	0	2939.247	2
2849.399	0	2895.925	1	2939.796	0
2851.225	1	2896.638	3	2940.057	3
2853.433	0	2897.820	1	2940.474	3
2854.173	4	2898.650	3	2942.366	1
2854.465	0	2898.845	1	2942.823	0
2854.820	0	2899.817	1	2943.593	1
2855.454	0	2901.890	1	2944.035	3
2855.995	2	2902.223	1	2944.294	0
2856.153	2	2902.969	1	2845.591	0
2857.367	1	2903.180	2	2945.775	4
2857.770	0	2904.825	0	2946.670	0
2858.693	0	2905.756	3	2947.102	4
2860.114	4	2905.952	1	2949.612	4
2860.491	0	2906.424	3	2950.080	0
2861.508	5	2908.590	0	2950.650	1
2861.833	1	2909.352	1	2951.516	2
2862.963	0	2910.542	2	2952.599	2
2863.112	2	2912.451	0	2953.116	0
2864.726	0	2912.555	0	2954.371	0
2866.743	5	2912.866	0	2954.594	4
2868.286	2	2913.286	3	2955.463	2
2868.426	2	2914.403	2	2955.714	0
2868.662	0	2915.736	2	2955.960	0
2869.047	0	2916.351	6	2957.297	0
2870.322	2	2917.249	2	2958.118	3
2871.296	3	2917.353	0	2958.993	0
2871.756	4	2917.880	2	2959.855	2
2872.468	2	2919.276	0	2961.097	2
2874.161	2	2919.723	4	2961.803	3
2875.104	5, u	2920.369	1	2962.442	0
2877.197	2	2921.068	2	2962.705	0
2877.930	2	2921.276	0	2963.523	2
2879.466	0	2924.760	0	2963.829	3
2879.853	3	2925.189	0	2964.415	0
2880.637	0	2925.685	0	2965.286	4
2881.373	1	2925.890	0	2965.670	3
2882.222	2	2926.913	0	2965.820	1
2882.697	2	2927.232	2	2966.674	1
2883.701	3	2927.858	0	2967.456	2
2884.601	2	2928.608	2	2968.233	0
2886.640	4	2929.027	0	2968.564	4
2887.224	0	2933.367	0	2969.069	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2969.850	0	3018.158	2	3057.468	3
2972.594	0	3019.472	2	3058.762	2
2973.743	0	3019.876	0	3058.909	1
2974.099	3	3020.360	0	3059.284	3
2974.454	2	3020.989	2	3060.346	0
2975.253	1	3025.212	0	3062.155	2
2976.707	4	3027.195	2	3064.958	4
2977.048	3	3027.361	0	3068.355	4
2977.346	2	3027.678	0	3069.289	2
2977.596	0	3027.910	0	3071.586	2
2978.760	2	3028.785	0	3071.721	0
2979.847	3	3030.801	2	3071.824	0
2980.065	3	3030.890	2	3073.440	4
2981.080	0	3032.026	2	3075.412	1
2982.045	4	3032.771	0	3076.886	2
2986.104	0	3033.562	4	3077.175	2
2986.453	1	3034.167	4	3077.657	2
2988.047	1	3035.578	3	3078.209	1
2988.224	1	3036.580	1	3079.953	0
2989.079	6	3037.845	2	3980.292	4
2989.451	2	3038.078	2	3081.009	4
2989.770	2	3038.289	2	3081.218	1
2990.413	2	3038.851	0	3081.489	0
2992.080	0	3039.586	0	3081.946	0
2993.070	1	3040.071	2	3083.252	3
2993.387	3	3040.418	3	3084.631	2
2995.083	5	3042.025	1	3084.728	0
2997.011	3	3042.598	3	3085.597	0
2997.743	2	3042.953	2	3086.181	4
2998.446	3	3043.161	1	3086.631	2
2999.011	1	3044.077	0	3086.888	1
3000.341	2	3045.630	0	3087.039	2
3001.756	3	3045.833	4	3088.050	0
3002.188	0	3046.114	2	3088.177	2
3003.600	2	3046.356	2	3088.362	0
3004.708	2	3047.108	0	3089.252	4
3006.094	2	3048.442	0	3089.915	4
3006.708	4	3048.606	4	3090.341	2
3008.387	2	3048.897	4	3091.004	2
3008.695	0	3049.174	0	3091.974	2
3008.911	2	3050.309	1	3092.085	0
3009.798	0	3050.504	0	3092.351	0
3010.623	2	3051.704	2	3094.500	2
3012.003	0	3051.974	0	3095.640	0
3013.040	3	3052.445	1	3096.062	0
3013.172	0	3053.450	0	3096.672	6
3013.477	3	3055.042	4	3097.706	4
3014.312	0	3056.192	4	3098.954	0
3016.818	0	3056.877	0	3099.390	5
3017.356	5	3056.971	0	3100.953	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3104.070	0	3140.201	1	3189.835	3
3104.570	2	3140.596	3	3190.088	4
3105.382	2	3141.081	4	3191.303	1
3105.524	2	3143.764	0	3191.900	2
3105.910	0	3144.369	4	3192.171	2
3106.942	3	3144.820	2	3193.617	2
3107.373	0	3146.183	2	3195.137	0
3107.698	0	3147.323	2	3195.438	1
3107.829	3	3147.547	0	3196.718	4
3108.526	2	3148.138	0	3197.603	0
3110.147	0	3148.593	2	3198.437	2
3110.641	4	3150.283	1	3199.238	0
3112.012	3	3150.803	4	3201.372	2
3112.408	2	3151.780	1	3201.604	3
3112.782	2	3153.927	4	3202.703	2
3113.502	2	3154.543	2	3205.428	2
3113.756	0	3156.733	0	3207.751	0
3115.536	0	3156.917	2	3208.405	0
3116.945	1	3157.739	2	3208.542	3
3117.181	0	3159.003	4, Ca?	3208.865	1
3117.563	0	3160.036	4	3209.758	1
3118.170	4	3163.186	0	3210.287	2
3118.792	4	3164.939	0	3213.098	3
3120.650	1	3165.086	0	3214.475	2
3122.108	1 d?	3165.307	1	3215.613	0
3122.970	0	3165.507	0	3216.641	4
3123.610	0	3167.514	0	3219.274	1
3124.277	4	3168.355	1	3220.195	1
3124.481	2	3168.648	5	3220.899	2
3124.709	2	3170.196	2	3221.303	2
3126.068	4	3171.352	2	3221.493	1
3126.730	2	3172.778	0	3223.393	4
3127.387	1	3173.221	2	3223.723	0
3127.643	0	3173.500	2	3224.772	2
3128.539	2	3174.243	4	3225.418	0
3129.574	0	3176.401	3	3226.497	5
3129.717	2	3177.159	4	3227.016	2
3129.935	3	3178.843	1	3228.021	3
3130.709	0	3179.380	2	3228.276	2
3132.122	1	3180.569	0	3228.651	4
3132.988	4	3181.126	0	3228.850	0
3133.800	2	3181.312	0	3229.881	2
3134.895	1	3185.276	0	3230.738	2
3135.170	0	3185.553	2	3231.869	0
3136.044	2	3186.171	4	3232.180	1
3136.451	1	3186.867	1	3232.881	4
3136.663	3	3188.057	2	3233.650	0
3137.036	0	3188.463	5	3234.920	2
3138.884	2	3188.713	2	3235.230	2
3139.379	2	3189.418	2	3235.431	0

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3236.101	2	3272.366	0	3316.523	6
3238.132	0	3273.217	5	3317.045	1
3238.667	5	3273.765	0	3318.012	4
3238.904	2	3274.831	5	3318.992	6
3239.745	3, d	3276.820	0	3319.655	1
3241.362	4	3277.699	4	3319.944	1
3241.643	0	3279.521	2	3321.385	2
3241.884	0	3280.599	2	3321.634	0
3242.283	2	3280.678	1	3322.368	2
3242.978	2	3281.735	0	3323.226	4
3243.638	2	3281.995	2	3324.077	2
3244.475	0	3282.744	0	3324.509	0
3244.585	1	3285.067	4	3325.136	4
3244.719	0	3285.505	2	3325.373	2
3245.746	0	3286.040	1	3327.831	4
3246.380	0	3289.389	2, u	3328.583	2
3247.501	0	3291.250	2	3332.186	4
3248.977	2	3291.789	2	3332.483	0
3250.065	2	3292.390	2	3332.768	2
3250.146	2, Rh	3294.269	6	3334.764	0
3250.605	1	3294.926	0	3335.822	4
3251.464	3	3296.252	4	3336.296	2
3252.031	3	3296.786	2	3336.774	3
3252.400	0	3297.393	3	3337.963	4
3252.683	2	3298.096	3	3338.849	2
3253.038	1	3298.559	4	3339.092	0
3253.136	2	3299.479	2	3339.691	6
3254.674	4	3299.926	0	3339.932	2
3254.856	4	3301.726	5	3341.230	2
3255.173	0	3302.312	1	3341.361	1
3255.356	1	3304.141	4	3341.809	4
3256.477	4	3304.418	0	3342.854	0
3256.746	0	3304.634	2	3342.999	0
3258.176	3	3304.772	0	3344.666	4
3259.111	0	3304.948	2	3344.934	2
3259.811	4	3305.804	0	3345.450	4
3260.304	2	3306.305	4	3346.360	0
3260.494	5	3307.679	2	3347.748	4
3261.257	3	3308.122	4	3348.145	2
3263.740	0	3308.751	0	3348.833	2
3263.988	3	3309.965	0	3349.822	0
3264.692	3	3310.220	0	3350.236	2
3264.808	2	3311.090	4	3350.363	0
3266.588	4	3311.388	0	3350.681	2
3267.269	0	3312.068	0	3352.060	4
3268.345	5	3312.348	1	3353.122	1
3269.087	2	3314.203	2	3353.444	2
3269.336	2	3315.181	2	3353.776	4
3270.388	2	3315.365	3	3354.001	2
3271.746	0	3315.590	2	3355.803	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3356.327	2	3400.890	2	3444.574	1
3356.598	2	3401.304	0	3445.453	1
3358.110	0	3401.637	2	3445.675	0
3359.230	6	3401.878	3	3446.095	0
3361.295	2	3403.924	1	3446.227	2
3362.142	4	3405.426	0	3446.630	2
3362.457	2	3406.017	2	3449.105	4
3364.230	4	3406.736	2	3449.608	0
3364.933	1	3407.042	0	3451.014	0
3365.163	0	3409.420	5	3453.056	4
3365.470	0	3409.707	2	3453.373	0
3367.868	0	3411.768	4	3455.548	2
3368.053	0	3412.221	2	3455.888	2
3368.588	6	3412.947	3	3456.769	4
3369.433	2	3413.870	0	3457.849	0
3369.813	2	3414.130	0	3459.736	2
3370.720	2	3414.422	2	3462.208	2
3371.793	0	3414.787	3	3463.289	4
3371.990	4	3416.329	1	3463.751	0
3372.922	0	3417.493	7	3465.437	1
3374.115	2	3417.790	1	3467.190	2
3374.790	4	3418.125	2	3472.843	2
3375.036	2	3419.394	2	3473.900	5
3375.377	2	3420.243	4	3477.350	0
3376.186	1	3420.881	0	3480.295	2
3378.165	4	3422.578	2	3481.044	0
3379.402	2	3426.120	2	3481.465	4
3379.747	4	3427.717	0	3482.499	2
3380.301	4	3428.476	4, r	3483.317	2
3381.040	2	3428.790	2	3483.463	2
3383.053	0	3429.702	4	3486.360	2
3385.303	4	3430.568	0	3486.948	2
3385.609	2	3430.910	4	3489.895	1
3385.838	2	3431.905	0	3490.879	1
3386.390	2	3432.354	3	3492.256	1
3387.368	2	3432.560	0	3493.377	2
3387.967	0	3432.909	4	3494.410	3
3388.849	4	3433.406	4	3496.145	2
3389.250	0	3434.325	0	3496.293	2
3389.639	4	3435.340	4	3498.103	1
3391.042	2	3436.237	0	3499.098	10, r
3392.032	2	3436.481	2	3501.510	1
3392.654	4	3436.886	5, r	3502.578	2
3395.465	0	3438.522	4	3509.870	2
3396.060	0	3438.819	0	3513.807	2
3396.967	4	3439.835	2	3514.649	4
3398.470	0	3440.361	4	3514.911	1
3399.040	0	3441.942	0	3516.046	0
3400.116	0	3443.309	2	3519.795	3
3400.738	1	3443.818	0	3520.285	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3528.841	2	3637.614	4	3717.823	2
3531.545	3	3638.163	2	3719.474	4
3532.965	2	3640.791	4	3722.458	1
3535.529	2	3645.827	1	3724.110	4
3535.985	2	3646.266	3	3724.633	2
3538.100	3	3650.473	4	3725.115	4
3539.418	2	3652.465	3	3726.254	4
3539.518	2	3652.627	0	3727.077	4
3541.788	3	3652.816	0	3728.170	5
3547.136	1	3653.857	2	3730.587	7
3550.420	2	3654.559	4	3730.745	3
3554.002	1	3656.112	2	3731.045	2
3556.779	0	3657.315	2	3732.170	2
3557.203	0	3657.716	1	3733.187	2
3562.035	0	3660.964	3	3737.548	3
3564.517	0	3661.486	7	3737.904	2
3564.714	1	3661.727	2	3738.774	2
3564.945	0	3663.526	5	3739.058	2
3567.308	2	3668.890	1	3739.622	4
3570.743	2	3669.604	4	3742.435	5
3571.910	1	3671.363	2	3742.938	4
3574.744	3	3672.210	2	3744.367	2
3579.923	0	3672.525	2	3744.550	2
3587.344	2	3675.408	3	3746.372	2
3989.370	4	3676.817	3	3753.695	4
3591.044	1	3677.100	2	3755.241	3
3593.177	4, r	3678.140	2	3755.865	2
3596.315	5, r	3678.222	2	3756.083	4
3599.548	0	3678.465	4	3759.976	2
3599.913	4	3683.730	1	3760.178	4
3601.627	2	3685.204	2	3761.644	4
3605.792	3	3686.109	4	3764.179	1
3606.297	1	3686.742	1	3765.938	0
3608.862	2	3690.179	1	3767.500	4
3609.241	2	3693.740	2	3771.244	0
3614.486	1	3696.738	4	3773.306	0
3017.090	4	3697.921	3	3777.723	3
3619.334	4	3698.016	2	3781.313	3
3620.426	4	3700.487	1	3782.891	0
3623.804	4	3701.134	2	3786.193	5
5623.995	0	3701.457	2	3790.649	5
3625.345	5	3702.369	2	3795.327	0
3626.897	5	3703.344	2	3798.205	1
3627.425	2	3705.506	2	3799.040	4
3629.352	1	3712.443	3	3799.486	4, r
3631.860	3	3714.788	1	3812.874	3
3632.545	1	3715.703	3	3817.439	1
3934.063	4	3716.323	3	3819.184	2
3635.093	7	3716.583	1	3822.225	1
3635.661	4	3717.152	4	3825.075	1

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3828.859	0	3965.057	4	4068.529	4
3831.946	4	3969.936	2	4071.560	3
3838.215	0	3972.568	0	4073.147	2
3839.815	1	3974.646	4	4073.260	2
3857.689	1	3978.620	5	4076.900	5
3867.965	1	3979.591	4	4079.440	1
3872.386	1	3982.043	2	4080.777	7
3884.203	2	3982.372	3	4082.947	2
3884.849	2	3984.840	1	4085.567	5
3887.962	2	3985.011	5	4091.218	1
3890.350	4	3987.959	4	4097.185	2
3891.567	2	3989.344	2	4097.965	4
3892.366	4	3994.700	1	4100.533	2
3892.916	2	3996.136	4	4101.906	4
3894.387	2	3996.650	2	4102.438	2
3897.390	2	4005.789	4	4106.065	0
3898.500	3	4006.749	4	4108.003	4
3901.393	4	4007.680	3	4108.218	2
3906.141	1	4008.422	2	4109.796	0
3908.907	3	4011.882	2	4112.910	4
3909.229	5	4013.655	4	4113.532	2
3911.279	3	4013.871	2	4114.285	1
3912.248	3	4014.297	2	4118.678	2
3915.000	4	4018.891	1	4121.147	2
3919.711	0	4019.699	2	4121.287	2
3921.060	4	4021.146	3	4123.227	2
3922.476	1	4022.327	5	4127.611	2
3923.636	6	4022.837	2	4128.017	2
3924 776	2	4024.001	4	4137.410	3
3926.071	6	4024.449	2	4138.923	0
3926.581	0	4024.848	2	4144.335	4
3931.936	4	4026.650	1	4145.905	4
3932.444	0	4028.584	2	4146.956	4
3934.352	1	4031.147	3	4148.530	1
3938.045	3	4032.363	4	4150.475	1
3939.268	0	4032.650	1, u	4161.817	4
3941.811	3	4036.612	2	4167.030	0
3942.209	4	4037.892	2	4167.666	5
3944.341	2	4039.370	4	4170.218	2
3945.723	0	4040.620	2	4175.615	2
3946.456	2	4042.123	2	4182.621	2
3949.564	2	4049.570	2	4182.807	1
3950.192	2	4051.566	4	4182.994	0
3950.366	4	4052.356	4	4189.639	0
3950.548	3	4054.216	4	4197.038	2
3951.351	4	4063.021	1	4197.748	4
3952.436	1	4063.160	2	4199.039	4
3952.850	5	4064.262	2	4200.069	7
3957.376	0	4064.616	4	4206.178	4
3957.696	2	4067.777	4	4207.797	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4212.240	5	4315.219	4	4396.868	0
4214.610	4	4316.792	2	4397.956	4
4217.438	7	4318.596	4	4399.751	1
4220.838	4	4319.274	2	4405.809	0
4225.258	3	4320.045	5	4410.207	6
4226.825	0	4320.743	2	4412.058	0
4229.472	4	4320.972	0	4413.458	2
4230.470	6	4321.450	2	4414.607	1
4232.478	4	4323 120	2	4420.634	2
4236.838	4	4323.626	0	4421.006	4
4240.194	0	4325.215	4	4421.629	4
4241.231	6	4326.987	4	4423.143	1
4243.228	6	4327.489	2	4424.958	3
4244.997	4	4327.588	3	4426.182	1
4246.359	0	4328.712	2	4428.624	4
4246.522	4	4331.321	4	4430.478	1
4246.902	4	4332.655	2	4439.574	2
4248.304	2	4332.789	0	4439.938	5
4255.868	1	4336.584	2	4440.245	0
4256.049	0	4337.427	4	4444.674	4
4256.790	0	4338.829	2	4449.509	4
4259.152	5	4340.503	2	4460.209	6
4260.166	3	4341.204	2	4464.661	0
4263.551	2	4342.243	6	4465.649	1
4265.766	2	4343.178	0	4466.511	2
4266.157	0	4346.640	4	4467.427	2
4273.115	0	4349.868	5	4471.200	0
4277.415	2	4350.632	0	4474.093	4
4278.842	2	4354.300	6	4475.493	2
4282.093	2	4354.960	3	4480.603	4
4282.357	2	4357.031	1	4482.194	2
4284.502	6	4361.372	5	4488.550	4
4287.209	4	4361.581	2	4490.396	2
4290.692	2	4362.872	1	4491.846	2
4292.419	0	4364.270	2	4498.322	4
4293.441	4	4365.741	0	4508.192	1
4294.268	4	4370.580	2	4508.715	2
4294.955	5	4371.363	4	4510.251	4
4296.090	5	4372.381	5	4511.353	4
4296.860	2	4376.745	1	4516.421	2
4297.887	8	4381.421	2	4517.060	4
4301.297	1	4383.530	2	4517.977	4
4302.150	0	4385.563	5	4521.110	4
4307.748	4	4385.823	5	4525.616	0
4308.567	0	4386.431	4	4531.035	4
4309.361	2	4389.150	2	4542.848	1
4312.047	0	4389.547	0	4547.105	2
4312.632	2	4390.614	6	4547.463	4
4313.067	0	4391.191	4	4548.030	4
4314.468	4	4395.125	2	4549.589	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4550.112	3	4716.201	2	4902.033	0
4552.281	4	4718.228	0	4903.223	5
4554.696	6, r	4721.078	1	4905.179	1
4559.215	1	4731.504	3	4908.045	3
4560.157	4	4733.486	0	4910.384	0
4562.772	1	4733.710	4	4911.755	1
4564.862	2	4738.587	0	4921.233	4
4580.246	3	4743.205	1	4935.805	0
4584.632	4	4751.197	0	4938.587	3
4589.177	0	4753.280	0	4955.416	1
4589.734	0	4756.402	2	4960.022	0
4591.257	4	4758.043	6	4969.055	2
4591.717	2	4764.582	0	4974.255	0
4592.695	4	4767.315	0	4975.534	0
4593.161	0	4769.464	4	4976.351	2
4593.367	0	4773.325	0	4980.498	2
4596.879	4	4774.168	0	4987.412	1
4599.271	6	4781.937	1	4992.891	2
4601.933	3	4794.547	2	5003.697	0
4602.978	0	4795.721	2	5005.394	1
4605.833	2	4798.607	2	5010.765	1
4617.827	0	4801.343	1	5011.387	3
4626.184	1	4805.043	2	5019.140	1
4628.495	0	4806.375	0	5020.472	0
4635.849	4	4813.412	0	5026.343	3
4638.569	0	4814.895	0	5039.794	0, u
4639.490	0	4815.694	5	5040.521	1
4641.135	0	4817.512	1	5040.908	1
4642.548	1	4822.738	0	5041.528	0
4642.752	1	4828.865	0	5045.570	1
4645.264	4	4833.157	2	5047.471	2
4646.326	0	4839.174	3	5053.114	0
4646.967	0	4839.930	1	5057.487	4
4647.787	5	4844.720	4	5062.815	1
4648.293	0	4854.731	1	5073.141	2
4652.371	0	4862.024	2	5077.243	1
4654.489	4	4863.265	0	5077.484	3
4654.901	0	4865.253	1	5093.996	4
4662.663	0	4869.314	6	5101.553	2
4670.146	4	4869.952	1	5101.892	0
4674.821	4	4874.489	0	5107.230	4
4681.563	0	4875.188	0	5127.423	2
4681.966	4	4877.598	0	5134.059	2
4683.258	0	4882.832	0	5134.285	0
4684.196	4	4885.186	0	5136.717	5
4685.947	1	4895.474	1	5142.933	4
4690.284	4	4895.555	1	5147.401	4
4709.672	6	4895.745	4	5151.230	4
4712.146	1	4899.416	1	5153.364	2
4714.335	0	4901.234	0	5155.302	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
5160.167	2	5419.056	4	5653.005	0
5168.237	0	5427.815	4	5653.482	2
5168.793	0	5439.421	2	5657.127	2
5169.242	0	5439.618	2, u	5663.233	0
5171.193	6	5452.930	1	5665.370	4
5174.105	0	5455.018	6	5676.720	1
5176.361	0	5456.329	2, u	5679.790	4
5195.171	4	5471.755	0	5688.990	2
5200.040	3	5473.050	2	5692.288	1
5202.285	2	5475.377	2	5693.190	4
5209.667	2	5479.619	4	5694.626	2
5213.586	3	5480.507	3	5696.526	1
5214.247	1	5484.524	6	5699.224	9
5223.708	3	5484.850	2	5699.741	2
5235.774	1	5494.575	1	5702.522	4
5242.560	1	5596.899	4	5713.025	4
5243.109	2, u	5501.230	1	5714.391	2
5245.112	0	5507.151	0	5724.975	4
5245.612	2	5510.934	6	5725.895	4
5251.816	1	5512.593	2	5730.122	2
5257.240	2	5518.056	2	5734.606	0
5264.113	0, u	5531.220	2	5740.710	0
5266.642	1	5540.881	3	5745.776	1
5266.988	1	5549.960	2	5746.131	4
5275.240	1	5556.719	3	5747.623	5
5280.989	2	5559.962	6	5752.163	3
5284.256	4	5569.233	4	5753.772	1
5291.327	1	5570.906	2	5756.980	3
5305.030	4	5578.594	4	5758.875	0
5306.035	0	5578.914	2	5768.066	3
5306.624	1	5579.650	2	5771.352	0
5307.481	0	5582.501	2	5774.533	2
5309.440	4	5600.753	2	5782.511	2
5315.520	2	5603.370	2	5782.720	4
5333.114	3	5603.782	3	5790.741	1
5334.901	2, u	5606.958	3	5792.382	1
5336.110	3	5609.360	2	5804.461	4
5348.340	0	5619.558	0	5815.157	5
5361.967	5	5627.722	2	5826.018	0
5362.271	2	5629.984	1	5828.235	2
5365.799	2	5636.441	7	5828.580	1
5373.505	0, u	5641.848	2	5833.380	2
5378.042	3	5647.755	0	5833.561	0, u
5386.083	4	5648.058	1	5864.830	0
5401.234	5	5649.737	3	5887.371	0
5401.609	2	5650.981	2		

THE ABERRATION OF PARABOLIC MIRRORS.

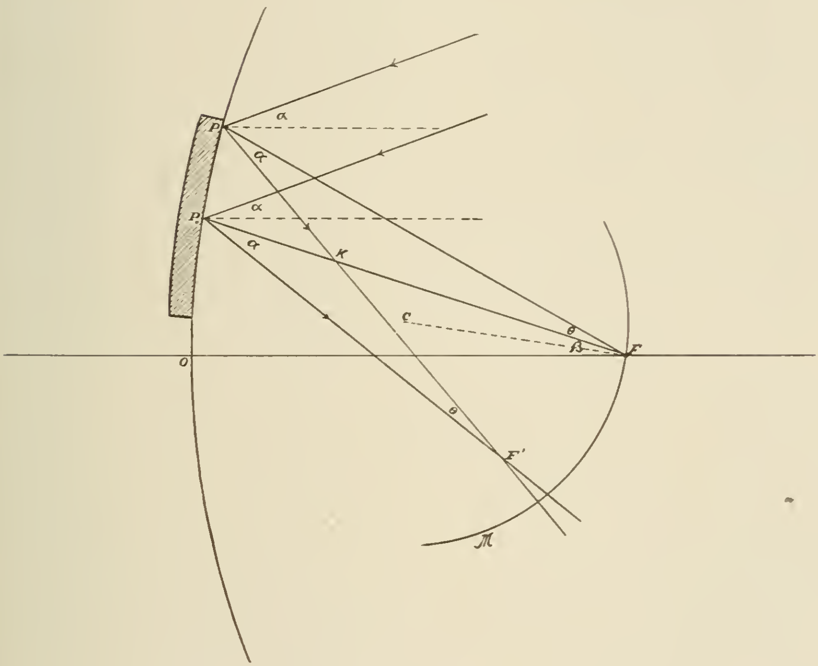
By Charles Lane Poor.

For some months I have been experimenting with parabolic mirrors for reflecting telescopes, being especially interested in attempts to grind and polish a new form of mirror, which promised, apparently, many distinct advantages over the ordinary form. These experiments led to an investigation of the theory of the optical properties of different portions of a paraboloid of revolution. To my knowledge no satisfactory investigation of these properties has been published. Astronomers and instrument makers have been satisfied with the fact that one portion of the field is theoretically perfect, and have assumed, apparently, that the field is sensibly flat and that the aberrations in different portions are insensible. These latter are far from being the facts, as is admirably pointed out in the paper by Professor Schaeberle in *Astronomical Journal*, No. 413.[1] The formulæ contained in his paper are approximate and are derived in an approximate and unsatisfactory manner. I have, therefore, put my results in shape and give them in the following paragraphs. These formulæ are rigorously derived and are accurate. They give rise to several interesting theorems concerning the field of a reflector and the various foci of different portions of the mirror.

Let the following figure represent a section of a paraboloid, OF being the axis and F the principal focus. Further let P_0 be the central point of a mirror formed from the paraboloid; β being the angular distance of the point, P_0, from the central point of the paraboloid: θ is the half angular aperture of the mirror.

[1] Professor Schaeberle does not seem to recognize that he is dealing with a simple case of what has hitherto been known in the theories of both lenses and mirrors as "aberration." The word, aberration, does not appear in his paper. He introduces a new and doubtful term, the so called "*blurring factor*."

Now put: f, the focal length of the parabola,
r_0, the radius vector drawn to P_0,
r, the radius vector drawn to P.



Suppose a beam of parallel light be incident upon the mirror and making an angle, α, with the axis of figure. The central ray of this beam is incident at P_0 and is reflected in the direction P_0F' making an angle, α, with the radius vector FP_0. The ray incident at P is reflected in the direction PF', making an angle, α, with the radius vector PF and intersecting the first ray at F'. The distance P_0F' is the focal length for the ray under consideration; call this distance f'. This distance will evidently be a function of α, β, and θ; and a general expression for it is derived as follows:

The two triangles PKF and P_0KF' are similar, hence their corresponding sides are proportional, whence:

$$f' = r\frac{P_0 K}{PK} \tag{1}$$

But in the triangle *PKF* we have:

$$PK = r\frac{\sin\theta}{\sin(\alpha+\theta)} \tag{2}$$

and

$$FK = r\frac{\sin\alpha}{\sin(\alpha+\theta)} \tag{3}$$

Again from the figure,

$$P_0K = r_0 - FK. \tag{4}$$

Substituting in equation (1) the values of PK and P_0K as found in (2), (3), and (4), we have:

$$f' = r_0\frac{\sin(\alpha+\theta)}{\sin\theta} - r\frac{\sin\alpha}{\sin\theta} \tag{5}$$

From the theory of the parabola we have:

$$r_0 = \frac{f}{\cos^2 \frac{1}{2}\beta} \tag{6}$$

$$r = \frac{f}{\cos^2 \frac{1}{2}(\beta+\theta)}$$

Substituting these values in equation (5) it at once takes the form:

$$f' = \frac{f}{\sin\theta\cos^2\frac{1}{2}\beta}\left[\sin(\alpha+\theta) - \sin\alpha\frac{\cos^2\frac{1}{2}\beta}{\cos^2\frac{1}{2}(\beta+\theta)}\right] \tag{7}$$

By writing,

$$\alpha = (\alpha+\theta) - \theta \tag{8}$$

and expanding the sine and substituting the result in (7), it may finally be put in the form:

$$f' = \frac{r_0}{2\cos^2\frac{1}{2}(\beta+\theta)}\left[\cos(\alpha+\theta) + \cos(\alpha+\beta+\theta) + \sin(\alpha+\theta)\tan\tfrac{1}{2}\theta\right] \tag{9}$$

And this is a rigorous expression for the focal length of oblique rays for a mirror cut from any portion of the paraboloid.

If in this equation we put α equal to zero, it reduces to,

$$f' = r_0$$

an expression independent of θ. This shows that when the incident beam is parallel to the axis of figure, the rays from all por-

tions of the parabola are brought rigorously to a single focus. This is, of course, the well-known property of the parabola.

If we put θ equal to zero our general equation becomes:

$$f_0' = \frac{r_0}{2 \cos^2 \frac{1}{2} \beta} \left[\cos \alpha + \cos (\alpha + \beta) \right] \qquad (10)$$

That is, as the angular aperture of the mirror is reduced, the focal length for oblique rays tends to the above as a limit. The locus of f_0', as α takes different values, is, therefore, the field of the mirror. For a given mirror, or what is the same for a given value of β, the locus of f_0', as given by the above equation, is a circle. If for a moment, we consider P_0 as origin and r_0 as the axis of X, then the coördinates of the center of the above circle and its radius, ρ, are given by:

$$\begin{aligned} x &= \tfrac{1}{2} r_0 \\ y &= - \tfrac{1}{2} r_0 \tan \tfrac{1}{2} \beta \\ \rho &= \tfrac{1}{2} r_0 \sec \tfrac{1}{2} \beta. \end{aligned} \qquad (11)$$

This circular field is shown on the diagram by the circle, FM, whose center is at C.

The field as actually used in any telescope is flat and is the plane tangent to the true field at the focus, F. A photographic plate is plane and is usually placed so that F is the center of the apparent field. The tangent plane at F is perpendicular to the radius CF, and, therefore, this plane is inclined at an angle, $90° - \frac{1}{2}\beta$, to the ray of light incident at its center F. For small values of β this probably does no harm, but for large values of β it renders the mirror useless for many astronomical purposes.

From the figure, as the triangles PKF and P_0KF' are similar, the four points, P_0, P, F, and F' lie on the circumference of a circle. This gives the following theorem:

"For incident light, oblique to the axis of figure, the intersection of the central ray with the ray from any point, P, of the parabola, lies on the circumference of the circle passing through the center of the mirror, the principal focus, and the reflecting point, P, of the mirror."

ABERRATION.

The aberration in different portions of the field of the parabolic section may be found rigorously in the following manner:

For oblique light the linear aberration in the direction of the central ray will be:

$$\Delta f' = f' - f_0'.$$

Substituting in this the values of f' and f_0', as given by (9) and (10), we have at once:

$$\Delta f' = \frac{r_0}{2\cos^2 \frac{1}{2}(\beta+\theta)} \Big[\cos(\alpha+\theta) + \cos(\alpha+\beta+\theta) + \sin(\alpha+\theta)\tan \tfrac{1}{2}\theta\Big]$$
$$- \frac{r_0}{2\cos^2 \frac{1}{2}\beta} \Big[\cos\alpha + \cos(\alpha+\beta)\Big].$$

Expanding the sines and cosines contained in the above parentheses, and simplifying the resulting expression, we shall finally obtain:

$$\Delta f' = -\frac{r_0 \sin\alpha \sin\theta \,[2 + 2\cos\beta + \cos\theta + \cos(\theta+\beta)]}{8\cos^2 \frac{1}{2}\beta \,.\, \cos^2 \frac{1}{2}\theta \,.\, \cos^2 \frac{1}{2}(\beta+\theta)} \qquad (12)$$

And this is the rigorous expression for the linear aberration of an oblique ray. We note in the above that the parenthesis and the denominator are both functions of β and θ alone. Calling these P and D respectively, we may write:

$$\Delta f' = - r_0 \sin\alpha \sin\theta \frac{P}{D} \qquad (13)$$

Now let σ represent the transverse linear aberration in the circular field, and we shall have:

$$\sigma = \Delta f' \tan\theta.$$

The injurious effect of the aberration is proportional to the angle which σ subtends at the center of the mirror, and calling this angle γ, we have finally:

$$\tan\gamma = \frac{\sigma}{f_0'}$$
$$= \frac{\Delta f'}{f_0'} \tan\theta.$$

Substituting in this the values of $\Delta f'$ and f_c' and reducing the result, we have

$$\tan \gamma = \frac{\sin \alpha \tan \tfrac{1}{2}\theta \tan \theta}{2\left[\cos \alpha + \cos (\alpha + \beta)\right] \cos^2 \tfrac{1}{2}(\beta + \theta)} \cdot P. \qquad (14)$$

And this is an expression for the angular size of a star disk in any portion of the field.[1]

If in this expression we put α equal to zero, then we have at once γ equal to zero; that is, the image of a star in the direction of the axis of figure is a point. And this is true for a mirror cut from any portion of the paraboloid; it is the well-known property of the parabola.

The equations so far derived are perfectly general, applying to all portions of the paraboloid. The properties of different portions may now be investigated by giving β proper values.

THE ORDINARY FORM OF MIRROR: β = 0.

If in our general equations we put β equal to zero, then the center of the mirror will coincide with o, and the mirror will be symmetrical with respect to the axis of figure of the paraboloid. This is the form in which nearly all mirrors for astronomical work have been made. Equations (9), (10), and (14) respectively reduce to:

$$f' = f \sec^2 \tfrac{1}{2}\theta \left[\cos (\theta + \alpha) + \tfrac{1}{2} \tan \tfrac{1}{2}\theta \,.\, \sin (\theta + \alpha) \right]$$

$$f_o' = f \cos \alpha$$

$$\tan \gamma = \tan \alpha \tan \theta \tan \tfrac{1}{2}\theta \left[1 + \frac{1}{2 \cos^2 \tfrac{1}{2}\theta} \right].$$

The second of these equations shows that the field is circular, and that the diameter of the circle is the focal length of the mirror. The complete mirror is formed by revolving the sec-

[1] The above formula gives the angular diameter of the star image measured in the plane passing through the axis of the paraboloid. The image will not be circular, although for stars near the center the images will not differ greatly from circles. The equations show that the light will not be uniformly distributed over the image. Professor Wadsworth has kindly given me valuable suggestions on this point, and I hope to discuss this subject in a subsequent paper.

tion about the axis of figure; hence the entire field will be spherical with f as diameter. The field used for photography, or for visual purposes, is plane; hence, even if the star images in the true field be points of light, their projections on the plate or in the eyepiece will be disks. The images in the true field are disks, and these will be, therefore, enlarged before they reach the flat plate. In every case the curvature of the field increases the injurious effect of the aberration.

The expression for γ gives the greatest diameter of a star disk in any portion of the true field. A very convenient and close approximation to it may be made by putting

$$\cos\theta = \cos \tfrac{1}{2}\theta - 1\,.$$

Doing this and also expressing the angle α in seconds of arc, we have

$$\gamma = \frac{3}{4}\frac{a^2}{f^2}(\alpha'') \qquad (15)$$

where a is the half linear aperture of the mirror. The size of the star disk is thus directly proportional to its angular distance from the center of the field and to the square of the proportions of the telescope (aperture divided by focal length).

The annexed table shows the results as obtained above and applied to reflectors of different proportions. The first column gives the ratio of aperture to focal length; the next two columns the size of a star disk on the field of least aberration for various distances from the center of the field; the final column gives the real size of a star disk as it would appear on a photographic plate and distant one degree from the center. This column contains the combined effect of aberration and curvature of the field (see table on next page).

As a result of this we see that in order to obtain satisfactory results the proportions of mirrors should be about the same as those of lenses: the ratio of aperture to focal length should be 1 to 15 or 20. Good definition cannot be obtained with the mirrors now in use, *i. e.*, those in which this ratio is 1 to 7 or 8.

TABLE I.[1]

Aperture. Focal length	Aberration for 5′	Aberration for 30′	Star disk 1° from center
1/5	2″.16	12″.95	33″.3
1/6	1 .54	9 .24	23 .9
1/7	1 .14	6 .82	18 .2
1/8	.87	5 .24	14 .4
1/10	.55	3 .30	9 .8
1/12	.39	2 .37	7 .3
1/15	.25	1 .48	5 .1
1/20	.14	.84	3 .2

UNSYMMETRICAL FORM: $\beta = 10°$ OR LESS.

This form has been suggested once or twice, and Dr. Draper constructed one or two such mirrors. These, however, were the results of accidents in grinding and polishing, and not the results of attempts to make such mirrors. In these cases β was about two degrees.

The general equations (9) (10) and (14) now apply and in these β must be given the proper value for each special case.

We note first that the field is circular and slightly flatter than in the ordinary form. But the field is now inclined, and to obtain the best results the plate should not be perpendicular to the central ray, but inclined to it at an angle of $90° - \frac{1}{2}\beta$. This inclination of the field will cause a distortion of the star images, but this effect will be very small for small values of β.

[1] To test the accuracy of these results a number of measures were made on a print of Roberts' photograph of the Orion nebula, taken December 18, 1886, with an exposure of fifteen (15) minutes. The star disks at various distances from the center were measured, stars of the 9.3 and 9.4 magnitudes alone being used. The images increase in size from center outward, and this increase is shown below, together with the approximate aberration as computed from my formula:

Distance from center of photograph	Measured increase in star image	Computed aberration
12′	6″	6″
27′	9″	12″
34′	13″	15″
42′	19″	19″

The amount of the aberration for mirrors of different proportions and for different values of β can be computed from equation (14). Results of such computations are contained in the annexed table. Mirrors of four different ratios are compared and the sizes of the star images computed for different values of β, from 0° to 10°. In each and every case the greatest diameter of the star disk $30'$ from the center of the field is given: the first column, $\beta = 0$, gives the size of star disk in the ordinary form of mirrors.

TABLE II.

Aperture. / Focal length	$\beta = 0^\circ$	$\beta = 2^\circ$	$\beta = 4^\circ$	$\beta = 6^\circ$	$\beta = 10$
1/ 8	9″.24	9″.26	9″.28	9″.30	9″.37
1/ 8	5 .24	5 .25	5 .26	5 .27	5 .31
1/10	3 .30	3 .31	3 .31	3 .32	3 .34
1/20	.84	.84	.84	.85	.86

From this table we see that such unsymmetrical mirrors perform nearly as well as the ordinary form. Thus two mirrors whose apertures are one eighth their respective focal lengths, the first symmetrical and the second cut at a point 10° from the axis, will give star images of $5''.24$ and $5''.31$ respectively. If the proportions are made one to twenty, then the oblique mirror will perform as well as the mirror of the usual type.

Such an oblique mirror would offer several advantages. In the first place it could be made of much greater focal length than the ordinary form, and again it would do away with the troublesome diffraction caused by the supports of the secondary mirror, or the photographic plate, as used in the ordinary form.

NINETY-DEGREE FORM: $\beta = 90^\circ$

This form was suggested some years ago by Professor Pickering. It offers many distinct advantages, especially in the resulting form of equatorial mounting; and this led me to an attempt to grind and polish a small mirror of this type. These experiments were described at the astronomical conferences

held in connection with the dedication of the Yerkes Observatory.

To investigate this form put β equal to 90° in the general equations and they will reduce to

$$f' = \frac{r_0}{1 - \sin\theta}\left[\cos(\alpha+\theta) + \sin(\alpha+\theta)(\tan \tfrac{1}{2}\theta - 1)\right]$$

$$f_0' = r_0 (\cos\alpha - \sin\alpha)$$

$$\tan\gamma = \frac{\tan\theta}{1 - \cos\alpha}\left[\tan \tfrac{1}{2}\theta + \tan\theta \tan(45° + \tfrac{1}{2}\theta)\right].$$

From the second of these we see that the field is circular and that the coördinates of the center of the field are, $\frac{1}{2}r_0$ and $-\frac{1}{2}r_0$. The field is therefore inclined at an angle of 45° to the central ray and to the axis of figure. It is, however, flatter than the field of the usual form, the radii of the circles for similar mirrors of the two types being in the ratio of 1 to $\sqrt{2}$. The inclination of the field renders this proposed form of mirror useless for many astronomical purposes, for only one portion of the field can be brought into focus with the eyepiece at any one time; but for some astrophysical investigations it may prove of value.

Again, from our final equation we find that the aberrations of a mirror of this form are very much larger than those of the ordinary symmetrical form. For a mirror of ratio one to ten the aberration at a distance of 30′ from the center of the field will be 6″.9 as against 3″.3 for the ordinary form.

JOHNS HOPKINS UNIVERSITY,
November 30, 1897.

ON THE CAUSES OF THE SUN-SPOT PERIOD.

By E. J. Wilczynski.

I.

In my inaugural dissertation, "Hydrodynamische Untersuchungen mit Anwendungen auf die Theorie der Sonnenrotation," Berlin, 1897, I have already mentioned briefly a very simple mechanical explanation of the Sun-spot period.

If every point of the fluid matter of which the Sun consists describes a circle, and taking into account the internal fluid friction, the following equations were found :[1]

$$-\omega^2 r = f\frac{\delta V}{\delta r} - \frac{1}{\rho}\frac{\delta p}{\delta r}\,, \qquad o = f\frac{\delta V}{\delta z} - \frac{1}{\rho}\frac{\delta p}{\delta z} \tag{1}$$

$$\frac{\delta \omega}{\delta t} = \frac{k}{\rho}\left(\frac{\delta^2 \omega}{\delta r^2} + \frac{3}{r}\frac{\delta \omega}{\delta r}\right) + \frac{\delta k}{\delta r}\frac{\delta \omega}{\delta r}\,, \tag{2}$$

where the axis of z is the axis of rotation, and $r = \sqrt{x^2 + y^2}$ is the distance from the axis. ω, ρ, p, k, V, denote respectively the velocity of rotation, density, pressure, coefficient of friction, and potential for the point whose coördinates are x, y, z. f is a constant depending only upon the units employed. If the surfaces of constant density and those of constant pressure coincide ω is a function of r and t only.

Our general view of the case is simply this: ω can be any function of r which verifies the above equations. The special form of this function has been determined by the initial conditions in the original solar nebula, just as the elements of a planetary orbit were determined by the position and by the magnitude and direction of the planet's velocity at the time of its formation. That the whole Sun should rotate as a solid body is then just as improbable as that a planet's orbit should be an exact circle.

[1] "Hydrod. Unters.," p. 7, of this Journal, 4, 101, August 1896, where equations (1) are also deduced. An error in this proof was corrected by Harzer, *A. N.*, 3386.

The tendency of friction is towards a rigid rotation of the entire body, *i. e.*, $\omega =$ const. But besides this secular change in ω there can be periodic changes, say with an eleven-year period. In order to demonstrate this we will not take the most general case, it being easy to see that our conclusions will admit of being applied to cases which are not covered by the rigid demonstration.

In equations (1) any function of r and t can be put for ω, so that these equations need not be investigated. In order to simplify equation (2) so as to enable us to study its solutions, we will put ρ and k equal to constants, corresponding to the case of an incompressible homogeneous fluid. Then (2) becomes

$$\frac{\delta \omega}{\delta t} = a^2 \left(\frac{\delta^2 \omega}{\delta r^2} + \frac{3}{r} \frac{\delta \omega}{\delta r} \right) \tag{3}$$

if

$$\frac{k}{\rho} = a^2. \tag{4}$$

Attempt to solve (3) by putting

$$\omega = e^{\phi(r)\, t + \psi(r)}$$

This will verify (3) if[1]

$$\phi(r) = a^2 \left[\phi''(r)\, t + \psi''(r) + \{ \phi'(r)\, t + \psi'(r) \}^2 + \frac{3}{r} \{ \phi'(r)\, t + \psi'(r) \} \right]$$

for all values of r and t. But this is only possible if the coefficients of like powers of t on the left and right hand member are equal. Hence

$$\begin{cases} \phi(r) = a^2 \left[\psi''(r) + \psi'(r)^2 + \frac{3}{r} \psi'(r) \right] \\ 0 = \phi''(r) + 2\, \phi'(r)\, \psi'(r) + \frac{3}{r} \phi'(r) \\ 0 = \phi'(r)^2 . \end{cases}$$

The last two equations are satisfied for $\phi(r) = c$, where c is any real or complex constant. Then ψ is determined by the equation

[1] $\phi'(r)$, $\phi''(r)$, etc., denote the first and second derivatives as usual.

$$\frac{d^2\psi}{dr^2} + \left(\frac{d\psi}{dr}\right)^2 + \frac{3}{r}\,\frac{d\psi}{dr} = \frac{c}{a^2} = q\,.$$

Put $\psi = \log u$, then

$$\frac{d^2 u}{dr^2} + \frac{3}{r}\frac{du}{dr} - q\,u = 0 \tag{5}$$

a well-known equation, closely related to Bessel's.

Now let c_k be any constant, let u_{k1}, u_{k2} be the independent solutions of (5), if instead of q we put $q_k = \frac{c_k}{a^2}$. Then the corresponding complete solution of (5) is

$$u_k = a_{k1}\,u_{k1} + a_{k2}\,u_{k2}$$

where a_{k1} and a_{k2} are arbitrary constants. A very general solution of our equation for ω will then be

$$\omega = \sum_{k=1}^{\infty} (a_{k1}\,u_{k1} + a_{k2}\,u_{k2})\,e^{c_k t}\,. \tag{6}$$

The real parts of c_k must not be positive since ω must be finite for $t = \infty$. If the real part is zero, the imaginary part must also be zero, since for $t = \infty$ we must assume ω to have assumed a value ω_∞, which is independent of t, and also of r, *i. e.*, it is the same all through the entire body, so that ω_∞ is an absolute constant. We can therefore put

$$\omega = \omega_\infty + \sum_{k=1}^{\infty} (a_{k1}\,u_{k1} + a_{k2}\,u_{k2})\,e^{c_k t} \tag{7}$$

where the real parts of the c_k's are all negative. If the quantity on the right member be a complex quantity, we must put ω equal to the real part. We will then have ω given as a series, all of whose terms have the following form :

$$A\,u\,e^{-c^2 t}\cos\lambda t \qquad \text{or} \qquad A\,u\,e^{-c^2 t}\sin\lambda t \tag{8}$$

where A, c, and λ are real constants, and u is a function of r verifying (5). The period of such a term is of course

$$\frac{2\,\pi}{\lambda}$$

after which time the term will again have its original value multiplied by

$$e^{-c^2 \frac{2\pi}{\lambda}}$$

which can be made as nearly equal to unity as may be required by simply choosing the arbitrary c small enough. Of course λ can be so chosen that the period is eleven years. But this will determine the value of q which occurs in (5). The solution of this equation gives u_1 and u_2, and thus will be obtained as a part of the expression for ω the real part of

$$(a_1 u_1 + a_2 u_2)\, e^{(-c^2 - \lambda\sqrt{-1})\,t}$$

If this does not represent the observations, another term must be taken, etc.

But far more important than this would be the solution of the following problem: For $t = 0$, let $\omega = \omega_0$ an arbitrary function of r, be given. Equation (3) being of the first order in t, ω is then determined for all values of t. Then, letting ω_0 represent the present law of solar rotation, the future variations of ω could be deduced and compared to the facts of solar physics, as for instance the eleven-year period of solar activity. Thus important conclusions could be drawn.

II.

We will not on this occasion enter upon the discussion of this problem. It suffices for our purpose to have shown the possibility that ω as a function of t may contain periodic terms. Now from (1) we have

$$\omega^2 = \frac{1}{r}\left(\frac{1}{\rho}\frac{\delta p}{\delta r} - f\frac{\delta V}{\delta r}\right)$$

and putting

$$p = c\rho T$$

according to the law of gases, where T denotes the absolute temperature in centigrade degrees, and where c is a constant, we obtain

$$\omega^2 = \frac{1}{r}\left[c\frac{\delta T}{\delta r} + c\frac{T}{\rho}\frac{\delta \rho}{\delta r} - f\frac{\delta V}{\delta r}\right]. \tag{9}$$

Now, suppose ω^2 to be a periodic function, then the bracket on the right-hand member must also have the same period. We will, moreover, assume that the density remains comparatively constant everywhere. Then T must be periodic, so that there must be a periodic change of temperature, and it is easy to see that this may have great influence upon all solar phenomena. If such a period of eleven years exists, this would seem to account for the observed periodicity of solar activity. But this is not the only cause acting in this manner. In assuming ω to vary, we still retain the assumption that every point describes a circle, although with varying velocity. But some observations seem to speak for the existence of regular currents toward and from the solar equator, and probably there are still other motions combining with these. We only treat of the periodic changes in ω, because of the mathematical difficulties of the more general case, and because this suffices to indicate the essential principles involved.

It will, perhaps, be remarked that any such periodic changes ought to have been noticed if they are great enough to have such important consequences. But while it is almost certain from Auwers' investigations that the diameter of the Sun is not subject to great variations, the same cannot be said of the rotation law. This is an important subject for future investigation. An estimate which we will now begin to make of the magnitude of these variations, which would be necessary to explain the Sun-spot period, allows us to hope that we may, perhaps, be able to find these variations with our instrumental means.

Let T' and ω' be the values of temperature and velocity of rotation for the time t', if T and ω represent these quantities at the epoch t. Then assuming ρ and V to be unaltered

$$\omega'^2 - \omega^2 = \frac{1}{r}\left[c\,\frac{\delta\,(T'-T)}{\delta r} + \frac{c}{\rho}\,\frac{\delta\rho}{\delta r}\,(T'-T)\right].$$

We shall now, in our ignorance of solar temperature conditions, be forced to assume that when the temperature changes from T to T' it changes by the same number of degrees everywhere, so that

$$\frac{\delta\,(T'-T)}{\delta\,r} = \frac{\delta\,(T'-T)}{\delta\,z} = 0\,.$$

Then

$$\omega'^2 - \omega^2 = \frac{c}{r}\,\frac{1}{\rho}\,\frac{\delta\rho}{\delta r}\,(T'-T)\,.$$

$\dfrac{\delta\rho}{\delta r}$ is certainly very small near the surface. In order to be able to compute this expression, I will assume, as I have done in my thesis, that the density is inversely proportional to the square of the distance R from the center, so that

$$\rho = \frac{\sigma}{R^2}\,,\quad R^2 = r^2 + z^2\,,$$

where σ is a constant. Then

$$\omega'^2 - \omega^2 = -\frac{2\,c}{R^2}\,(T'-T)\,.$$

If $T' > T$, this gives $\omega'^2 < \omega^2$, *i. e.*, under these conditions a slower rotation corresponds to a higher temperature.

Suppose the gas to be hydrogen at the point where we are investigating its motion. For 0°C. and atmospheric pressure, *i. e.*, in absolute units, for $T = 273$, $p = 1013650$ dynes per square centimeter we have the density of hydrogen

$$\rho = 0.00008998 = 8998 \cdot 10^{-8}\left[\frac{gr}{cm^3}\right].$$

Since $p = c\,\rho\,T$ we find

$$c = \frac{1013650}{273 \cdot 8988}\,10^8\,.$$

The Sun's radius in centimeters is $R = 7.10^{10}$, so that

$$\omega'^2 - \omega^2 = -17.10^{-15}\,(T'-T)$$

Let us suppose $T^1 - T = 100$, a change of temperature which may well be supposed to cause a considerable change in solar phenomena. Then

$$\omega^2 - \omega^{2\prime} = 17.10^{-13}$$

or

$$\omega - \omega' = \frac{17.10^{-13}}{\omega + \omega'} = \frac{17.10^{-13}}{2\,\omega}$$

if $(\omega-\omega^1)^2$ is neglected. Taking the equatorial rate of 862′ a day,

$$\omega = \frac{862}{86400} = 1'.10^{-2} = 3.10^{-6} \text{ radians per second.}$$

since $1' = 3.10^{-4}$ in a circle of radius unity. Therefore in one second

$$\omega - \omega' = \frac{17.10^{-13}}{6.10^{-6}} = 3.10^{-7}$$

and in one day

$$\omega - \omega' = \frac{864 \cdot 3.10^{-5}}{3.10^{-4}} = 86.'4$$

which is a very great difference between ω and ω^1. But if we remember that, in the regions where the spots are found, it is probably not hydrogen that plays the principal part, but the much denser metallic vapors, that the value we have taken for $\frac{\delta \rho}{\delta R}$ is almost certainly much too great, we shall find that the value of ω–ω' just found is about a hundred times greater than that which we would have to expect upon the Sun. In fact, if the vapor is iron, we must divide the above number at least by 56. We can thus hardly expect differences of more than $1'$ in the daily arcs described by any point at different times. Moreover, we have only taken into account the variations in ω, while other disturbances of the regular circular motion may be much more important.

At all events it will be seen that the Sun-spot period does not appear quite as mysterious as it has done. We can even see how this may influence terrestrial magnetism. For, if the Sun, as many think, is a magnet or electromagnet, changes in the distribution of the Sun's temperature and density cannot but affect the magnetic or electromagnetic field surrounding it.

Much work is needed to decide whether the theory here offered is sufficient. The exact motion of the solar phenomena in latitude as well as in longitude must be investigated, and all the details of the rotation law must be closely studied. The rotation law is the key to a scientific theory of the Sun.

CHICAGO, August 30, 1897.

RADIATION IN A MAGNETIC FIELD.

By A. A. Michelson.

Further analysis of the radiations emitted in a magnetic field shows that the phenomenon is much more complex than was supposed. An examination of the separate components of the "triplet" brings out the fact that in general these are multiple lines. The laws may be summarized as follows:

A.

1. All spectral lines are tripled when the radiations emanate in a magnetic field.

2. The separation is proportional to the strength of field and is approximately the same for all colors and for all substances.

3. Viewed in a plane perpendicular to the magnetic field the outer lines are polarized parallel with the field, and the central line perpendicular to the field.

4. Viewed in the direction of the lines of force, the central line vanishes, while the outer ones are circularly polarized, the component of shorter wave-length in the direction of the magnetizing current, the other in the opposite sense.

To these laws (which were verified by the examination of a dozen or more lines) the following must now be added.

B.

1. The "middle line" is a symmetrical triple, the distance between the components being one-fourth that of the "outer lines," and hence, also proportional to the strength of field.

2. The relative intensity of the components varies for different substances, and for different lines of the same substance; and accordingly the group may appear as a single line, or a double, or a triple.

3. The "outer lines" are unsymmetrical, but are symmetrically placed with respect to the "central line." The distance

between the components is usually one-fourth that between the "outer lines," but is in some cases one-sixth.

4. The intensity of the components varies for different spectral lines and these variations do not always correspond to those of the "central line." The outer groups may accordingly appear as single or double or triple or multiple lines.

FIG. 1 represents a plan of the arrangement of apparatus employed in the investigation. *S* is the source of light, either

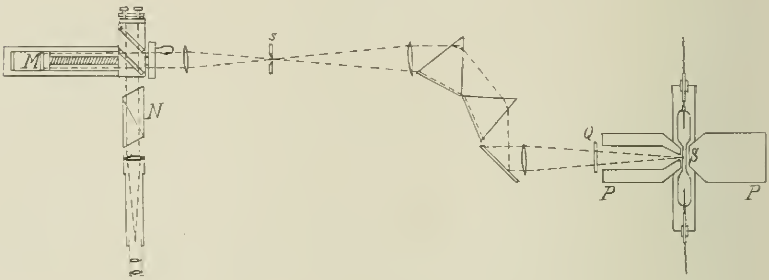


FIG. 1.

a small hand-blowpipe with a bead of the substance to be examined in the flame, or a vacuum tube which is usually placed in a metal box, for heating, of such form as to permit a close approach of the pole-pieces *P*, of an electromagnet. One of these is bored out to permit examination of the axial ray. The light from *S* undergoes a preliminary analysis by the spectroscopic train (two bisulphide prisms) the radiation to be investigated being isolated by the slit *s*. It then enters the interferometer, one of the mirrors of which, *M*, is moved on ways so accurately ground that no readjustment is necessary in any part of its path, that is, the mirror remains so nearly parallel with itself that the interference fringes (concentric circles) are always as clear as possible. The emergent beam then passes through the analyzer *N* to the observing telescope.

The clearness or "visibility" of the interference fringes is estimated at positions of the mirror *M* corresponding to increments of the difference of path of one, two, or five millimeters,

according to the nature of the curve. This, it must be admitted, leaves much to be desired in the way of precision, and in some cases there may be corrections of as much as 20 per cent. to reduce the observations to the value they should have, namely, $V = I_1 - I_2 / I_1 + I_2$, where I_1 is the maximum intensity and I_2 the minimum, for adjacent fringes. Doubtless much more accurate readings could be obtained by the use of comparison fringes,[1] but the process is so much more tedious that the form of the curve is liable to alter during the observations. The case is somewhat analogous to eye estimations of stellar magnitudes, which are but little inferior to photometric determinations and much less troublesome.

In any case it is always easy to distinguish ascending and descending slopes, and maxima and minima can be located with very great accuracy, and this is usually quite sufficient to permit a fairly accurate deduction of the distribution of light in the spectrum. It has been shown[2] that with the definition of visibility just given, if $y = \phi(x)$ is the intensity curve of the spectrum

$$P\,V = \sqrt{C^2 + S^2} \text{ in which}$$

$$P = \int \phi(x)\,dx,\ C = \int \phi(x) \cos kx\,dx, \text{ and } S = \int \phi(x) \sin kx\,dx,$$

the integration extending over the spectrum.

But by Fourier's formula

$$\phi(x) = \int_0^\infty C \cos kx\,dk + \int_0^\infty S \sin kx\,dk$$

so that if C and S are both known $\phi(x)$ can be determined. In general this is not the case unless another relation between C and S is given. Such a relation is furnished by the "phase curve" which gives the displacement of the fringes from the position they would occupy had the source been homogeneous. If δ is this displacement and $\theta = 2\pi\delta/\lambda$ then

$$C = V \cos\theta \text{ and } S = V \sin\theta .$$

[1] See *Phil. Mag.*, September 1892. [2] *Ibid.*

In general the θ curve is troublesome to observe on account of the difficulty in securing a sufficiently homogeneous comparison source, but in the present instance this is furnished by the non-magnetized radiations.[1] Usually, however, the assumption was made that the spectrum was symmetrical, and in only a few cases was the solution verified by the complete analysis. In this simpler form we have

$$\phi(x) = \int_0^\infty V \cos kx \, dk.$$

This integral may frequently be calculated when V can be expressed in simple analytical form as a function of k. In general this is not the case, and it was for the solution of such problems that the harmonic analyzer[2] was devised. The curve, $V = f(k)$ is "fed" to the machine, which then draws the curve $y = \phi(x)$, the whole operation taking but a few minutes.

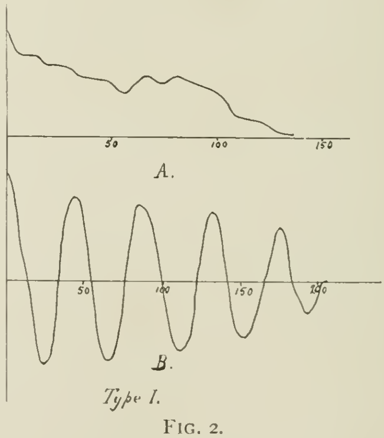


FIG. 2.

It was found on completing the analyses of some fifty or more visibility curves, that the resulting spectra could be classified under three types; there were some interesting variations which would merit a separate investigation, but most of the cases could be identified at a glance.

[1] These are not always sufficiently simple, as in the case of the green thallium line.

[2] *Phil. Mag.*, January 1898.

The three types of visibility curve are given in Figs. 2, 3, and 4. Those marked *A* referring to observations made with the

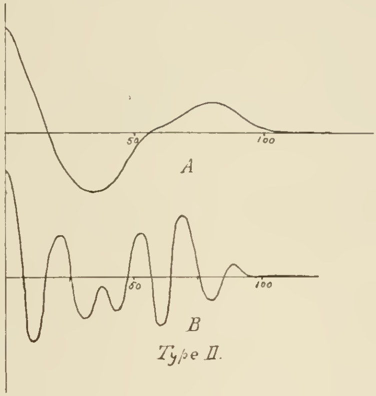


FIG. 3.

line of sight at right angles with the magnetic field and with the plane of polarization perpendicular to the lines of force; while *B* correspond to observations with the line of sight still

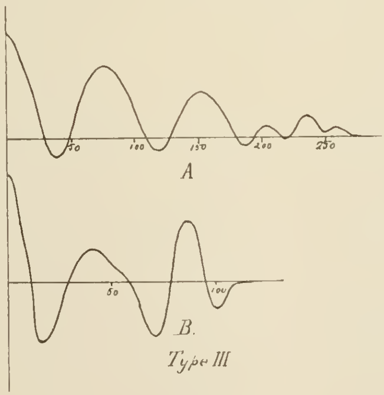


FIG. 4.

normal to the field, but plane of polarization parallel with the lines of force.

It was found that there was no appreciable difference between these last, and the observations with the line of sight parallel

with the field; but in this case it was possible to analyze either one of the outer groups separately, by the use of the quarter-wave plate *Q*, Fig. 1. This was done in a few cases, but no new result was obtained.

The abscissæ of the visibility curves are differences of path in millimeters, reduced to a field strength 10,000, as determined by a bismuth spiral.

Fig. 5 gives the intensity curves of the corresponding spectra, the abscissæ being in tenth-meters.[1]

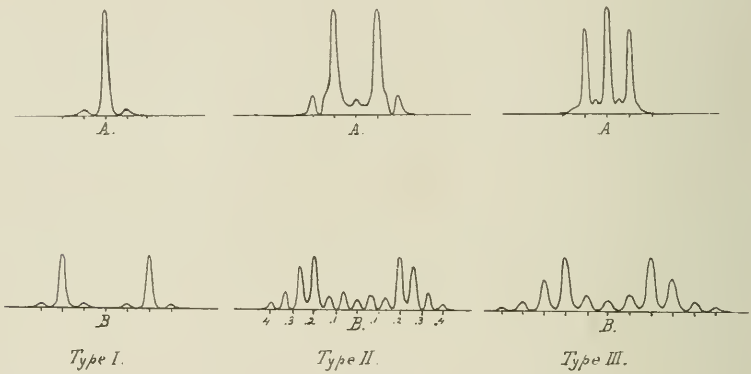


FIG. 5.

The following is a list of the radiations and their classification.

Mercury:	yellow lines	type I
	green line	type III
	violet line	type II
Cadmium:	red line	type I
	green line	type III
	blue line	type II
Zinc:	red line	type I
	green line	type III
	blue line	type II

[1] For this the abscissæ of the curve drawn by the analyzer are multiplied by the square of the wave-length.

Sodium:	yellow lines	type II
Thallium:	green line	type II (doubtful)
Lithium:	red line	broadened
Hydrogen:	red line	broadened
	blue line	broadened
Helium:	yellow line	broadened
	green line	broadened

The following table shows that the law *A* 2 is only approximately true. In fact, owing to the complexity of the spectra there is considerable latitude in the choice of the distance between the outer groups. If this correspond to the brightest components the law can hardly be said to hold at all; but if the distance be taken between the centers of gravity of the light areas, a fair agreement is found. The table gives separation in tenth-meters for a field 10,000. The lines marked with an asterisk are less accurate than the others on account of broadening.

* Hydrogen	red	0.48
* Lithium	red	0.60
Cadmium	red	0.42
Zinc	red	0.42
Mercury	yellow	0.36
* Sodium	yellow	0.50
* Helium	green	0.37
Mercury	green	0.40
Cadmium	green	0.41
Zinc	green	0.40
* Thallium	green	0.36
Cadmium	blue	0.40
Zinc	blue	0.33
Mercury	violet	0.33

Taking into account the uncertainty alluded to, the results show on the whole a fair agreement, from which it may be concluded that the separation is independent of the radiating substance and of the color.

It is possible that some of the resemblances in the preceding tables are due to the fact that the substances in question are chemically related, and perhaps it is scarcely justifiable to generalize from such a limited number; and it may well be that a wider range of elements would show other peculiarities.

I desire to express my hearty appreciation of the efficient service rendered in this work by Mr. C. R. Mann, and especially to recognize the patience and skill shown in the tedious and delicate process of preparation of the vacuum tubes, to which in great measure the success of the investigation is due.

Ryerson Physical Laboratory,
January 1898.

Minor Contributions and Notes.

A VARIABLE BRIGHT HYDROGEN LINE.[1]

The presence of the bright hydrogen line $H\beta$ in the spectrum of the star *A. G. C.* 9181 was found from the Draper Memorial photographs in 1895 and was announced in the Astrophysical Journal, 1, 411. From a comparison of photographs of this object taken on different dates Miss A. J. Cannon finds that this line is variable. On October 5, 1892, it was invisible. On November 28, 1894, it was about half as bright as the corresponding line in *A. G. C.* 9198, ω Canis Majoris. On April 27 and 30, 1895, the line in *A. G. C.* 9181 was distinctly the brighter of the two, while in January 1897, it was again invisible. From a large number of photographs of this object taken recently it appears that this line, which was bright in October 1897, is now, December 27, invisible. Edward C. Pickering.

January 1, 1898.

A NEW SPECTROSCOPIC BINARY.[1]

From an examination of the Draper Memorial photographs Mrs. Fleming finds that the star *A. G. C.* 20263, β Lupi, is a spectroscopic binary. The period has not yet been determined but photographs are being taken for this purpose.

Measures of the spectroscopic binaries μ^1 Scorpii and *A. G. C.* 10534 show that the relative velocities of the components are approximately 460^{km} and 610^{km} respectively. The velocities are, therefore, much greater than in the case of ζ Ursae Majoris and β Aurigae. The separation of some of the lines amounts to as much as nine tenth-meters. Edward C. Pickering.

January 1, 1898.

PHOTOGRAPHIC MAGNITUDES.[2]

In determining the photographic magnitudes of the stars it is a matter of great importance to know how much their relative brightness will

[1] *Harvard College Observatory Circular No. 21.* [2] *Ibid., No. 22.*

vary on different plates, or on different portions of the same plate. It is especially important to determine the amount of this error, since it is not easily eliminated and has been supposed to be large by some persons not familiar with stellar photographs. A moment's investigation of photographs of the same portion of the sky shows that this source of error is small, so small that it is not readily determined by direct measurement. The uniformity of different portions of the film is shown by allowing the stars to trail over the plate. The different portions of the trails appear equally intense, and no variation is perceptible to the eye. A much more delicate test was found in the discussion of a series of measures, made by Miss E. F. Leland, of the variables discovered by Professor Bailey in the cluster Messier 5. Sixty-three of these variables were compared on 41 plates by Argelander's method, with a sequence of comparison stars. Estimates were made of the difference in grades of each variable from the next brighter and the next fainter star of the sequence. The sum of these differences gives the interval between the comparison stars and combining all the results gives, in general, several measures of each interval on each plate. Each comparison star in turn may then be regarded as a variable and its changes in light determined from the next brighter and next fainter star of the sequence. Comparatively few measures were made of the six brightest and the three faintest stars of the sequence. The five intermediate stars were measured on 41, 39, 38, 30 and 30 photographs respectively. The corresponding ranges in the measures derived from each plate, none being rejected, were 0.14, 0.10, 0.12, 0.15, and 0.08 magnitudes, and the average deviations, 0.02, 0.01, 0.02, 0 03 and 0.02. The largest residual was 0.10 and this depended upon two estimates only. We find, therefore, that on the average, five stars were measured on 35 plates with a range of 0.12 magnitudes and an average deviation of 0.02 magnitudes. The total number of estimates from which these results are derived is 4294. The average deviation 0.02 includes: (1) The errors of observation, which are increased by the fact that four estimates enter into each determination, but are diminished since on the average twelve determinations were made of each interval on each plate. (2) Errors due to neglecting hundredths of a magnitude, the computation so far being made only to tenths. (3) Errors due to irregularities in the film, which enter with their full value into the result. Since the combined effect of these three sources of error is only $\pm$ 0.02, it is evident that neither of them can be large. The errors due to the film are in fact so small that there is no

evidence that they exist and more delicate methods of measurement are required to render them perceptible.

EDWARD C. PICKERING.

January 4, 1898.

THE VARIABLE STAR U PEGASI.[1]

MUCH difficulty has been experienced in determining the nature of the variations in the light of this star. Mr. Chandler states that he at first supposed that it was a star of the Algol type having a period of $2^d.06$ or $2^d.07$. It was then observed by Mr. Yendell, who confirmed the variability and still regarded it as of the Algol type, but with a period of $0^d.69$. Mr. Chandler, under date of October 26, 1895, announces in the *A. J.*, **15**, 181, that the period is $5^h\ 31^m\ 9^s.0$ which "is probably only a moderate fraction of a second in error," that it is not of the Algol type, but that the times of increase and decrease are equal. Mr. Yendell (*A. J.*, **16**, 78) states that the time of increase varies from $1^h\ 28^m$ to $3^h\ 41^m$. Again, Mr. Chandler (*A. J.*, **16**, 107) announces that the period is $5^h\ 32^m.25$, that it is perfectly regular, and that previous discrepancies are due to a large subjective error amounting to $0^m.6p$, in which p is the parallactic angle. The correction for this error will sometimes increase and sometimes diminish the observed time of minimum by as much as half an hour. He also states that the decrease in light is more rapid than the increase, and, referring to the short period variables η Aquilæ and δ Cephei, that "we may, therefore, regard U Pegasi, provisionally, as a type of variability distinct from this class of stars, as it evidently is from those of the Algol type."

Owing to these uncertainties it seemed desirable to determine the true form of the light curve photometrically, especially as such observations are free from the subjective error mentioned above, since the images compared are constantly interchanged. Assuming the light curve to be constant, it appeared possible to determine its form from observations made during a single evening. Accordingly, measures were made by Mr. O. C. Wendell with the polarizing photometer (this JOURNAL **2**, 89) attached to the 15-inch equatorial telescope of the Harvard College Observatory. The results for a single evening, December 28, 1897, are contained in the annexed table, the first column giving the Greenwich Mean Time, and the second the pho-

[1] *Harvard College Observatory Circular No. 23.*

tometric magnitude found by adding the observed difference in magnitude between U Pegasi and the comparison star, + 15°4916, to 8.90, the photometric magnitude of the latter star. These observations are also indicated by the crosses in the figure. The third column gives the residuals found by subtracting from the magnitude given in the second column that derived graphically from all the observations.

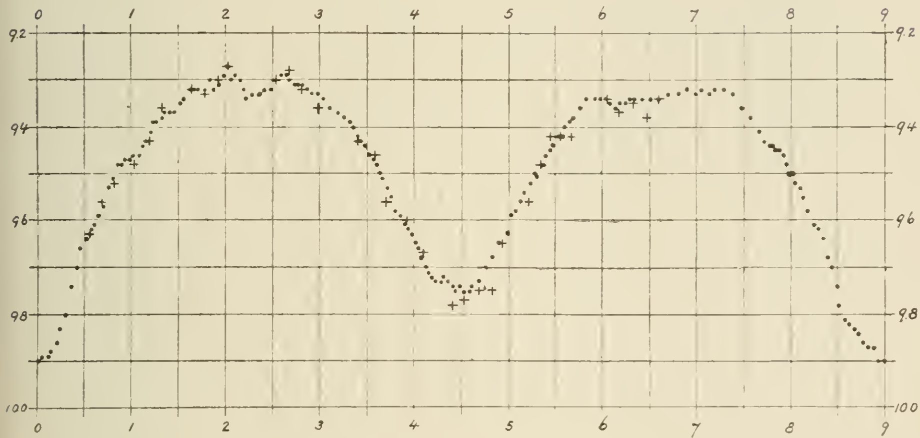
OBSERVATIONS ON DECEMBER 28, 1897.

G. M. T.	Mag.	O—C	G. M. T.	Mag.	O—C
10^h $26^m.9$	9.63	.00	13^h $48^m.7$	9.60	—.01
10 34 .2	9.56	—.01	13 58 .6	9.67	—.02
10 42 .2	9.52	+.01	14 17 .5	9.78	+.04
10 55 .3	9.48	+.02	14 24 .5	9.77	+.02
11 4 .7	9.43	+.02	14 34 .0	9.75	+.02
11 12 .4	9.36	—.02	14 43 .0	9.75	+.07
11 31 .3	9.32	—.01	14 50 .0	9.65	.00
11 40 .0	9.33	+.02	15 7 .0	9.56	+.03
11 48 .6	9.30	.00	15 15 .0	9.48	.00
11 55 .4	9.27	—.02	15 21 .0	9.42	—.02
12 25 .8	9.30	.00	15 26 .8	9.42	.00
12 33 .4	9.28	—.02	15 33 .2	9.42	+.04
12 41 .2	9.32	+.01	15 56 .9	9.34	.00
12 51 .1	9.36	+.03	16 3 .0	9.37	+.02
13 17 .4	9.43	+.01	16 13 .3	9.35	+.01
13 27 .2	9.46	—.01	16 21 .2	9.38	+.04
13 36 .0	9.56	+.03	16 29 .6	9.34	.00

Each observed magnitude is derived from the mean of four sets of four settings each, the photometer being reversed between the second and third sets, and the positions of the images of the stars interchanged after the second setting of each set. The magnitudes derived from each pair of sets differ on the average ± 0.04 mag. The average values of this quantity during the seven successive hours of observation were 0.06, 0.03, 0.04, 0.03, 0.04, 0.04, and 0.04, respectively. As they show no progressive increase it is evident that the fatigue of the observer did not sensibly affect the accuracy of the observations. The accuracy is also unaffected by the altitude, which was only 5°.6 at the end of the series. No correction has yet been applied for differential absorption, which is small but sensible, although the distance between the stars compared is only 15′. The computed probable error of a single set of four settings from these differences is ± 0.024 mag.

The average value of the residuals in the third column is ± 0.017, only one being greater than 0.04.

It is obvious from an inspection of these observations that the star had nearly the same brightness after an interval of $4^h\ 30^m$, at first



indicating that the variable had this period and not $5^h\ 31^m$. But later observations showed that the phenomena were not quite so simple. Assuming the period $4^h\ 30^m$ it soon appeared that the even minima were fainter than the odd. Observations continued through even minima on four nights gave the minimum brightness of U Pegasi, including errors of observation, 9.94, 9.94, 9.92, and 9.85, respectively, while corresponding measures of odd minima gave the magnitudes 9.78, 9.78, and 9.75. It thus became evident that the period should be doubled, and plotting the observations a light curve was formed closely resembling that of β Lyrae. The formula *J. D.* 2413514.6157 + $0^d.37478E$ has been adopted, which closely represents the observations recently made and also a few less accordant observations made here in 1895. Changing the period by two seconds, to $0^d.3748$, would alter the times of minima in 1895 by about an hour, while the uncertainty is only a small fraction of this amount. The period may, therefore, be assumed to be $8^h\ 59^m\ 41^s$. The magnitude at maximum is 9.30, at primary minimum 9.90, and at secondary minimum, 9.75.

The above figure represents the observations on a scale such that in the ordinates one division corresponds to a tenth of a magnitude,

and in the abscissæ to 30 minutes. All the observations made by Mr. Wendell on eight nights are included, none being rejected for discordance. In fact, the most discordant set of sixteen settings differs from the mean curve by less than a tenth of a magnitude. Each dot represents 80 settings, the method of overlapping means, in common use in meteorology, being employed. The observations of December 28, as stated above, are represented by crosses, each representing the result of sixteen settings. The total number of settings was 2784 and the entire time of observation, including rests, about 30 hours.

If we neglect the difference between the primary and secondary minima, reduce the half period to fractions of a day, and multiply it by 16 we obtain the product $2^d.99824$, or very nearly three days. Therefore, the phases recur at nearly the same times every three days. If we multiply the period $5^h\ 32^m\ 15^s$ by 13 we obtain $2^d.99948$, or very nearly the same quantity. In either case, therefore, the phases would recur at the same times every three days, but with the second period the interval between successive minima would be greater than with the first period in the ratio of 16 to 13. If then we used the second period we should find the observed minima in error by an amount equal to three-thirteenths of the hour angle, or expressed in minutes $0^m.92h$, in which h is the hour angle expressed in degrees. Since the latitude of Cambridge is $+42°.4$ and the declination of U Pegasi is $+15°.4$ it follows that when h is small we may write $h = 0.61p$ and $0^m.92h = 0^m.56p$, which agrees very nearly with $0^m.6p$, the correction found empirically by Mr. Chandler from his observations. The coefficient becomes larger with large hour angles having the values 0.61, 0.74, and 0.91 for $h = 15°, 30°$, and $45°$, respectively. It therefore follows that by the application of the correction $0^m.6p$ not only were the observed times of minima changed in some cases by more than half an hour, but an error of more than an hour was introduced into the half period. An arbitrary correction proportional to the parallactic angle is not to be recommended, since by assuming different values of the coefficient, very different values of the period will be found.

From the above discussion it appears that U Pegasi is no longer the variable star having the shortest period known. This position appears to be held by the variable ω Centauri 19, discovered by Professor Bailey, who finds its period to be $7^h\ 11^m$. Although U Pegasi can no longer be regarded as an example of that peculiar class of short

period variables having a single maximum in which the decrease is more rapid than the increase, this class is still represented not only by S Antliæ, but by ω Centauri 24, which Professor Bailey finds to decrease twice as rapidly as it increases, while ω Centauri 45 increases at least five times as fast as it diminishes.

EDWARD C. PICKERING.

January 14, 1898.

REPORT ON A NEW EDITION OF THE NORTHERN DURCHMUSTERUNG CHARTS.[1]

(From a communication by Dr. F. Küstner.)

I.

WITH regard to the *first* edition, the following information has been obtained:

1. The original stones have not been preserved. Probably only a few stones were used and repolished for different charts.

In the case of the SD. the stones are preserved.

2. The contract between Argelander and the publisher, Marcus, made the latter the proprietor of the charts, although the government paid one-half of the expenses of engraving and printing.

3. The *Northern Durchmusterung* was sold for ninety marks, but the publication was not lucrative. In the case of the SD. there has probably been a loss, because, within ten years of the publication of the charts, less than 100 copies were sold, and at present the sales average about two copies a year.

II.

About the *new* edition, the following facts are reported:

1. While the Observatory at Bonn has no rights in the new edition, it considers it a point of honor to coöperate with the publisher in making the edition free from error as far as possible. For this purpose an extended list of errors has been compiled, and the corresponding corrections have been made on the charts wherever they would change the configuration of the stars. On some charts the number of corrections amounted to about fifty. Notice of erroneous or doubtful stars will be gladly received, and the matter will be looked up in the original records, as has been the custom heretofore at the Bonn Observatory.

[1] Report of a committee appointed at the Yerkes Observatory Conferences.

2. From the many experiments made at Bonn, it was found that while the mere reproduction of the charts was easy, it was very difficult, technically, to get prints which were free from specks or spurious stars.

3. The actual printing of the charts had been going on for about a year when the printing firm failed. Dr. Küstner says that a year's labor in reading and correcting the proofs has thus been lost.

4. Flittner, Marcus' successor. is trying to engage another printing firm, and is now corresponding with the government printing office at Berlin. It is the publisher's intention to call for subscriptions as soon as a sufficient number of some of the charts has been reproduced. The selling price will depend on the number of subscribers, and, according to Dr. Küstner's view, the coöperation of our committee should be directed principally towards making this number as large as possible.[1]

J. G. Hagen, S. J.

Georgetown College Observatory,
January 6, 1898.

A NOTE ON THE FIGURING AND USE OF THE ECCENTRIC AND UNSYMMETRICAL FORMS OF PARABOLIC MIRRORS.

In a valuable paper which is published elsewhere in the present number of this Journal[2] Professor Poor has investigated in a very complete manner the question of the spherical aberration at various points in the field of a parabolic mirror, and has derived general equations which determine rigorously the diameter (maximum) of a star image in the principal plane of the parabolic section. As I have pointed out elsewhere, these results, although valuable in themselves, do not, when taken alone, enable us to draw any very complete or definite conclusions as to the definition and resolving power of mirrors

[1] Since the above report was put in type circulars asking for subscriptions to the new edition of the charts have been received from the publishers. It is stated that the new edition will be published provided at least one hundred copies of the Atlas are ordered by May 1, 1898. As the price is only 70 marks, and as every astronomer must feel it a duty to assist in bringing these invaluable charts within the reach of all observers, there should be no difficulty in securing the requisite number of subscriptions. Intending subscribers should address A. Marcus und E. Weber's Verlag, Bonn.—Eds.

[2] See p. 4.

in different portions of the field, because these qualities are determined more by the *effective distribution of light* in the image than by the mere linear extent or "spreading" of the latter in the focal plane.[1] In a general way, however, they support the conclusions reached by the writer on quite different grounds of comparison[2] that mirrors are inferior to the ordinary reflector, and the photographic doublet in the respective fields of visual observations and general photographic work. But it is not in either of these fields that mirrors should be employed if their peculiar excellencies are to be best utilized and their peculiar defects minimized in importance. Reflecting telescopes are, as has often been pointed out,[3] preëminently adapted to stellar spectroscopic and allied branches of astrophysical work, and it is to such work (which is now certainly important enough in itself to warrant the construction and erection of large telescopes especially designed for its most efficient prosecution) that they should therefore be devoted.

Now in stellar spectroscopic work we have to deal only with the image of a point at or very near the center of the field, where the definition is sensibly perfect with any form of mirror, and the particular objections, on theoretical grounds, against even the highly eccentric form (90°) considered by Professor Poor, vanish. Could the practical difficulties of figuring the surface be overcome, this latter form would, on account of the simplicity and beauty of the mounting, particularly the small number of reflections involved in making the instrument of the *coudé* form, be of the greatest value in astrophysical work. These difficulties, however, are most formidable, although I am inclined to think after discussing the matter with Professor Poor, that they may be overcome by special methods. It is to be greatly

[1] See *Pop. Astron.* **5**, 528, Feb. 1898. In this paper the general equations were given which enable us to determine the distribution of light in the image of a star or other point source at any part of the field of a parabolic reflector.

[2] See papers in *Pop. Astron.* **5**, 200, August 1897; *A. N.*, No. 3439; *Knowledge*, August and September 1897, pp. 193, 218; *Obs'y.* **20**, 303, 365, 404, September, October, and November 1897.

[3] See paper by HALE "On the Comparative Value of Refracting and Reflecting Telescopes for Astrophysical Investigation," *Ap. J.*, February 1897, p. 119; also various papers by the writer—in the *Phil. Mag.*, July and October 1894, pp. 137 and 337; *Astron. and Astrophysics*, December 1894, p. 835; *Ap. J.*, January 1895, p. 52, March 1894, p. 232, November 1894, p. 264, March 1896, pp. 169, 182, 183, May 1896, p. 347, November 1896, p. 274, February 1897, p. 132, etc.

hoped, therefore, that Professor Poor will continue his experiments in this direction and succeed in carrying them to a successful issue.

In this connection it might not perhaps be out of place to refer to certain suggestions of my own of somewhat the same character as that of Professor Poor. Some time ago I proposed to use mirrors cut from one side of a paraboloid of revolution for use in certain forms of astronomical and laboratory spectroscopes and spectrobolometers and in the spectroheliograph.[1] In these cases the mirrors required are small, and no difficulty would be experienced in obtaining them, figured with as great accuracy as desired, by cutting them out of one side of a larger paraboloid of revolution figured and polished by the usual method. The portion cut out and used would moreover be quite near the center of figure (only far enough to one side to allow the incident beam to clear the slit-plate, observing eyepiece or bolometer strip), and the size of the original mirror would not therefore need to be much more than twice the diameter of the part utilized. Such a figured surface would furnish at least four smaller mirrors of the form required. I have also considered the possibility of grinding a single large disk for a telescope objective so that the center of the parabolic figure is at, or near, one edge, instead of at the center of the disk. This might perhaps be accomplished by present methods, by embedding the glass disk eccentrically in a suitable matrix of the same general nature and hardness as the glass itself (perhaps furnace slag or broken glass, finely ground and then recemented together, under pressure in a mold of proper shape, would be found to answer well), and then figuring the whole surface into a paraboloid of revolution as usual.

Such a speculum, if it could be made, would realize the advantages of increased light-gathering power possessed by the Herschelian form of reflecting telescope without the disadvantage of working considerably out of the optical axis, as is necessary in the latter form of instrument.

It will be readily seen, from the preceding considerations, that, in the opinion of the writer, the question of the aberration of mirrors for points some distance from the optical axis is of comparatively little practical importance; only the center of the field being used in those lines of work for which reflecting telescopes are best adapted. The investigation of this question is, nevertheless, of great theoretical interest, and I am very glad to see that Professor Poor has taken it up.

[1] See papers in ASTROPHYSICAL JOURNAL already referred to.

The results he has already obtained (for the size of the distorted image), when supplemented by the investigation which he proposes to undertake[1] of the distribution of light in the image will furnish us for the first time with data which will make it possible to draw definite conclusions as to the practical defining and resolving powers of various forms of parabolic mirrors at points in the field some distance from the optical axis.

F. L. O. Wadsworth.

Yerkes Observatory,
Dec. 8, 1897.

[1] See footnote on page 119.

Reviews.

Ueber einige Emissionsspectra des Cadmiums, Zinks, und der Haloid Verbindungen des Quecksilbers und einiger anderen Metallen. A. C. Jones. *Wied. Ann.* **62**, 30–53, 1897.

The Multiple Spectra of Gases. John Trowbridge and Theodore Wm. Richards. *Am. Jour. of Sci.*, **3**, 117–120, 1897.

On the Propagation of Waves along Connected Systems of Similar Bodies. Lord Rayleigh. *Phil. Mag.*, **44**, 356–362, 1897.

On a Relation between the Spectrum of Hydrogen and Acoustics. A. S. Herschel. *The Observatory*, June 1896.

Of the known spectroscopic properties of the elements, two of extraordinary importance and of increasing interest have been established within the last third of the century. One of these is the fact that, under different physical conditions, the same element yields radically different spectra; the other is that, under the *same* physical conditions, many, possibly each, of the elements yield "series" of vibrations, *i. e.*, vibrations which are related to one another in a definite, and apparently simple, manner.

Neither of these phenomena have yet been "explained;" by which we mean, in the first case, that the exact physical conditions under which different spectra of one and the same element are produced, have not yet been "described;" and in the second case, that the mechanical analogue of a system vibrating in the "series" mode has not yet been "described."

Professors Trowbridge and Richards have made a long step in showing that the oscillatory discharge gives one class of spectra, the deadbeat discharge another: but, even yet, we are left in the dark as to whether this change of spectrum is due more immediately to a change of temperature, or to some chemical change which is wrought by the one discharge and not wrought by the other; or as to whether it may not be due to still other causes, perhaps mere changes in the time and space distribution of electric potential in the luminous gas. And still another alternative must be mentioned. For so many groups and sub-

groups of lines have been distinguished in the spectra of single elements, that it is not impossible that we may have purely thermal, purely electrical, and purely chemical spectra overlapping, and *present together* in the ordinary spark or arc-spectrum.

In illustration of these groups and sub-groups, I need only mention lines which fall into series and lines which do not (Kayser and Runge); hydrogen of Balmer's series and hydrogen in ζ Puppis (Pickering); lines which "shift" under pressure and lines which do not (Humphreys and Mohler); lines which depend upon the presence of another element and lines which do not, *e. g.*, aluminum and oxygen; lines which are strong in the arc-spectrum and weak in the spark-spectrum; lines which are rapid in making their appearance in the Plücker tube and lines which are less rapid (Schuster, *B. A. Report*, 1897).

In the first of the articles under review, the author, Mr. A. C. Jones, has undertaken to discover just what effect is produced upon the spark-spectra of zinc, cadmium, mercury and their respective halogen compounds when the electric discharge varies in *intensity* but not in *kind*.

This variation of intensity is accomplished either by narrowing the bore of the vacuum tube, or by introducing a spark gap, or by the use of both methods.

The changes in intensity of the lines corresponding to changes in intensity of the spark-discharge are summarized in tables.

This interesting paper is marked by one error whose very frequency of occurrence calls for comment in this place, *i. e.*, the identification of *resolving power* of the spectroscope (auflösende Kraft, p. 35) with the *scale* of the spectrum.

Lord Rayleigh has pointed out in the most unmistakable manner that the resolving power of a prism cannot be described without giving the difference of thickness of prism traversed by the two extreme rays, or giving some statement which is mathematically equivalent.

The beautiful investigation of Professors Trowbridge and Richards regarding the red and blue spectrum of argon has been extended so as to include nitrogen, hydrogen, and helium. The following single sentence states the purpose of the work and summarizes their results. "It is the object of this paper to emphasize anew the importance of the electrical conditions of the circuit, and to call attention once more to the fact that the behavior of most elementary gases is in every respect similar to that of argon." That is to say, in general, the oscil-

latory discharge produces the blue and linear spectra, while the dead-beat discharge gives the red and channeled spectra.

Some exceptions to this statement are most instructive, *e. g.*, "no considerable effect" is produced upon the spectrum of helium when the discharge is changed from the oscillatory to the continuous.

Again, if the continuous discharge be produced by placing impedance in a circuit which is already oscillatory, the spectrum obtained differs very markedly from that given by the battery alone without any condenser in the circuit.

As a further extension of this work, the authors propose to use their powerful battery to determine "whether the oscillatory discharge produces its effect simply by increasing the temperature, or because of some inherent property in the manner of discharge." An experimental answer to this very fundamental question will be awaited with unusual interest; for such an answer is one of the great needs of modern spectroscopy.

The next paper, that of Lord Rayleigh on vibrating systems, has a most important bearing on the second of the general problems stated at the outset.

It is a study of the modes of vibration of a system of similar bodies distributed at equal intervals, either along a straight line of infinite length, or in a closed chain.

By the use of generalized coördinates, the equations of motion are elegantly derived in a linear form with constant coefficients. Following the general solution is a study of some half dozen special cases, in each of which the object is to express the frequency in terms of the wave-length; or, what is the same thing, since in all cases $v = n\lambda$, to determine the relation between speed and wave-length.

Among these special cases is a generalized expression of the ordinary formula for the frequency of a stretched string. This generalized expression includes also Lord Kelvin's wave model as a special case.

A linear system of magnets — a row of compass needles — is next considered. Following this is Fitzgerald's system of pulleys belted together by means of rubber bands. The last case considered is that of an open chain of magnets for which the frequency, n, is found to be

$$n = 2 \cos \frac{s\pi}{2m} = 2 \cos \frac{a\pi}{2\lambda}$$

where a is the distance between the successive magnets, λ is the wave-

length, and m the total number of magnets in the chain, s being some integer.

If N be the highest possible frequency of the system (corresponding to a configuration in which the magnets are all parallel to each other) the above expression takes the form

$$n = N \cos \frac{s\pi}{2m} = N\left(1 - \frac{s^2 \pi^2}{8m^2}\right)$$

where, if s (the number of waves in the length of the chain) is made to vary, the frequency will vary in the same general way as in a spectral series.

But the most interesting feature in the whole discussion, from the spectroscopic standpoint, is that when s is held constant, say $s = 2$, and m is varied, the frequencies are distributed in a series almost identical with that of Balmer, which is as follows:

$$n = N\left(1 - \frac{4}{m^2}\right).$$

But when m assumes different integral values, beginning with 3, we have for each different value of m a different mechanical system. This allotment of each line in the series to a different, if not an entirely independent, dynamical system is an entirely new and suggestive point of view.

One's hopes for immediate progress along this line are, however, somewhat dampened by the closing paragraph, which must be quoted in full.

"There is one circumstance which suggests doubts whether the analogue of radiating bodies is to be sought at all in the ordinary mechanical or acoustical systems vibrating about equilibrium. For the latter even when gyratory terms are admitted, give rise to equations involving the square of the frequency; and it is only in certain exceptional cases, *e. g.*, (31) that the frequency itself can be simply expressed. On the other hand, the formulæ and laws derived from observation of the spectrum appear to introduce more naturally the *first* power of the frequency. For example, this is the case with Balmer's formula. Again when the spectrum of a body shows several doublets, the intervals between the components correspond closely to a constant difference of frequency, and could not be simply expressed in terms of squares of frequency. Further the remarkable law discovered independently by Rydberg and Schuster connecting the con-

vergence frequencies of different series belonging to the same substance, points in the same direction.

"What particular conclusion follows from this consideration, even if force be allowed to it, may be difficult to say. The occurrence of the first power of the frequency seems suggestive rather of kinematic relations[1] than those of dynamics."

To those who are accustomed to look upon work of this kind as dreamy speculation we recommend a careful and minute reading of the original paper, in confidence that the reader will there find only a very accurate and very beautiful description of the behavior of some six or eight actual dynamical systems.

The avowed object of the next paper is to produce an acoustical, and hence mechanical, analogue of Balmer's series. While more speculative than the preceding, it is full of suggestion.

The author first points out that if, while an open organ pipe be sounding the m^{th} harmonic of its fundamental, a wind current be driven through the pipe, the resonance for its m^{th} harmonic will be destroyed: and if the speed of the wind be the $\left(\frac{n}{m}\right)^{\text{th}}$ part of the speed of sound in air, the vibration will no longer be the m^{th} harmonic, but will have a frequency related to that of the m^{th} harmonic as follows:

$$\frac{1}{\lambda} = A\left(1 - \frac{n^2}{m^2}\right).$$

For a wind whose speed is $\left(\frac{2}{m}\right)^{\text{th}}$ that of sound, we have

$$\frac{1}{\lambda} = A\left(1 - \frac{4}{m^2}\right).$$

But one is not to be deceived by the outward resemblance between this expression and that of Balmer; for the limiting frequency, A, is here *not* a constant, as in Balmer's expression, but it is itself a function of m, viz., the product of m by the frequency of the fundamental.

Accordingly the author takes refuge in a more complex atom. He imagines the open organ pipe replaced by a circular tube—a hollow annulus—in which the wind current is replaced by a current of ether, flowing with a speed which is $\left(\frac{2}{m}\right)^{\text{th}}$ that of light. These annuli are then

[1] *E. g.*, as in the phases of the Moon.

assembled to form a vortex-ring atom, each stream-line of the ordinary vortex ring being replaced by one of these resonant ring tubes.

If now we *assume m* to vary from one tube of flow to another, and if we *assume* farther that each annulus can respond to and emit *one harmonic only*, then we have a fair approximation to Balmer's series.

But there is here evidently much of speculation and we are no longer on the solid foundation of experiment and mathematical deduction.

H. C.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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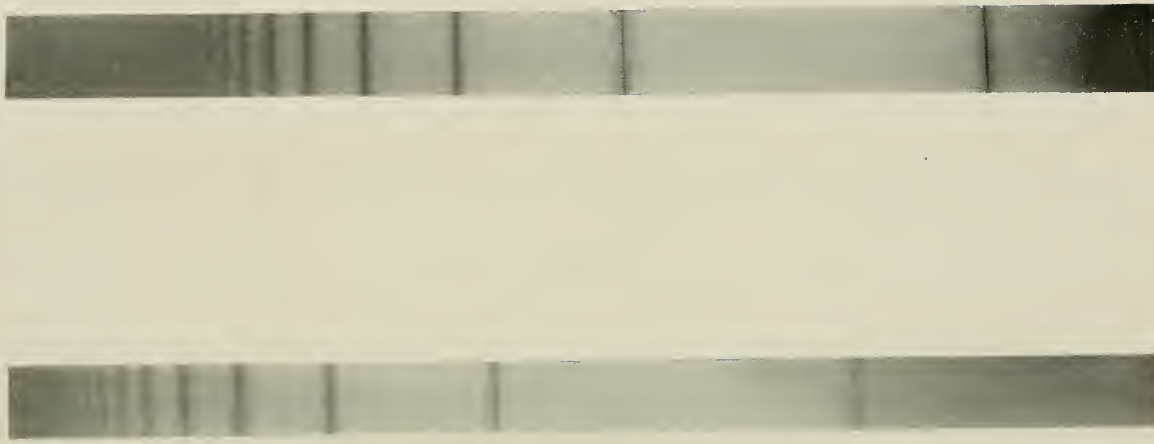
PLATE III.



PHOTOGRAPHS OF THE SPECTRUM OF SIRIUS TAKEN WITH A CONCAVE GRATING.

THE

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THE CONCAVE GRATING FOR STELLAR PHOTOGRAPHY.

By Charles Lane Poor and S. Alfred Mitchell.

The concave grating has proved a most powerful instrument for spectroscopic research, but heretofore it has not been successfully applied to stellar spectroscopy. Experiments are now being carried out at the Johns Hopkins University under the direction of Dr. Charles Lane Poor with the view of thoroughly testing the various methods of using the concave grating for astronomical purposes. The methods, originally suggested by Professor Rowland, were developed, and the formulæ derived by Dr. Poor, and the preliminary apparatus constructed under his direction: the experiments and the photographs were made by Mr. S. Alfred Mitchell. As some promising photographs have been obtained the following notes are now published in regard to methods and results.

There are two radically different methods of using the concave grating for stellar work.

First: In connection with an objective; the concave grating merely replacing the ordinary stellar spectroscope. This was tried by Professor Crew at the Lick Observatory in 1892, and a few results obtained.

Second: Direct; the grating is the objective and the spectroscope combined: the light from the star being reflected

directly from the grating to the photographic plate. In 1892 Dr. Poor had a rough apparatus made to test this method, and in 1892-3 he introduced this way of using the grating into his lectures on "Theory of Instruments." In the ASTROPHYSICAL JOURNAL of January 1896, is an article by Professor F. L. O. Wadsworth[1] in which this method is treated of and the equations given.

It is our purpose to test fully all the various methods so far as it is possible to do so at the Johns Hopkins University. Our best results, so far, have been obtained by the direct method, and this paper will be confined to an explanation of that method, to the derivation of the necessary formulæ and to a few notes in regard to the photographs already taken.

From the theory of the concave grating we have the general equation:

$$r = \frac{R\,\rho\cos^2\mu}{R(\cos\mu + \cos\nu) - \rho\cos^2\nu} \qquad (1)$$

(See Rowland, *American Journal of Science*, Vol. XXVI, Aug. 1883.)

In this equation ρ is the radius of curvature of the grating, and the axis of the grating is the line of reference for angular measurements; R and ν are the spherical coördinates of the source of light; r and μ those of the curve on which the spectra are brought to a focus.

This equation may be put into the following form:

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\nu - \frac{\rho}{R}\cos^2\nu} \qquad (2)$$

If now the source of light be placed at an infinite distance, then R is equal to infinity, and the equation (2) reduces to:

$$r = \frac{\rho\cos^2\mu}{\cos\mu + \cos\nu} \qquad (3)$$

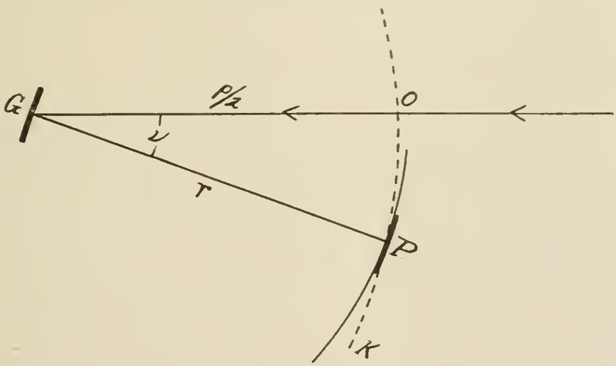
This is the general equation for the case under consideration.

[1] Professor Wadsworth's mathematical treatment on pp. 57–58 does not apply to our method of using the grating. He finds the variation in wave-length corresponding to changes in the angle of incidence of the light from the star. In our mounting this angle is constant, and within certain limits the resulting spectrum is "normal."

We may use the grating in a number of different ways, depending upon the position of the photographic plate and of the source of light in reference to the axis of the grating. One position is by far the best for general work, and in this preliminary note the formulæ for that case alone are given. The position is that in which the center of the photographic plate is on the axis of the grating. For this we have $\mu = 0$, and our general equation reduces to:

$$r = \frac{\rho}{1 + \cos \nu} \tag{4}$$

For any given value of ν, r is constant, and those parts of the spectra, for which we can assume $\cos \mu$ equal to unity, are brought to a focus on a circle whose radius is r as above given. The equation itself is that of a parabola, so that when ν is changed, μ being kept equal to zero, in order to bring different parts, or different orders, of spectra to the center of the plate, the value of r will vary and will correspond in value to that radius vector of the parabola which corresponds to the value of ν. This case is shown in the following figure:



G is the grating, *P* the photographic plate. The light comes in from the star in the direction of *OG*. The curve *OPK* is a parabola, *OG* being the half parameter and is equal to $\frac{1}{2}\rho$. For a constant value of ν those spectral lines on the photographic plate for which $\cos \mu$ can be assumed equal to unity, are brought to a focus on a circle whose radius is r.

To investigate this case fully we must return to the general equation (3). For a fixed value of ν, all the spectra are brought to a focus on the curve:

$$r = \frac{\rho \cos^2 \mu}{\cos \mu + \cos \nu}$$

in which $\cos \nu$ is constant. When μ is so small that its cosine may be taken as unity, this curve reduces to a circle, as above described.

By the theory of diffraction (Rayleigh, *Encyc. Brit.*, "Wave Theory of Light") we have:

$$\lambda = \frac{\omega}{N} (\sin \nu + \sin \mu) \tag{5}$$

where ω is the grating space and N is the order of the spectrum. From this we have at once:

$$\frac{d\lambda}{d\mu} = \frac{\omega}{N} \cos \mu \tag{6}$$

To find the change in wave-length as we pass along the focal curve, we have:

$$\frac{d\lambda}{ds} = \frac{d\lambda}{d\mu} \cdot \frac{d\mu}{ds}$$

and

$$\frac{d\mu}{ds} = \frac{1}{\sqrt{r^2 + \left(\frac{dr}{d\mu}\right)^2}}$$

Differentiating the equation of the focal curve, we find:

$$\frac{dr}{d\mu} = \frac{r \sin \mu - \rho \sin 2\mu}{\cos \mu + \cos \nu} = \phi(\mu) \tag{7}$$

whence substituting we finally find:

$$\frac{d\lambda}{ds} = \frac{\omega}{N} \cdot \frac{\cos \mu}{\sqrt{r^2 + [\phi(\mu)]^2}} \tag{8}$$

and this is the general formula for change in wave-length along the focal curve.

If now we put $\mu = 0$, this reduces to,

$$\frac{d\lambda}{ds} = \frac{\omega}{N} \cdot \frac{1}{r_0} \tag{9}$$

a constant. Hence at this point the spectrum is "normal."

Within the limits, therefore, to which we can take cos μ as equal to unity, the spectrum may be considered as normal.

For a grating of medium dispersion an entire spectrum will be practically normal, provided the center of the plate is on the axis of the grating; *i. e.*, μ equal zero for the middle of the spectrum.

In the grating used in our experiments the entire first order spectrum subtends an angle of about 6°; and by computation we find from the above formulæ that the scales of different portions of the spectrum differ by less than three parts in a thousand. To be more exact, at a point 3° from the axis, the scale is smaller than at the center of the plate; the ratio of the latter to the former being 1.0025. On a plate of the solar spectrum as taken by the usual twenty-one foot Rowland mounting the scales of the middle and end differ by one and one-half parts in a thousand for the same variation of 3°.

The advantages of this method of working are thus apparent. The photographic plate should be bent to conform with the focal curve as given by equation (3); within the above limits, however, this differs but little from a circle.

In order to test the above method a small Rowland concave grating with a ruled surface of 1×2 inches was used. The grating has a radius of curvature of one meter, and is ruled with 15,000 lines to the inch. The apparatus for mounting the grating is extremely simple, consisting of a light box clamped to the tube of the equatorial; the telescope being used merely as a finder. The light from the star falls directly on the grating, is diffracted and brought to a focus on a photographic plate. The grating is mounted in an ordinary holder which can be adjusted by side and back screws. The plate holder holds a plate 1×5 inches, bent as closely as possible to the proper radius. The holder is capable of adjustments, so that the plate and grating can be made parallel, in order to procure a normal spectrum. These adjustments are made with very little difficulty. The box is clamped to the telescope in such a way that the lines of the grating are parallel to the equator, and accordingly, by regulating the driving clock of the telescope to run a little too slow or too fast the spectrum can be made of any convenient width.

For our trials Sirius was the star principally used, and exposures ranged from ten minutes to one hour according to the width of the spectrum. All the photographs were made with the first order spectrum, and Seed's Gilt Edge plates were used.

The spectra are about 5^{cm} long, and vary in width from $0^{mm}.1$ to $1^{mm}.5$, depending upon the exposure and the rate of the clock. Details of a few specimen plates follow:

Sirius. Nov. 27. Exposure 40 minutes, width $1^{mm}.5$, 8 hydrogen bands and H and K lines.

Capella. Dec. 9. Exposure 40 minutes, width $0^{mm}.2$. F, G, *h*, H, K, and about 50 fine lines.

Procyon. Dec. 15. Exposure 40 minutes, width $0^{mm}.15$, 6 hydrogen bands, H and K and about 20 fine lines.

Rigel. Dec. 28. Exposure 85 minutes, width $0^{mm}.1$, 14 hydrogen bands, H and K, and 6 other lines.

Sirius. Jan. 3. Exposure 40 minutes, length 5^{cm}, width $0^{mm}.1$, 16 hydrogen bands, H and K lines and 15 other distinct fine lines.

The accompanying plate gives the enlargement of two of our photographic plates of Sirius. The lower spectrum is the enlargement of one taken December 15, 1897, showing 13 hydrogen lines, H, K, and ten others. The upper spectrum is the enlargement of the plate taken January 3 (noted above). Since our original spectra are extremely narrow, considerable difficulty is experienced in widening out the spectra without introducing spurious lines. Although some of the finer lines are spurious, the plate shows the general character of our photographs in that it shows clearly the hydrogen lines, K, and many other lines which can be easily identified.

All these experiments were carried on in the Observatory, which is on the fifth floor of the Physical Laboratory, and is subject to the jar of street cars and city traffic as well as to dust and to the glare of electric lights. We are confident that much better results will be obtained under better conditions, and think that this method promises to become of great value to stellar spectroscopy.

Johns Hopkins University,
January 5, 1898.

ON CERTAIN NEW RESULTS RELATING TO THE PHENOMENA DISCOVERED BY DR. ZEEMAN.

By M. A. CORNU.

SUCCESSIVE improvements in the method of observing the phenomena discovered by Dr. Zeeman have led me to certain results which are not in agreement with the earlier observations, and which may modify our ideas on the mechanism of these phenomena.

The general method of conducting the experiment is that which I have previously described: the luminous source (oxy-hydrogen flame saturated with saline vapors, induction spark, etc.) is placed between the two poles of a powerful electro-magnet, and the image of this source is projected upon the slit of a spectroscope of high dispersion, provided with the necessary doubly refracting appliances.

1. *Observations in the direction of the lines of force.*

My earlier conclusions regarding the resolution of the ray of ordinary light into two circularly polarized rays are not affected.[1] But micrometric measures have shown that the magnitude of this doubling does not depend exclusively on the wave-length of the line observed; the observations may be summarized as follows:

The effect of the magnetic field on the period of vibration of the radiations of a luminous source seems to depend not only upon the chemical nature of the source, but also upon the nature of the group of spectral lines to which each radiation belongs, and on the part which it plays in this group.

There thus remains little hope of the possibility of expressing the magnitude of the magnetic doubling of the lines of a given spectrum as a simple function of the wave-length, as had been hoped at the outset.[2] It is, however, this very point of view of

[1] M. A. CORNU, this JOURNAL, **6**, 378, December 1897; *C. R.*, **125**, 555.

[2] H. BECQUEREL, *C. R.*, **125**, 679.

the existence of essential differences among the lines of the same spectrum—differences already recognized under various circumstances (spontaneously reversible lines,[1] hydrogen groups,[2] etc.)—which has led me to pursue the detailed study of the Zeeman phenomenon as offering a new means of bringing to light those families of lines, the existence of which certain optical peculiarities have already led us to suspect.

As a matter of fact, the observation of groups well known by their regular geometric arrangement reveals under the action of magnetism peculiarities analogous to their unequal facility of spontaneous reversal. Thus visual observations of the magnesium group *b* and photographs of the group of three blue lines of zinc show that the magnitude of the magnetic doubling of their components increases rapidly with the refrangibility, while the difference in wave-length of the various lines is insignificant.

Contrary to what one would be led to expect from the experiments of Messrs. Egoroff and Georgiewski, it is the most easily reversible line which shows the least doubling effect.

2. *Observations in the direction normal to the lines of force.*

The principal result obtained in this case profoundly modifies in an important particular the early conclusions of Messrs. Zeeman and Lorentz.

(1) *Under the influence of the magnetic field in the direction normal to the lines of force a single spectral line becomes* QUADRUPLE (and not TRIPLE, as has been previously announced). *The two outer lines are polarized parallel to the lines of force, the two intermediate lines perpendicular to this direction.*

(2) *The quadruplet thus formed is symmetrical with reference to the original line, and the separation of the two similarly polarized lines is sensibly proportional to the intensity of the magnetic field.*[3]

[1] M. A. Cornu, *C. R.*, 123, 332.

[2] M. A. Cornu, *C. R.*, 100, 1181.

[3] I have also found that for equally intense magnetic fields the distance between the two lines polarized parallel to the lines of force is sensibly equal to the distance between the circularly polarized lines; but the precision of the optical or magnetic measures is still insufficient to render possible a certain demonstration of this equality.

It is the improvement of the optical apparatus rather than the increased strength of the magnetic field which has permitted me to effect the doubling of the central line of Zeeman's *triplet;* this doubling must already have been seen by many observers; but imperfect images have caused it to be mistaken for a simple *reversal.* Moreover, it is usually very small and always very unequal, varying with the line observed, even in very compact groups.

The most striking example, which is at the same time the easiest to observe, is that offered by the sodium group D_1, D_2.

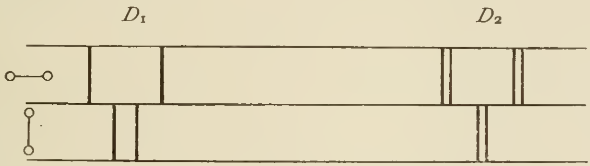


FIG. 1.

The line D_1 (Fig. 1), which is the narrower and the less refrangible of the two, is transformed into a well-defined *quadruplet,* the two intermediate lines being separated by half the amount of the two outer ones. The line D_2, which is broader and more easily reversible, becomes a triplet, because the central line remains single. It is true that there is some indication of a faint dark line in the middle of this central component; but as the two other oppositely polarized components have the same appearance, the doubling remains uncertain. Thus the essential difference in nature between D_1 and D_2, long ago exhibited by their unequal facility of spontaneous reversal, is demonstrated here by a well-defined characteristic, *i. e.*, unequal separation of the central components. This separation is very great in D_1, very small in D_2; the distance between the exterior components is, on the contrary, sensibly the same for both lines. May one not be permitted to suppose that the action of the magnetic field affects one of the most essential elements of the mechanism concerned in the genesis of these radiations?

The magnesium group b, composed of three lines b_1, b_2, b_4,[1] is equally instructive. One might expect to see the narrowest line transformed into a quadruplet; but this is not the case, it becomes a simple triplet. The intermediate line b_2, on the contrary, is sharply separated into four. The first line b_1, the most easily reversible of the three, is also separated into a quadruplet, but it is too diffuse to show the phenomenon in a satisfactory way.

The green line of thallium is also too broad to show satisfactorily the doubling of the central line. The green line (No. 4) of cadmium also separates into four, but an intense magnetic field is required to show this subdivision well.

If one were inclined to be in doubt, from the few observations made in the direction of the lines of force, regarding the special effect of the magnetic field on the radiations emitted, the results just cited, obtained in the direction normal to these lines, must remove all question. There is, moreover, no reason to fear errors due to imperfect adjustment of the optical apparatus; in fact, in the direction normal to the lines of force I have utilized as a separating apparatus only a small rhomboid of Iceland spar. As for the magnetic field, the uniformity of which is never perfect, I have convinced myself (by giving to the pole-pieces the most diverse forms) that if the mean intensity of the field varies with the form of the poles the relative distance of the components of the quadruplet nevertheless remains unchanged; the phenomenon thus in no way depends upon the particular form of the equipotential surfaces of the field.[2]

It might finally be objected, not without some reason, that the small scale of the deviations obtained up to the present time renders the interpretation of the appearances very uncertain.

[1] The line b_3 in the b group of the solar spectrum belongs to nickel.

[2] In this connection I have found a very curious method for rendering *visible* the equipotential magnetic surfaces in the neighborhood of the pole-pieces in very intense fields; I do not know whether it is known, but it is, in any case, very convenient. It consists in causing the uncondensed spark of a powerful induction coil to pass between two well-separated metallic electrodes placed in the field to be explored. The line of sparks is not deviated, but the violet halo is *blown* aside; it spreads out on one side

But this objection is not applicable to my experiments; thanks to the various precautions resulting from successive attempts, I obtain very sharp and brilliant images separated by well-defined dark intervals.[1]

This result is due to the use of the excellent plane grating which was employed in my solar spectroscopic studies,[2] and which I owe to the kindness of Professor Rowland. With this I have constructed a spectroscope of high dispersion,[3] in which the third order spectrum is particularly bright, so that the observed deviations are relatively large. I give below measures obtained in an observation made with a magnetic field of about 13,000 C. G. S. units.

Distance between the outer lines of the quadruplet D_1 . .	$0^t.54$ of the ocular micrometer.
Distance between the interior lines of the quadruplet D_1 . .	0 .26
Distance between the lines D_1, D_2 in their ordinary state .	3 .61

The pitch of the micrometer screw is half a millimeter.

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only in the form of a luminous mantle, veined in concentric curves, which closely follow the form of the equipotential surface, passing through the point where the discharge occurs, and its area increases with the intensity of the magnetic field at this point.

This mantle changes from one side to the other when either the direction of the induced current or that of the lines of force is reversed.

With easily volatile electrodes (thallium, metallic sodium, etc.), the phenomenon is especially brilliant.

If the electrodes are very close together a second mantle, symmetrical but narrower, appears on the other side, the whole forming a butterfly with unequal wings; it is evidently due to the discharge of the induced low tension direct current.

[1] As a particular instance I may mention that the sodium lines, D_1, D_2, are obtained by varying the proportion and the pressure of the oxyhydrogen gases impinging upon a globule of sodium glass; with a little skill one succeeds in producing at will all the known spectral appearances, lines faint and diffuse, lines bright and well-defined, with or without reversal.

In the induction spark passing between two poles of metallic sodium, the metal does not take fire even with a strong condensed discharge; but the lines are bright and reversed, and the quadruplet appears dark on a bright field.

[2] *Ann. Chim. et Phys.*, (6) 7, 5.

[3] *Jour. de Phys.*, (2) 2, 53. The spectroscope described in this article gave excellent results, but by replacing the flint prism with the grating the definition was considerably improved.

The greatest distance between the components separated by the magnetic action thus amounted to nearly one-sixth of the distance between the lines D_1 and D_2.

Remark.—This unexpected quadrupling of the vibratory period of a monochromatic source, normal to the lines of force, at first sight contradicts the simplicity of the elegant kinematic interpretation corresponding to the formation of the triplet, which led to the conclusion that the amplitude of vibration of the radiations is not modified in the direction of the lines of force. But on reflection I am convinced that the new experimental result, which we are forced to recognize, nevertheless agrees perfectly with the idea which may be formed of a line of magnetic force, which is defined by a *vector* or *directed quantity ;* the properties of the complex system which it represents thus depend upon the direction in which it is directed. Now the amplitude of vibration is also a directed quantity ; it is thus natural that the reciprocal influence of two parallel elements, both characterized by vectors, may be of two kinds according as the vectors in play are of the same or contrary sign. This is evidently a somewhat abstract argument, but one which nevertheless imposes the necessary condition. The resultant effect may be *nil;* this is what appeared from the earlier imperfect observations ; but not being *nil*, it necessarily has two equal values of contrary sign. This is exactly what the new observations show, that is to say, a variation of period symmetrical on either side of the original period.

If the kinematic interpretation of the phenomenon becomes a little more complex, it at the same time acquires a very suggestive symmetry regarding the constitution of the magnetic field.

Like the vibratory components normal to the lines of force, the component parallel to this direction is doubled and the periods of the two parts are altered by quantities respectively equal, of contrary sign, and proportional to the intensity of the field.

From what precedes it is evident how many important questions regarding the relationship of electricity with light are raised

by these new experiments. Although the observations are very delicate and still very incomplete, I have thought it desirable to make them known, with the intention of pursuing them when the necessary means which I hope to have at my disposal shall permit me to increase still further the magnitude of the effects and consequently the precision of the measures.

PARIS, January 21, 1898.

RÉSUMÉ OF SOLAR OBSERVATIONS MADE AT THE ROYAL OBSERVATORY OF THE ROMAN COLLEGE DURING THE SECOND HALF OF 1897.

By P. TACCHINI.

I GIVE below a résumé of the solar observations made at the Royal Observatory of the Roman College during the second half of 1897. The results for the spots and faculæ are brought together in the following table:

1897	Number of days of observation	Relative frequency		Relative areas		Number of spot groups per day
		of spots	of days without spots	of spots	of faculæ	
July	31	8.16	0.00	31.3	66.1	2.5
August	28	6.42	0.00	31.1	73.8	1.6
September	28	14.43	0.00	43.9	65.4	4.7
October	27	3.82	0.22	5.6	93.1	1.8
November	22	2.05	0.50	4.1	73.1	0.5
December	21	8.71	0.05	42.7	75.0	2.1

The season was very favorable, and it will be seen that the spots have continued to decrease, particularly in area. A rather marked minimum occurred in October and November, after the September maximum; a similar fluctuation may be found in the preceding series for the months of April, May and June.

For the prominences the following results have been obtained:

1897	Prominences			
	Number of days of observation	Mean number	Mean height	Mean extent
July	30	2.57	29″.9	1°.1
August	27	3.96	36 .1	1 .5
September	26	5.23	37 .2	1 .3
October	20	4.90	37 .7	1 .3
November	18	4.95	39 .2	1 .7
December	17	3.00	39 .0	1 .3

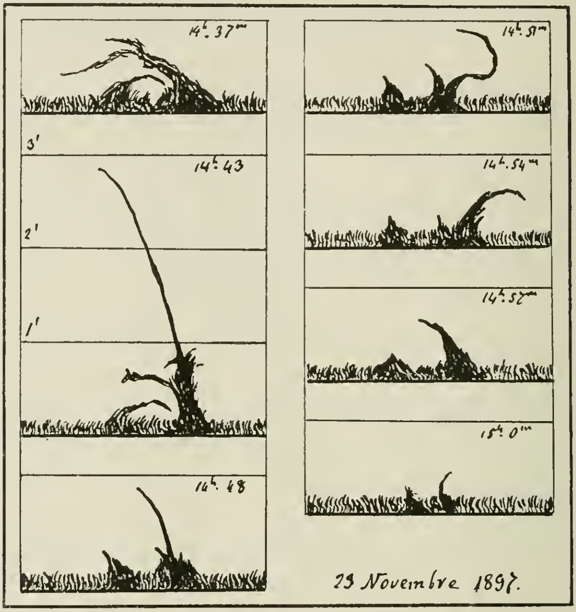
Comparing these numbers with the results obtained for the first half of the year we conclude that the prominences have remained practically stationary in activity.

The results for the distribution in latitude of the various phenomena are given by quarters and by zones in the following table:

1897 Latitude	Prominences		Faculæ		Spots	
	Third quarter	Fourth quarter	Third quarter	Fourth quarter	Third quarter	Fourth quarter
90° +80°	0.003	0.008				
80 +70	0.006	0.004				
70 +60	0.000	0.020				
60 +50	0.086	0.096				
50 +40	0.089 } 0.390	0.076 } 0.448	0.007	0.004		
40 +30	0.027	0.044	0.036	0.017		
30 +20	0.068	0.080	0.080 } 0.396	0.054 } 0.480		
20 +10	0.057	0.072	0.135	0.161	0.104 } 0.271	0.162 } 0.676
10 + 0	0.054	0.048	0.138	0.244	0.167	0.514
0 —10	0.140	0.104	0.215	0.219	0.373 } 0.729	0.243 } 0.324
10 —20	0.128	0.120	0.236	0.198	0.354	0.081
20 —30	0.116	0.100	0.109 } 0.604	0.095 } 0.520		
30 —40	0.030	0.032	0.029	0.004		
40 —50	0.092 } 0.610	0.044 } 0.552	0.015	0.004		
50 —60	0.083	0.132				
60 —70	0.012	0.016				
70 —80	0.006	0.004				
80 —90	0.003	0.000				

The prominences have continued to show themselves in almost all zones, with a maximum of frequency between the equator and the parallel of −20°. But it should be remarked that

two secondary maxima occurred at the same distance from the equator, *i. e.*, in the zones ($\pm 40° \pm 60°$). The spots have been confined within the region between the equator and $\pm 20°$, as was



the case during the second quarter. The only eruption observed during the entire period of six months was that of November 23, on the west limb, at latitude $+8°.2$. A very bright jet suddenly formed and rose to a height of 168″, disappearing in twenty minutes, as is indicated in the figures.

Rome, January 31, 1898.

ON THE ARC-SPECTRA OF THE ELEMENTS OF THE PLATINUM GROUP. II.

By H. Kayser.

IV. RHODIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2308.88	2	2432.755	1	2488.547	1
2318.432	2	2433.346	0	2489.986	0
2319.173	2	2436.974	0	2490.860	3
2328.737	2	2437.174	2	2492.395	2
2334.762	1	2439.338	0	2493.733	1
2345.597	1	2440.427	2	2494.604	4, u
2368.380	3	2442.830	0	2499.095	2, u
2369.654	2	2443.221	0	2500.668	2
2370.642	2	2443.812	0	2500.740	0
2382.969	2	2444.337	4, u	2501.115	1
2383.490	2	2444.843	0	2502.546	2
2384.751	2	2445.714	2	2502.843	1
2386.222	4	2448.378	0	2503.458	0
2386.489	0	2448.923	2	2503.939	1
2396.617	0	2450.660	4	2504.384	4, u
2399.044	0	2453.898	0	2505.189	2
2406.472	0	2455.521	0	2505.758	2
2407.974	2	2455.788	2	2507.342	0
2408.100	0	2456.277	1	2508.743	0
2408.275	1	2459.004	2	2509.788	2
2408.745	0	2459.237	1	2510.747	2
2409.626	0	2461.120	2	2511.133	2
2410.348	0	2463.670	4, u	2512.180	2
2412.613	1	2469.203	1	2513.464	2
2414.433	0	2470.486	2	2515.833	2
2414.662	0	2470.860	0	2518.561	0
2414.927	3	2471.561	2	2520.623	2
2417.523	0	2472.571	2	2522.988	2, u
2418.718	3	2473.199	2	2525.221	0
2420.271	2	2474.116	0	2526.092	1
2420.947	0	2474.677	1	2526.244	2
2421.060	2	2475.097	4	2526.744	0
2422.237	0	2475.749	0	2530.284	0
2424.021	2	2475.978	0	2531.053	0
2424.521	0	2477.618	0	2531.369	0
2427.193	3	2480.596	0	2531.920	2
2427.777	2	2480.921	0	2532.743	2
2429.053	2	2481.686	0	2533.687	2
2429.268	0	2483.423	2, u	2534.170	2
2429.610	2	2485.688	2	2534.682	0
2431.936	2	2487.581	4	2536.803	4

IV. RHODIUM — *continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2537.155	3	2607.831	2	2666.498	2
2537.721	2	2608.639	2	2667.317	0
2539.860	4, u	2609.266	0	2667.453	0
2541.096	2	2610.156	0	2669.419	0
2543.648	0	2612.315	0	2671.144	4
2544.317	2	2613.145	0	2671.529	1
2545.794	4	2613.689	4, u	2674.059	2
2547.366	0	2615.735	2	2674.287	2
2548.679	2	2616.178	2	2674.525	2
2551.289	2	2618.596	3	2676.200	4
2553.426	0	2621.099	2	2676.573	2
2555.010	0	2622.661	4	2680.379	2
2555.449	4	2622.756	1	2680.717	4
2556.172	1	2624.821	0	2681.873	3
2558.714	4	2624.948	0	2682.624	2
2560.322	2	2625.309	2	2683.660	0
2562.741	0	2625.496	1	2684.301	2
2564.900	0	2625.973	3	2685.551	0
2565.888	2	2626.776	2	2686.608	4
2566.137	2	2627.042	0	2687.015	4
2566.960	2	2628.222	0	2687.411	2
2567.374	4	2630.003	2	2688.173	2
2569.171	0	2630.509	2	2689.022	0
2570.206	2	2633.373	2	2689.716	0
2573.577	2	2633.523	2	2692.390	2
2574.332	2	2634.605	0	2692.463	2
2574.751	2	2635.082	4	2693.726	2
2576.330	2	2635.407	1	2694.405	4
2579.487	2	2636.744	1	2697.955	2
2579.650	0	2637.484	0	2700.384	2
2580.043	0	2638.388	0	2700.688	1
2581.100	2	2638.839	2	2702.158	2
2581.790	0	2639.097	0	2702.337	2
2584.016	1	2639.327	0	2702.621	0
2586.897	2	2642.857	0	2703.820	6
2587.245	2	2643.077	4	2705.059	0
2587.353	0	2643.691	2	2705.718	3
2588.545	0	2647.375	4	2706.135	2
2589.892	1	2648.681	2	2707.320	2
2592.247	0	2649.686	1	2707.896	0, u
2596.134	0	2650.985	0	2709.613	3
2597.014	3	2651.973	0	2714.499	4
2597.484	0	2652.750	5	2714.881	0
2597.774	2	2656.000	2	2715.149	2
2598.166	2	2658.515	0	2715.399	2
2599.352	0	2659.098	2	2716.645	0
2601.926	0	2659.573	2	2716.912	2
2603.500	4	2659.937	1	2717.606	3
2605.807	2	2663.389	0	2718.111	0
2606.540	4	2663.764	2	2718.648	2

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2720.235	3	2794.587	0	2852.809	0
2720.622	3	2795.366	1	2854.237	0
2722.243	2	2795.824	2	2854.848	2
2722.389	0	2796.743	3	2855.273	4
2725.961	0	2799.536	0	2859.735	2
2726.934	0	2799.705	0	2859.908	3
2729.034	6	2800.021	0	2860.208	0
2729.611	0	2801.674	3	2860.774	3
2731.874	0	2802.113	0	2860.886	4
2732.261	0	2804.020	2	2861.877	0
2734.906	2	2805.908	2	2862.572	0
2736.860	3	2806.212	1	2863.057	6
2737.509	2	2807.270	2	2864.517	4
2737.717	2	2809.853	0	2865.755	2
2738.359	2	2810.999	3	2867.973	1
2739.845	1	2814.817	0	2868.400	2
2740.027	1	2816.979	1	2869.746	0
2740.304	2	2819.367	2	2870.108	2
2740.487	0	2819.742	3	2870.551	2
2740.647	2	2820.946	3	2871.489	5, u
2743.568	0	2821.620	1	2873.104	0
2751.140	0	2822.850	0	2873.742	4
2751.450	2	2822.979	2	2874.115	2
2752.941	2	2823.504	2	2874.507	0
2754.845	0	2823.756	0	2875.764	2
2757.005	1	2823.988	0	2876.592	0
2760.541	2	2826.532	4	2878.139	0
2762.311	0	2826.798	4	2878.770	4
2762.938	2	2827.433	4	2879.628	0
2764.909	2	2828.259	0	2880.775	1
2767.832	4	2829.421	2	2880.912	2
2768.336	4	2829.664	2	2881.400	2
2770.277	1	2831.398	0	2882.497	4
2771.615	4	2832.893	2	2884.683	2
2773.397	2	2833.981	1	2885.364	0
2774.557	2	2834.233	4	2886.112	4
2775.869	2	2834.990	1	2887.082	0
2778.162	4	2835.671	1	2888.986	0
2778.967	3	2836.799	4	2889.222	4
2779.654	3	2838.425	2	2889.623	1
2780.439	3	2839.666	0	2889.962	4
2781.184	1	2841.909	4, U	2892.320	4
2783.140	5	2842.270	4, u	2892.817	0
2785.920	0	2844.463	4, u	2893.142	1
2786.934	2	2844.917	0	2895.823	1
2790.493	2	2845.868	2	2897.171	0
2790.872	2	2849.461	2	2897.806	0
2791.270	4	2850.608	1	2899.800	2
2792.886	2	2851.526	0	2900.080	4
2794.020	2	2852.459	1	2902.975	0

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2903.428	2	2961.805	2	3036.483	0
2903.960	0	2963.664	2	3038.583	2, u
2904.440	0	2965.018	0	3043.586	0
2905.106	2	2965.801	0	3045.887	3
2907.335	3	2968.790	6	3046.304	2
2907.835	1	2970.807	1	3046.871	4
2909.837	0	2971.741	0	3047.440	0
2910.281	4	2974.156	3	3048.095	2
2912.746	4	2975.935	2	3049.003	0
2913.185	0	2977.809	5	3049.334	2
2913.474	0	2981.238	2	3049.919	0
2913.715	2	2982.514	3	3050.050	0
2914.114	4	2983.194	4	3050.842	2, u
2914.691	0	2984.135	0	3051.780	2
2915.534	4	2984.593	0	3053.988	2
2917.028	0	2986.330	7	3054.980	0
2920.296	1	2987.117	5	3055.755	0
2921.229	0	2987.568	3	3056.452	0
2923.239	4	2988.487	0	3057.996	4
2924.140	4	2988.977	0	3058.974	1
2926.160	0	2989.302	0	3059.473	2
2926.322	0	2990.048	2	3060.001	0
2926.953	0	2990.158	0	3061.782	2
2927.062	0	2991.881	2	3062.544	0
2928.559	0	2995.828	0	3063.700	1
2929.256	4	3001.582	0	3065.800	0
2932.065	4	3004.565	5	3066.333	0
2934.988	0	3005.929	2	3066.475	0
2937.285	2	3009.103	1	3067.395	6
2938.403	2	3010.369	0	3069.034	2
2939.588	2	3011.021	0	3070.467	1
2940.175	0	3014.352	2	3071.134	3
2941.246	3	3015.960	0	3071.716	1
2942.116	0	3016.930	1, u	3073.550	0
2946.042	2	3017.225	1	3074.806	2
2948.388	0	3018.194	0	3076.736	2
2949.475	1	3019.569	2	3078.905	0
2950.023	3	3019.664	2	3080.449	0
2951.957	1	3019.928	0	3081.714	0
2955.395	2	3022.117	0	3084.078	4
2955.541	2	3022.673	0	3085.790	2
2955.942	0	3023.164	0	3087.180	0
2956.229	0	3024.018	3	3087.534	4
2956.406	1	3025.517	2	3088.428	2
2958.504	0	3027.053	2	3089.480	0
2958.899	4	3027.817	1	3089.775	0
2959.478	1	3028.545	4	3090.506	2
2959.769	4	3028.975	0	3091.840	0
2960.686	0	3031.573	0	3093.592	0
2960.773	0	3034.474	0	3094.691	2

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3096.722	0	3177.020	0	3255.104	4
3096.834	1	3177.201	4	3258.352	0
3099.567	0	3178.517	4	3259.994	0
3102.634	4	3179.833	5	3260.938	2, u
3105.110	4	3181.330	3	3263.280	8
3105.756	1	3182.519	0	3263.924	2
3108.405	2	3183.012	0	3264.313	0
3115.027	5	3183.558	0	3266.511	1
3119.846	0	3184.485	0	3267.605	1
3120.714	0	3185.702	5	3268.597	5
3121.381	0	3187.265	0	3270.702	3
3121.879	6	3187.740	0	3271.748	8
3123.818	6	3187.998	1	3274.908	4
3124.508	2	3188.408	1	3276.122	4
3125.000	0	3189.162	5	3278.620	2
3126.990	2	3190.466	3	3280.680	4, r
3130.918	4	3191.313	6	3281.827	4
3134.047	1	3192.112	0	3282.932	0
3134.710	0	3192.336	0	3283.705	4, r
3135.590	2, u	3193.633	1	3284.151	0
3137.450	4	3193.963	2	3285.964	2
3137.825	5	3194.671	4	3286.520	4
3138.506	1	3197.257	4	3288.159	2
3140.355	0	3199.979	1	3289.274	5
3140.549	1	3206.202	4	3289.739	5
3140.963	0	3207.390	2	3292.531	0
3141.314	0	3211.504	3	3293.012	0
3145.518	1	3212.667	0	3293.533	0
3145.734	2	3214.440	4	3294.400	5
3146.327	0	3214.628	0	3294.843	1
3147.274	0	3214.984	4	3296.847	4
3147.736	4	3218.009	4	3297.409	2
3148.350	1	3218.395	4	3297.667	0
3149.580	0	3218.655	0	3299.066	2
3149.978	0	3220.893	2	3300.133	0
3150.385	4	3221.193	0	3300.479	2
3152.724	6	3221.422	1	3300.604	4
3159.453	0	3221.589	0	3301.820	0
3155.489	6	3232.627	4	3303.474	0
3158.063	2	3233.440	0	3303.872	0
3159.001	2	3234.656	0	3304.258	2
3159.354	2	3235.910	2	3305.298	4
3162.388	1	3237.781	4	3307.091	1
3162.608	0	3240.644	0	3307.474	0
3163.551	1	3240.998	0	3308.067	3
3167.072	0	3241.602	0	3309.663	2
3170.379	0	3242.111	0	3314.665	2
3171.625	2	3242.820	1	3316.670	0
3172.392	4	3250.151	2	3323.232	6, r
3176.666	0	3253.457	2	3331.223	4

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3331.393	4	3399.823	7	3485.031	2
3332.648	1	3401.109	3	3487.366	3
3335.328	0	3403.247	0	3487.621	3
3336.842	0	3404.021	2	3491.216	3
3338.672	7	3406.690	5	3491.365	3
3340.987	0	3407.387	2	3494.585	5
3343.036	5	3407.884	2	3498.887	7
3343.573	2	3408.990	0	3502.686	8, r
3344.337	5	3410.074	0	3505.559	4
3347.437	1	3410.625	1	3507.471	4, r
3347.660	0	3412.425	6	3508.754	1
3352.510	2	3415.824	0	3509.444	3
3353.834	2	3416.901	0	3511.696	3
3354.853	4	3420.307	4	3511.942	4
3356.670	1	3422.430	3	3513.258	4
3357.560	0	3423.699	0	3519.690	2
3357.980	2	3424.533	4	3525.805	2
3358.962	0	3428.559	2	3528.183	7, r
3360.043	6	3432.234	2	3538.269	3
3360.952	8	3435.037	10, r	3538.409	3
3362.321	5	3440.675	4	3542.068	4
3363.382	0	3442.243	0	3544.122	5
3364.281	0	3442.781	4	3549.681	5
3365.138	0	3443.001	2	3550.165	0
3365.652	0	3446.202	0	3564.290	2
3368.518	6	3447.897	6	3590.688	1
3368.914	3	3448.715	5	3593.685	3
3369.824	5	3450.437	5	3594.054	0
3372.379	7	3451.294	4	3596.183	4
3372.672	2	3454.617	0	3596.343	4, r
3372.930	0	3455.369	4	3597.300	6, r
3373.879	0	3455.595	4	3598.057	3
3375.735	0, u	3456.284	0	3600.911	4
3376.017	0	3457.219	5	3606.029	5
3377.275	5	3458.070	6	3608.246	4
3377.742	2	3458.815	0	3612.621	5, r
3377.850	4	3459.375	3	3614.099	1
3380.775	4	3462.191	5, r	3614.674	1
3381.208	0	3469.355	0	3614.934	4
3381.578	4	3469.774	5	3620.621	5
3385.919	6	3470.505	1	3626.759	7
3387.174	2	3470.817	4, r	3627.342	4
3387.960	0	3472.402	5	3627.958	4
3389.340	3	3472.994	0	3639.684	6
3390.608	1, u	3474.939	5, r	3643.301	0
3391.847	2	3477.354	1	3644.363	0
3391.935	2	3478.646	2	3651.516	2
3392.230	1	3479.064	4, r	3655.044	5
3395.014	3	3480.658	0	3658.148	8, r
3396.956	8, r	3484.186	4	3661.760	2

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3662.027	3	3856.167	0	4177.780	1
3666.381	7	3856.663	4, r	4196.672	3
3667.070	4	3865.291	1	4206.770	2
3674.924	5	3870.140	2	4211.306	5, r
3681.205	6	3872.534	0	4218.142	2
3690.872	4, r	3877.470	2	4228.002	0
3691.481	2	3888.475	2	4230.354	2
3692.506	10, r	3891.953	0	4244.598	4
3695.105	2	3904.362	2	4258.608	1
3695.674	5	3905.423	1	4270.696	2
3698.415	3	3912.971	2	4273.578	4
3698.758	5	3913.657	4	4276.962	2
3699.461	2	3922.340	4	4278.744	3
3701.057	8, r	3934.384	4, r	4288.883	7
3713.156	4, r	3935.123	2	4296.926	4
3713.593	3	3935.982	4	4308.982	2
3714.989	4	3942.862	5	4315.126	2
3725.091	2	3953.214	2	4325.584	0
3735.429	4	3958.313	4	4336.181	1
3737.448	4	3959.006	5, r	4342.608	1
3744.325	4	3964.688	4	4345.247	2
3748.383	5	3968.320	2	4345.629	3
3754.269	4	3975.472	5	4349.336	2
3754.441	3	3976.240	2	4362.393	0, u
3755.290	1	3984.555	5	4373.212	5
3755.748	2	3995.768	4	4374.976	7, r
3760.554	2	3996.313	5	4376.350	1
3765.232	5, r	4023.302	5	4380.097	5, u
3770.130	4	4026.089	1	4388.224	1
3771.779	2	4048.572	3	4402.725	1
3775.864	2	4049.188	3	4410.449	0
3778.279	4	4053.602	3	4420.178	0, u
3788.633	5	4056.491	3	4421.383	0
3793.366	4, r	4077.739	4	4423.835	1
3799.466	4, r	4080.690	1	4424.215	3
3806.071	4	4081.961	2	4433.495	4
3806.920	4	4082.942	5	4484.015	2
3809.655	2	4084.442	2	4492.644	4
3812.599	2	4087.950	4	4503.955	4
3815.169	1	4088.646	4	4506.815	1
3816.611	1	4097.690	5	4528.904	5
3817.524	0	4116.496	4	4530.763	3
3817.990	0	4119.855	4	4544.447	4
3818.345	5	4121.870	4	4551.828	6
3822.397	4, r	4125.063	2	4557.343	4
3827.505	0	4129.080	6	4558.897	4
3828.623	2	4135.448	4, r	4561.062	5
3833.733	0	4137.008	0	4565.373	4
3834.016	5, r	4154.495	2	4568.538	2
3834.893	2	4158.615	1	4569.181	6

IV. RHODIUM — *continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4570.489	2	4918.953	2	5237.918	1
4571.466	4	4919.823	2	5248.918	0
4572.794	2	4922.633	2	5251.549	2, u
4599.553	0	4944.975	2	5259.382	3
4601.792	2	4960.318	1	5268.092	0
4608.294	2	4961.012	0	5269.429	3
4620.059	5	4963.831	4	5280.250	2
4626.105	1	4966.511	2	5292.279	4
4634.017	4	4977.869	4	5314.911	3
4639.526	4	4985.107	2	5329.571	0
4643.337	6	4996.012	0	5329.890	4
4666.261	2	4997.919	1	5331.237	2
4675.187	7	5012.538	0	5336.794	0
4677.532	4	5025.692	1	5339.845	0
4683.093	3	5028.492	2	5349.463	2
4689.610	1	5046.583	2	5354.573	7
4696.463	1	5057.576	2	5356.638	3
4704.230	5	5064.475	4	5359.850	0
4707.108	1, u	5073.607	0	5364.290	0
4719.545	2	5085.676	4	5369.470	1
4721.148	6	5088.949	0	5379.275	5
4724.483	2	5090.795	5	5381.683	0
4731.333	1, u	5110.115	2	5384.214	0
4745.276	6	5120.824	1	5390.622	5
4750.007	0	5130.903	2	5404.898	4, u
4755.717	4	5145.110	2	5408.972	2
4770.938	3	5155.691	5	5423.483	2, u
4771.687	2	5157.224	2	5424.910	4
4777.304	2	5157.814	5	5425.636	4, u
4791.164	3	5160.464	0, u	5431.813	2, u
4791.640	0	5165.561	0	5432.224	2, u
4794.364	0	5174.883	0	5439.783	4
4798.829	4	5176.110	6	5441.547	4, u
4801.517	1, u	5177.396	3	5444.508	2, u
4803.393	0	5178.311	0	5445.424	4, u
4810.645	6	5184.342	4	5468.288	3, u
4813.678	1	5185.172	1	5468.921	2, u
4817.233	0	5187.088	0	5471.040	5, u
4833.627	0	5193.276	7	5475.318	2, u
4842.556	4	5197.697	1	5480.997	0
4844.145	6	5203.468	2	5481.602	2, u
4851.777	6	5207.099	3	5484.421	4, u
4856.614	0	5211.637	4	5492.048	2, u
4861.497	2, u	5212.866	4	5497.197	0
4861.808	0, u	5213.491	2	5503.776	2, u
4865.922	4	5214.913	3	5504.845	4, u
4888.045	0	5222.783	4	5534.074	1, u
4898.022	1	5225.706	1	5535.235	5, u
4908.744	2	5230.752	4	5542.260	0
4913.649	2	5237.284	5	5544.797	6, u R

IV. RHODIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
5555.288	0	5659.924	2, u	5797.668	2
5556.968	3	5686.543	4	5803.482	2
5557.364	1, u	5695.823	1	5807.058	4
5568.495	0	5700.628	4, u	5821.991	2
5595.053	2, u	5708.930	0, u	5831.730	4
5599.620	6, u	5713.799	1, u	5833.808	1, u
5605.214	0	5718.038	0	5871.947	1
5607.898	3	5726.875	1, u	5899.128	1
5608.541	4	5727.466	3	5907.478	1
5626.254	2	5730.600	2	5918.698	1
5632.954	2	5742.985	0	5941.743	1
5634.847	2	5755.894	0	5952.791	0
5651.466	1, u	5792.824	4	5983.830	4
5659.791	4	5795.936	2		

V. OSMIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2325.636	0	2371.270	1	2408.764	2
2327.081	0	2376.398	0	2409.010	1
2329.356	0	2377.128	2	2409.476	0
2332.288	1	2377.704	0	2410.282	0
2334.640	1	2378.842	0	2411.536	1
2336.876	1	2379.482	1	2411.992	0
2338.723	1	2379.730	0	2414.042	0
2340.732	0	2379.931	0	2414.198	0
2342.043	0	2382.595	0	2414.639	1
2343.831	1	2384.715	0	2415.436	0
2345.855	0	2387.378	2	2418.081	1
2347.480	0	2391.248	0	2418.457	0
2350.323	0	2393.986	0	2418.618	1
2351.678	0	2394.379	0	2420.137	0
2351.826	0	2395.969	0	2421.268	0
2355.378	0	2396.855	0	2421.949	0
2356.999	0	2397.730	0	2422.106	0
2357.344	0	2398.300	0	2423.158	2
2362.498	0	2401.219	2	2424.102	0
2362.855	2	2402.328	0	2424.655	1
2363.128	0	2402.620	0	2424.820	0
2363.421	1	2403.944	1	2426.297	0
2367.434	2	2405.176	0	2426.907	0
2369.346	1	2405.531	0	2427.280	0
2370.796	1	2406.053	0	2427.386	0

V. OSMIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2427.997	0	2491.106	2	2547.289	0
2429.025	0	2491.789	2	2548.196	2
2429.801	0	2492.477	2	2548.930	1
2431.299	1	2493.710	0	2550.873	0
2431.699	1	2493.935	0	2554.558	2
2434.605	0	2496.425	1	2555.205	1
2434.731	0	2498.512	1, u	2555.378	1
2437.798	0	2500.821	1	2555.902	1
2440.913	0	2501.016	0	2556.179	0
2442.104	0	2501.963	0	2557.868	0
2445.980	0	2502.382	2	2558.191	1
2446.125	1	2503.766	2	2560.308	0
2449.987	0	2504.486	2	2560.578	0
2450.581	0	2504.603	0	2560.831	0
2450.833	1	2506.481	0	2562.771	1
2451.290	0	2506.767	0	2563.257	2
2452.869	0	2507.282	0	2564.287	1
2453.392	0	2508.707	1	2564.469	0
2453.989	0	2509.809	0	2565.261	2
2454.278	0	2510.024	2	2565.816	0
2455.002	1	2510.591	0	2566.595	3
2455.422	0	2512.970	2	2567.335	0
2455.716	0	2513.340	2	2568.937	2
2456.555	1	2515 140	2	2570.855	0
2457.273	0	2518.006	2	2571.244	0
2457.804	0	2518.533	2	2571.611	0
2459.940	0	2519.886	1	2571.878	2
2461.508	3	2520.156	0	2572.572	1
2464.577	1	2524.879	0	2573.198	0
2466.535	0	2526.833	0	2573.601	0
2467.420	0	2527.174	0	2574.852	1
2468.209	0	2527.335	0	2577.141	0
2470.925	0	2527.832	1	2578.284	1
2472.378	1	2529.047	0	2578.430	2
2473.756	0	2532.083	1	2579.839	0
2475.064	0	2532.732	0	2580.120	2
2475.769	0	2534.270	1	2581.154	2
2476.179	0	2535.484	0	2582.027	4
2476.923	2	2536.184	0	2586.995	0
2477.100	0	2538.087	4	2587.575	1
2480.825	0	2538.174	1	2588.517	0
2481.892	1	2538.500	0	2589.495	0
2482.524	2	2539.751	0	2589.595	0
2485.424	0	2540.230	2	2590.859	4
2486.326	3	2540.835	2	2592.082	2
2488.415	1	2541.747	0	2594.000	0
2488.640	4	2542.592	4	2594.238	2
2488.890	0	2543.892	0	2596.101	2
2489.113	0	2544.067	4	2596.474	0
2489.370	0	2546.261	2	2596.783	2

V. OSMIUM — *continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2597.092	0	2650.754	0	2696.709	0
2597.319	1	2651.562	0	2697.338	2
2597.664	0	2652.369	0	2698.321	0
2597.990	0	2653.068	2	2699.688	4
2600.008	1	2653.388	1	2700.840	2
2600.560	1	2653.860	2	2703.203	0
2600.855	0	2655.297	0	2704.551	2
2602.444	1	2655.879	1	2704.695	0
2603.323	0	2656.774	2	2705.547	0
2603.554	0	2657.203	0	2706.804	2
2604.701	2	2658.682	4	2707.519	2
2605.051	0	2659.924	2	2708.276	2
2608.342	0	2661.011	1	2709.953	2
2609.303	2	2662.069	2	2712.848	0
2609.669	2	2662.653	2	2713.300	0
2610.881	2	2663.314	2	2714.744	3
2611.410	2	2663.950	0	2714.997	0
2612.732	2	2664.390	0	2715.471	2
2613.167	4	2664.879	4	2715.726	2
2614.158	0	2665.370	0	2717.162	0
2615.122	0	2666.079	2	2717.488	0
2617.062	0	2666.295	2	2717.839	0
2617.895	0	2667.593	0	2718.796	1
2618.435	0	2669.158	0	2720.130	4
2618.923	0	2669.606	2	2720.578	1
2620.035	4	2670.640	0	2721.959	4
2620.723	2	2672.145	0	2722.700	0
2621.473	0	2674.654	2	2722.867	0
2621.912	2	2674.793	0	2727.357	0
2623.711	0	2674.969	2	2728.364	2
2624.677	0	2677.473	0	2729.093	0
2625.436	0	2678.870	0	2730.782	4
2628.377	2	2679.457	0	2731.467	1
2632.994	1	2679.825	1	2731.931	0
2634.375	2	2680.806	0	2732.905	4
2634.547	1	2682.279	2	2735.848	0
2637.223	4	2683.974	0	2736.479	1
2638.081	0	2684.497	2	2738.427	0
2638.428	0	2685.973	0	2738.636	2
2639.533	0	2686.624	0	2740.414	2
2640.079	0	2686.777	0	2740.701	2
2640.625	0	2687.277	0	2740.862	2
2641.271	1	2688.174	2	2742.801	0
2641.700	2	2689.447	2	2744.981	0
2643.132	1	2689.904	4	2745.632	1
2643.727	2	2691.483	0	2748.003	2
2644.211	4	2692.021	0	2748.964	2
2645.207	0	2692.790	2	2750.970	0
2647.817	2	2694.615	2	2751.246	2
2649.428	2	2694.854	0	2751.875	0

V. OSMIUM —*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2753.792	2	2796.833	2	2846.507	2
2754.780	0	2799.692	1	2846.707	2
2755.680	0	2802.039	1	2847.408	0
2756.095	0	2804.055	0	2848.360	2
2757.902	2	2804.185	2	2849.175	2
2758.775	0	2805.576	0	2849.427	0
2758.923	2	2807.025	5	2850.877	4
2760.168	0	2807.600	0	2853.441	0
2761.184	2	2807.910	0	2853.971	0
2761.530	2	2808.357	0	2855.455	1
2762.745	0	2809.045	4	2857.117	0
2763.371	2	2809.815	0	2857.659	2
2764.032	2	2810.468	0	2858.210	0
2764.637	0	2810.680	0	2858.733	0
2765.143	2	2811.683	2	2860.184	2
2765.541	1	2813.130	0	2861.075	4
2766.650	0	2813.904	2	2861.895	0
2767.236	1	2814.318	3	2864.366	2
2768.369	0	2814.602	0	2865.131	0
2769.385	3	2814.962	2	2865.802	2
2769.975	1	2815.380	1	2865.892	0
2770.213	1	2815.895	2	2867.216	1
2770.825	4	2818.897	0	2872.529	3
2771.150	1	2819.349	1	2873.126	0
2771.869	0	2819.601	0	2873.534	3
2773.176	2	2820.298	2	2874.700	1
2773.592	0	2820.682	2	2875.083	4
2774.125	2	2821.367	2	2875.930	0
2774.257	0	2823.687	0	2876.602	0
2774.488	2	2824.051	0	2877.464	3
2775.004	2	2824.283	2	2878.524	3
2777.011	2	2824.918	0	2879.095	0
2779.197	1	2825.013	1	2879.956	0
2779.584	0	2825.437	0	2880.327	2
2780.269	0	2827.038	0	2880.477	0
2780.970	0	2827.670	0	2884.064	1
2781.972	1	2829.138	0	2884.537	2
2782.658	4	2829.390	2	2884.967	0
2785.147	2	2829.468	1	2885.295	0
2786.061	1	2831.693	2	2886.182	1
2786.414	4	2832.345	2	2886.368	0
2786.904	2	2837.542	2	2886.622	2
2787.153	1	2838.283	3	2889.280	1
2789.620	0	2838.751	5	2889.654	0
2791.007	2	2839.792	0	2890.970	2
2792.844	0	2840.557	2	2891.961	1
2794.091	2	2841.711	4	2892.466	1
2794.309	2	2844.501	4	2893.014	0
2795.275	1	2844.802	2	2896.183	3
2796.221	0	2845.067	0	2898.023	0

V. OSMIUM — *continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2899.372	0	2937.111	0	2979.555	2
2901.308	0	2938.491	0	2979.802	0
2901.455	2	2938.590	0	2980.453	0
2903.193	2	2939.519	0	2982.252	2
2903.354	2	2940.208	0	2982.680	2
2905.862	2	2940.694	0	2983.032	3
2906.103	2	2940.873	0	2984.419	1
2906.909	0	2941.985	0	2984.751	0
2908.150	2	2942.267	1	2985.084	0
2908.468	0	2942.348	2	2985.752	2
2909.185	6	2942.692	0	2988.396	2
2909.797	2	2942.981	2	2989.253	2
2910.801	1	2943.291	2	2989.655	2
2911.269	0	2943.756	1	2989.963	0
2911.466	2	2945.437	0	2990.763	1
2911.695	0	2946.705	0	2992.240	3
2911.939	0	2947.277	0	2993.698	2
2912.470	2	2948.328	4	2994.908	0
2913.969	2	2949.635	3	2995.298	0
2914.341	1	2949.930	1	2995.762	2
2914.841	2	2950.986	1	2996.385	0
2915.382	0	2951.357	1	2997.777	3
2915.586	0	2952.412	2	3000.234	1
2916.193	0	2955.128	1	3003.605	2
2917.383	4	2956.629	2	3004.872	0
2917.946	3	2957.214	2	3005.064	0
2919.053	0	2957.774	0	3005.878	0
2919.380	0	2958.467	1	3008.022	2
2919.935	4	2961.140	4	3012.902	1
2920.204	1	2961.526	0	3013.194	4
2920.974	0	2962.272	4	3014.068	2
2921.193	2	2962.465	2	3015.158	0
2922.818	0	2962.819	0	3015.772	2
2923.109	0	2963.005	1	3017.380	3
2923.298	2	2963.178	0	3018.169	4
2924.617	2	2964.190	4	3018.440	0
2925.414	2	2964.890	0	3018.744	0
2925.708	3	2965.215	1	3019.498	3
2927.370	0	2966.217	0	3020.782	3
2929.646	2	2966.428	0	3021.226	0
2930.334	2	2966.685	0	3022.382	0
2930.704	2	2967.860	0	3024.434	0
2931.416	4	2969.938	0	3027.659	1
2931.879	0	2970.825	0	3027.790	0
2932.585	2	2971.098	3	3028.032	2
2934.111	3	2975.461	2	3029.496	2
2934.420	0	2976.470	0	3030.817	4
2934.779	3	2977.757	3	3031.122	2
2935.083	0	2978.338	2	3031.418	2
2936.817	2	2978.645	2	3031.828	1

V. OSMIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3032.924	2	3074.192	4	3116.593	2
3033.331	2	3074.771	0	3117.215	0
3033.843	0	3075.074	4	3118.014	1
3036.668	2	3076.845	1	3118.242	2
3040.184	1	3077.167	3	3118.450	2
3041.021	5	3077.557	2	3119.196	0
3042.860	2	3077.834	4	3120.016	2
3043.622	2	3078.227	2	3120.777	0
3043.793	2	3078.496	2	3121.307	0
3044.040	1	3080.614	0	3121.592	0
3044.191	2	3080.907	0	3124.142	1
3044.525	2	3081.313	0	3125.643	0
3045.031	2	3083.565	0	3127.620	0
3045.430	2	3084.715	2	3128.677	1
3045.898	2	3085.004	2	3129.348	2
3046.200	0	3085.982	0	3130.125	2
3047.574	1	3086.394	2	3131.021	0
3049.172	2	3087.125	0	3131.227	4
3049.580	3	3087.868	2	3131.600	1
3050.517	3	3088.385	2	3131.995	0
3051.280	2	3088.545	0	3133.953	1
3052.540	2	3090.205	2	3134.805	0
3053.004	0	3090.416	2	3135.126	0
3053.743	0	3090.613	2	3136.334	0
3054.091	2	3091.368	2	3136.785	0
3054.620	1	3092.613	0	3137.421	0
3054.780	0	3093.704	3	3137.636	2
3055.086	2	3194.192	1	3138.157	1
3055.326	2	3102.503	1	3139.745	0
3055.726	0	3102.835	2	3140.431	2
3056.315	0	3103.412	0	3141.056	2
3057.014	1	3105.098	2	3143.169	2
3058.782	6	3106.114	3	3144.471	2,d?
3060.248	0	3106.762	0	3146.074	2
3060.412	3	3107.119	0	3146.843	0
3061.814	1	3107.495	2	3147.601	0
3062.039	0	3107.989	2	3149.365	0
3062.297	4	3108.098	2	3149.927	1
3062.584	1	3108.846	0	3150.260	0
3062.803	2	3109.102	3	3150.730	0, U
3063.480	1	3109.504	4	3151.005	0
3065.391	0	3109.800	2	3152.181	3
3065.783	0	3110.538	1	3152.806	4
3066.225	2	3110.743	2	3153.727	4
3066.715	1	3111.196	3	3154.666	0
3066.945	2	3112.630	0	3155.450	1
3070.049	3	3113.405	0	3156.365	6
3070.374	1	3114.932	2	3156.878	3
3071.974	1	3115.150	1	3157.102	0
3072.681	0	3115.838	0	3157.342	1

V. OSMIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3159.477	0	3220.318	1	3275.320	4
3160.397	0	3220.408	0	3276.533	0
3160.540	0	3220.895	4	3278.086	4
3161.547	1	3221.444	0	3279.590	1
3161.837	1	3223.987	1	3281.028	2
3164.550	0	3226.579	0	3281.778	0
3164.718	2	3227.409	2	3284.680	0
3165.772	2	3229.336	0	3288.616	0
3166.611	4	3230.525	0	3288.960	2
3168.390	2	3231.410	0	3289.387	4
3171.249	0	3231.543	2	3291.259	1
3173.306	2	3232.072	2	3298.374	0
3173.609	0	3232.196	4	3301.692	7
3174.037	4	3232.672	1	3301.990	1
3174.284	1	3234.318	2	3304.980	0
3175.781	0	3234.651	0	3305.501	2
3177.522	1	3234.858	0	3306.352	2
3178.184	4	3238.304	1	3311.035	4
3178.357	2	3238.751	4	3312.178	0
3180.237	1	3239.398	0	3315.555	2
3181.907	1	3241.159	3	3315.816	2
3183.341	0	3241.642	2	3316.822	2
3183.661	1	3241.933	0	3317.420	0
3183.905	0	3242.108	1	3317.998	0
3184.458	0	3243.700	0	3318.284	0
3185.304	0	3248.106	0	3318.724	0
3185.439	3	3250.695	0	3322.175	0
3186.516	2	3250.974	0	3322.734	1
3186.643	2	3255.038	3	3324.486	4
3187.096	4	3255.139	0	3324.876	0
3187.443	2	3255.414	0	3325.518	2
3189.566	3	3257.051	4	3325.644	0
3193.986	2	3259.530	0	3327.562	4
3194.350	4	3260.420	3	3329.252	0
3194.805	3	3260.683	1	3333.986	0
3195.494	2	3262.428	6	3334.295	2
3196.082	1	3262.880	4	3336.282	4
3196.152	0	3264.820	2	3339.601	0
3197.310	0	3266.565	2	3340.851	0
3202.956	1	3266.890	0	3342.018	2
3204.155	2	3267.338	2	3348.791	2
3204.646	0	3268.080	6	3351.853	2
3205.909	0	3269.340	4	3354.042	1
3212.240	2	3270.025	0	3358.095	4
3212.840	2	3271.002	0	3359.876	1
3213.418	3	3271.320	0	3361.280	3
3216.340	0	3272.118	0	3361.905	0
3217.177	1	3272.301	3	3362.716	0
3218.153	0	3272.607	0	3364.250	1
3219.260	0	3273.513	1	3364.486	0

V. OSMIUM — *continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3368.617	2	3469.517	0	3965.106	1
3370.340	2	3477.798	0	3969.832	2
3370.725	4	3478.670	2	3975.596	3
3371.602	1	3482.269	2	3977.389	4
3372.929	0	3482.380	2	3979.524	0
3373.337	0	3487.387	2	3988.340	2
3375.262	0	3487.610	2	3988.785	0
3377.088	2	3488.915	2	3995.103	0
3380.674	0	3490.464	2	3996.979	0
3381.814	2	3498.686	2	3999.110	0
3383.042	2	3501.314	2	4003.652	2
3384.732	2	3504.811	4	4004.184	2
3386.077	2	3513.145	1	4015.203	0
3386.277	3	3513.791	2	4018.425	0
3387.970	4	3528.743	3	4035.249	0
3388.794	1	3598.260	2	4036.640	0
3391.401	1	3601.984	0	4038.009	0
3395.862	2	3604.624	2	4038.813	0
3396.973	2	3616.726	2	4042.081	2
3397.910	0	3630.099	0	4048.216	0
3398.713	0	3640.487	4	4051.584	0
3400.264	0	3648.962	1	4053.417	2
3401.315	2	3653.873	0	4055.647	1
3402.002	4	3654.631	2	4055.859	0
3402.643	3	3657.048	2	4066.460	0
3402.855	0	3671.040	3	4066.862	3
3406.423	2	3675.599	1	4071.020	0
3406.816	2	3681.705	2	4071.169	0
3408.906	2	3689.191	4	4073.768	2
3412.908	0	3691.750	0	4074.829	0
3412.946	0	3700.688	1	4088.598	0
3414.390	1	3703.391	2	4091.980	2
3421.558	0	3746.612	0	4097.087	1
3421.837	2	3895.331	0	4098.233	0
3422.800	1	3900.541	2	4100.436	0
3427.590	0	3901.851	2	4112.177	2
3427.816	5	3915.543	0	4124.760	0
3434.023	4	3925.253	1	4129.114	0
3437.150	2	3926.923	1	4135.955	4
3437.642	0	3928.557	0	4138.021	1
3438.792	0	3928.691	0	4152.448	0
3439.639	1	3930.148	0	4172.708	1
3444.616	2	3931.660	2	4173.391	2
3445.695	3	3938.739	2	4175.783	1
3449.352	4	3939.704	1	4190.059	2
3455.172	2	3949.925	0	4201.541	0
3459.163	2	3952.904	0	4212.028	3
3462.335	1	3960.656	2	4219.005	0
3465.029	0	3961.159	2	4229.531	0
3465.585	2	3963.774	4	4233.630	1

V. OSMIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4241.682	0	4377.070	1	4525.035	1
4251.321	0	4385.068	0	4529.848	1
4252.718	0	4386.485	1	4540.093	2
4261.011	4	4390.406	0	4548.836	1
4264.893	2	4391.251	2	4550.584	4
4269.526	0	4395.040	4	4551.461	3
4269.767	2	4397.424	3	4595.206	3
4270.952	1	4400.751	1	4597.321	2
4273.984	0	4402.901	3	4616.948	4
4275.074	0	4404.375	1	4632.000	4
4277.315	1	4410.899	1	4634.930	1
4281.535	0	4411.298	1	4642.010	0
4286.056	2	4420.639	5	4663.977	4
4294.105	2	4428.059	1	4692.220	2
4296.382	2	4432.584	2	4738.215	1
4297.556	0	4436.490	3	4738.508	2
4299.870	0	4437.258	1	4744.050	2
4309.041	1	4439.808	2	4755.332	1
4311.561	4	4445.582	1	4763.263	0
4317.754	0	4445.854	0	4794.177	5
4319.513	0	4447.535	3	4816.105	2
4326.413	2	4459 646	0	4865.759	2
4328.838	3	4459.790	0	4899.386	0
4338.913	3	4462.473	1	4912.771	1
4342.681	1	4466.134	1	4937.522	0
4351.695	2	4479.974	2	5031.988	1
4354.631	1	4484.935	2	5103.670	2
4358.157	1	4488.771	1	5149.895	2
4358.318	1	4503.474	0	5202.789	3
4361.126	0	4507.590	0	5523.786	2
4365.835	3	4514.445	0	5728.735	2
4370.826	3	4519.050	0		

VI. IRIDIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2321.481	0	2395.974	0	2456.882	0
2321.622	0	2398.824	0	2457.123	2
2324.006	0	2401.866	2	2457.312	2
2324.754	0	2402.379	1	2462.454	0
2325.029	1	2403.113	0	2463.118	1
2328.046	0	2405.955	0	2464.462	0
2328.324	0	2406.115	0	2467.382	3
2328.598	0	2409.465	1	2468.263	0
2328.790	0	2410.264	2	2468.705	1
2329.469	0	2410.818	1	2469.594	0
2333.372	2	2414.473	0	2469.848	0
2333.917	2	2415.950	2	2470.143	0
2334.406	0	2416.334	0	2470.607	0
2334.575	2	2416.672	0	2472.709	0
2337.628	0	2418.190	2	2474.170	1
2342.573	0	2418.657	0	2475.209	4
2342.763	1	2420.698	1	2478.190	1
2343.062	0	2421.306	0	2479.255	0
2343.255	2	2422.286	0	2480.685	0
2343.684	2	2424.406	0	2481.262	3
2347.329	1	2424.741	0	2482.383	0
2349.400	0	2424.971	1	2486.463	0
2349.790	0	2425.069	2	2486.826	0
2350.136	0	2425.744	2	2488.325	0, u
2351.492	1	2426.622	1	2489.293	0
2352.705	0	2426.875	0	2491.778	0
2355.082	2	2427.189	0	2492.406	0
2356.122	0	2427.694	2	2493.163	2
2356.388	0	2427.878	0	2495.680	1
2356.674	2	2429.830	0	2495.951	0
2357.623	0	2431.331	2	2496.360	2
2358.245	1	2432.021	2	2500.357	0
2359.668	0	2432.439	1	2502.710	2
2360.790	2	2432.667	0	2503.068	4
2363.134	4	2433.433	0	2504.446	2
2365.849	1	2434.107	0	2505.308	0
2367.469	0	2436.513	0	2505.814	1
2368.120	4	2445.184	0	2507.712	2
2368.486	0	2445.436	2	2508.434	0
2370.462	2	2446.926	0	2509.798	0
2372.856	4	2447.583	0	2512.016	1
2375.195	2	2447.850	2	2512.191	0
2381.714	2	2448.316	1	2512.665	2
2383.270	1	2449.112	1	2513.799	2
2383.840	0	2449.916	0	2515.448	0
2386.665	1	2452.893	3	2521.175	0
2386.981	2	2454.212	1	2523.290	0
2390.706	2	2454.945	0	2524.953	0
2391.282	3	2455.691	2	2526.856	0
2394.404	0	2455.949	2	2527.868	0

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2528.011	0	2589.057	0	2653.124	3
2529.559	1	2589.231	0	2653.853	2
2529.870	0	2589.470	0	2654.033	2
2530.200	0	2590.296	0	2654.670	0
2530.498	0	2591.129	1	2656.898	3
2530.786	0	2591.927	1	2657.587	0
2532.290	0	2592.146	4	2657.799	2
2534.103	0	2593.224	1	2657.993	0
2536.760	0	2595.188	0	2660.040	0
2537.309	2	2595.914	2	2660.163	0
2537.770	1	2599.129	2	2661.080	0
2538.548	0	2599.224	0	2662.080	6
2538.949	0	2602.122	2	2662.706	4
2540.483	1	2604.645	2	2663.400	2
2541.556	1	2606.081	0	2664.871	5
2542.097	2	2606.668	0	2665.144	0
2544.059	5, u	2607.608	2	2667.540	2
2545.620	1	2608.314	4	2668.362	0
2545.868	0	2609.996	0	2669.070	2
2547.278	1	2610.198	0	2670.006	3
2550.987	0	2611.384	3	2671.930	4
2551.475	2	2612.136	0	2672.888	0
2554.480	2	2612.344	1	2673.694	4
2555.425	2	2614.287	1	2675.376	0
2555.955	1	2615.064	2	2676.911	2
2556.860	1	2616.090	2	2677.899	0
2557.285	0	2617.177	0	2679.506	0
2558.821	0	2617.514	0	2681.184	2
2559.643	0	2617.872	2	2682.536	2
2562.999	0	2618.352	0	2683.387	0
2563.365	1	2619.967	2	2688.381	0
2564.253	4	2620.102	0	2689.769	0
2564.922	0	2621.610	0	2691.154	2
2566.442	0	2622.203	0	2691.998	0
2568.407	0	2623.736	1	2692.267	0
2569.962	2	2625.396	2	2692.429	4
2572.156	2	2626.844	2	2692.964	2
2572.459	2	2628.271	0	2693.571	2
2572.784	3	2629.498	2	2694.320	6
2573.338	0	2634.340	3	2695.550	0
2577.622	0	2634.513	0	2696.010	1
2578.794	2	2635.353	2	2698.688	2
2579.008	2	2636.967	0	2701.200	2
2579.573	2	2637.407	0	2704.117	3
2579.860	0	2639.073	2	2704.722	0
2581.019	0	2639.510	2	2705.213	0
2581.523	0	2640.462	2	2705.296	0
2583.261	1	2644.279	2	2705.453	0
2584.867	0	2646.334	2	2705.632	1
2586.146	0	2650.584	0	2706.985	0

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2707.265	0	2767.764	2	2824.546	6
2708.752	0	2771.711	3	2826.316	0
2710.177	2	2772.547	4	2827.259	2
2711.402	0	2774.685	2	2829.720	1
2712.817	4	2775.073	2	2830.264	3
2713.195	1	2775.646	4	2830.601	2
2714.643	1	2777.149	0	2830.964	0
2716.612	0	2777.536	2	2831.455	2
2717.730	0	2777.645	0	2831.912	1
2719.906	0	2779.752	1	2832.874	2
2720.534	2	2780.507	0	2833.337	4
2721.443	0	2781.047	2	2833.777	0
2723.248	0	2781.401	4	2835.408	0
2723.849	2	2782.342	0	2835.762	2
2724.884	0	2782.885	0	2836.197	1
2726.566	1	2783.492	0	2836.506	4
2728.224	0	2783.797	0	2837.421	2
2728.494	1	2785.319	4	2839.287	6
2729.638	2	2787.099	1	2840.332	4
2730.500	0	2787.687	0	2841.798	2
2731.954	0	2789.066	0	2842.390	2
2732.752	2	2790.795	0	2845.009	0
2734.596	0	2793.907	0	2845.245	1
2735.165	1	2794.189	2	2846.753	0
2736.509	0	2796.558	2	2848.557	0
2738.875	0	2797.456	4	2849.557	0
2739.413	2	2798.283	4	2849.848	6
2740.085	2	2799.522	0	2850.906	0
2740.166	0	2799.835	3	2851.161	0
2740.267	2	2800.755	1	2851.518	1
2740.432	1	2800.923	4	2851.648	2
2743.477	0	2804.300	0	2852.605	0
2743.769	0	2806.479	2	2853.416	2
2744.091	4	2806.772	0	2854.722	0
2747.383	0	2807.754	2	2855.931	2
2747.602	2	2808.249	0	2856.048	2
2748.395	0	2810.657	2	2857.058	2
2749.075	0	2812.896	3	2859.138	0
2753.954	0	2814.532	2	2860.126	0
2756.206	1	2814.966	2	2860.767	3
2758.325	2	2815.744	0	2862.455	1
2759.100	2	2816.409	0	2863.955	4
2759.405	2	2817.039	2	2866.798	4
2760.009	2	2817.284	0	2869.815	3
2760.207	0	2819.848	0	2870.304	0
2760.474	0	2820.614	0	2870.698	0
2761.227	0	2820.738	2	2872.227	0
2761.700	0	2823.280	5	2873.929	0
2763.287	0	2823.831	0	2875.721	4
2767.423	3	2824.228	1	2876.096	4

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2877.108	0	2931.821	0	2982.962	0
2877.781	4	2933.252	2	2985.921	4
2878.632	2	2934.748	4	2988.335	0
2879.515	4	2935.305	0	2990.746	3
2879.878	0	2935.427	0	2991.520	1
2880.174	0	2936.814	3	2993.184	2
2880.324	2	2937.371	0	2993.751	0
2881.270	2	2937.656	0	2996.202	4
2882.742	4	2938.097	0	2996.785	0
2882.970	0	2938.606	3	2997.314	3
2883.549	2	2938.877	0	2999.155	0
28 5.615	0	2939.390	0	3000.149	2
2887.240	2	2940.548	0	3001.383	0
2889.688	1	2940.669	2	3002.086	1
2890.634	0	2941.197	2	3002.375	4
2892.371	1	2943.287	5	3003.761	4
2893.785	0	2947.093	4	3004.429	0
2894.388	0	2949.882	3	3005.338	3
2895.705	0	2950.606	1	3007.745	0
2897.070	2	2950.883	2	3007.838	0
2897.260	4	2951.266	2	3008.753	1
2897.783	0	2951.363	2	3010.020	3
2898.455	2	2952.686	0	3011.812	3
2899.055	0	2953.205	0	3012.695	2
2899.733	3	2954.909	1	3012 984	1
2900.165	0	2956.301	0	3014.585	1
2900.492	2	2956.699	0	3014.854	1
2902.430	0	2959.049	0	3016.550	3
2903.852	0	2959.573	0	3017.450	4
2903.995	0	2961.009	2	3018.151	2
2904.913	4	2961.595	2	3019.350	4
2905.744	2	2962.580	1	3020.125	4
2907.353	4	2963.111	4	3022.536	3
2909.669	4	2965.095	0	3022.807	3
2909.912	0	2965.329	3	3024.410	2
2913.592	0	2966.245	2	3026.489	1
2915.625	0	2967.360	0	3029.487	4
2915.793	0	2968.334	2	3030.365	1
2916.479	4	2971.205	2	3030.568	0
2917.347	0	2972.119	0	3032.528	3
2917.885	2	2972.646	0	3033.744	3
2918.683	3	2974.220	1	3034.675	2
2919.299	0	2974.659	1	3036.361	0
2921.237	0	2975.062	4	3037.861	3
2924.912	7	2976.857	0	3039.378	5
2926.212	0	2978.056	2	3040.580	4
2927.129	2	2980.375	0	3041.056	1
2927.833	0	2980.578	0	3041.979	1
2930.298	2	2980.776	4	3042.429	0
2930.743	3	2981.042	2	3042.760	2

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3043.671	0	3088.163	4	3154.679	2
3044.255	0	3089.660	0	3154.874	3
3045.768	0	3090.277	2	3156.274	2
3047.277	4	3090.871	0	3157.614	2
3047.905	1	3091.254	0	3157.836	0
3048.783	1	3094.144	2	3159.280	5, d?
3049.559	4	3094.326	1	3159.644	2
3050.134	1	3097.147	0	3159.992	1
3051.243	3	3097.482	0	3161.477	2
3052.288	3, d?	3097.931	2	3161.948	2
3053.709	3	3098.555	0	3162.445	0
3054.351	0	3099.055	2	3162.871	0
3054.570	1	3100.586	2	3162.953	0
3056.770	0	3101.288	2	3163.972	1
3057.398	2	3103.667	1	3164.376	0
3057.590	2	3103.875	2	3165.323	1
3058.087	0	3104.301	0	3165.833	1
3058.438	0	3106.072	0	3166.886	2
3059.858	1	3108.670	2	3167.328	3
3060.114	1	3112.475	2	3167.792	0
3060.460	0	3113.259	1	3168.297	3
3060.950	2	3113.908	1	3168.404	1
3061.515	4	3114.170	4	3168.673	0
3064.216	0	3114.669	4	3169.010	5
3064.622	4	3117.457	0	3171.812	2
3064.904	3	3117.645	2	3172.915	3
3065.292	0	3117.968	0	3173.222	0
3065.944	0	3118.967	1	3173.466	1
3066.167	0	3119.422	0	3176.106	0
3066.766	0	3120.885	5	3177.325	0
3068.507	1	3121.894	4	3177.712	4
3069.005	4	3122.509	4	3178.811	2
3069.220	2	3123.334	2	3179.328	3
3069.825	2	3124.024	0	3180.487	2
3072.078	0	3124.203	2	3182.514	0
3072.904	0	3128.510	4	3182.924	1
3073.390	2	3133.210	2	3186.030	0
3073.800	0	3133.432	5, u R	3186.184	0
3074.864	2	3135.358	0	3186.667	1
3075.577	0	3136.418	0	3187.267	0
3076.800	4	3139.704	2	3188.487	0
3077.996	1, u	3141.947	1	3188.702	1
3078.793	2	3142.371	1	3189.486	2
3079.892	0	3142.994	0	3193.240	1
3081.709	1	3143.668	0	3193.345	2
3082.828	0	3147.860	2	3195.882	0
3083.085	1	3148.346	0	3198.226	2
3083.343	4	3150.128	0	3199.058	5
3085.088	1	3150.727	2	3200.166	2
3086.564	4	3151.748	2	3201.027	2

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3202.023	0	3245.510	0	3303.771	3
3202.250	0	3246.431	2	3304.460	0
3204.230	0	3246.951	0	3305.057	3
3204.587	2	3247.417	1	3305.787	1
3205.227	4	3249.638	2	3305.980	2
3205.837	0	3249.866	3	3307.774	2
3208.287	2	3253.497	1	3308.581	0
3209.050	0	3254.542	4	3308.939	0
3210.131	2	3256.194	2	3309.535	2
3212.240	3	3256.346	1	3310.052	0
3212.350	2	3257.916	2	3310.674	4
3212.629	0	3262.147	4	3311.161	2
3213.681	4	3262.852	2	3311.365	0
3216.431	0	3263.062	2	3312.268	4
3216.905	1	3263.436	2	3313.472	0
3217.301	0	3265.399	0	3316.129	0
3217.700	0, u	3266.580	6	3316.534	0, u
3218.593	4	3267.236	1	3316.771	4
3220.924	6, u	3268.663	0	3317.457	2
3221.415	3	3269.835	0	3317.664	0
3222.600	1	3271.372	4	3318.596	2
3222.854	0	3271.936	4	3318.812	0
3223.138	0	3272.772	0	3319.231	2
3223.645	2	3274.686	2	3319.680	0
3224.016	2	3275.167	2	3320.504	1
3224.637	0	3275.452	1	3321.901	0
3226.840	3	3275.735	2	3322.750	4
3227.675	0	3276.291	1	3323.011	4
3228.672	0	3277.422	4	3326.056	0
3229.412	5, u	3280.011	0	3326.245	2
3230.903	5	3280.705	1	3326.687	0
3232.145	5, u	3282.024	0	3327.039	2
3232.342	1	3282.458	2	3327.688	0
3232.618	0	3284.456	1	3330.968	0
3235.370	0	3284.695	1	3333.600	0
3235.537	0	3285.721	0	3334.318	4
3237.115	0	3287.198	4	3335.185	0
3238.003	0	3287.726	4	3336.195	2
3238.414	1	3290.640	0	3337.637	0
3238.675	0	3291.010	0	3337.985	1
3240.351	4	3291.187	0	3338.535	4
3240.688	2	3294.150	0	3339.028	0
3241.395	0	3294.251	0	3339.532	3
3241.640	4	3295.220	2	3340.485	2
3242.132	1	3297.655	2	3342.930	0
3242.462	2	3300.732	0	3343.182	0
3242.734	2	3301.502	0	3343.745	0
3243.568	0	3301.735	0	3344.360	2
3244.887	0	3301.900	1	3346.609	1
3245.022	2	3303.236	2	3347.695	2

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3348.015	1	3420.111	0	3557.325	3
3352.987	0	3420.646	2	3559.160	3
3353.696	1	3420.895	0	3568.156	1
3355.739	0	3421.923	2	3573.888	3
3355.942	2	3424.854	4	3594.308	2
3356.342	1	3425.526	1	3594.557	4
3356.697	0	3429.026	2	3596.356	0
3359.262	0	3429.748	0	3598.936	4
3360.038	6	3430.197	2	3601.568	4
3360.950	7	3430.941	0	3605.958	2
3364.380	2	3431.476	1	3609.933	4
3365.273	0	3432.930	0	3617.378	4
3365.678	0	3433.475	2	3619.326	2
3367.063	2	3434.915	2	3623.976	1
3367.210	2	3435.200	0	3625.872	3
3368.640	6	3435.554	0	3626.460	4
3370.785	2	3437.189	6	3628.843	5
3371.594	4	3437.670	4	3629.317	2
3372.958	1	3438.244	2	3629.911	3
3374.597	2	3445.682	0	3636.370	4
3374.942	0	3446.476	4	3641.037	1
3376.146	0	3446.793	2	3645.468	1
3377.288	0, u	3448.621	0	3647.857	1
3378.119	0, u	3449.133	6	3653.358	1
3378.550	0, u	3450.916	1	3657.774	0
3379.993	2	3455.949	2	3661.527	2
3381.151	3	3465.390	4	3661.867	5
3383.474	0	3468.749	2	3664.780	4
3383 917	0	3476.182	0	3675.160	4
3385.272	2	3476.611	3	3688.321	1
3385.752	2	3477.930	1	3689.476	0
3386.330	2	3481.254	1	3692.851	3
3386.417	0	3482.760	3	3696.308	2
3386.678	0	3484.256	2	3698.261	2
3388.023	1	3484.649	4	3701.107	2
3388.158	2	3485.660	3	3707.147	3
3389.473	1	3488.727	2	3712.630	3
3391.032	1	3492.217	0	3721.628	1
3395.129	3	3494.787	3	3722.904	3
3401.927	4	3496.580	1	3725.536	3
3402.182	2	3499.272	1	3731.504	4
3402.962	2	3503.088	2	3734.900	1
3409.931	2	3508.731	1	3738.682	2
3410.180	2	3510.793	2	3742.948	1
3411.730	2	3512.054	1	3747.352	4
3412.762	2	3512.356	2	3750 539	2
3415.408	3	3513.807	4	3768.817	2
3415.906	2	3516.110	2	3794.211	0
3418.533	0	3522.191	2	3799.047	2
3419.592	4	3552.223	2	3800.243	2

VI. IRIDIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3817.385	0	4172.736	1	4478.649	4
3889.715	0	4182.626	1	4491.523	2
3902.632	3	4200.031	2	4492.333	1
3902.807	2	4212.197	0	4495.525	3
3909.219	0	4212.383	2	4496.200	1
3915.055	2	4217.908	2	4533.003	2
3915.538	4	4218.243	0	4538.819	1
3923.634	2	4218.428	1	4545.837	4
3924.573	1	4220.950	2	4548.645	4
3931.903	0	4223.327	0	4550.941	2
3934.063	2, u	4230.486	0	4568.246	4
3935.005	4	4240.644	0	4570.183	2
3941.242	0	4241.198	0	4604.629	3
3944.534	1	4243.944	0	4614.342	0
3946.420	4	4257.528	2	4616.549	6
3948.459	1	4259.280	4	4640.231	2
3950.259	0	4261.408	2	4656.329	4
3952.099	2	4262.051	0	4669.130	2
3956.262	0	4265.450	2	4702.751	0
3962.926	2	4266.202	1	4709.034	2
3976.466	5	4266.532	0	4729.005	4
3978.240	0	4268.251	4	4732.014	1
3985.003	2	4269.101	0	4756.613	4
3987.963	2	4286.776	2	4758.107	2
3989.575	2	4300.802	1	4778.330	4
3992.277	6	4301.776	4	4795.827	3
3996.602	0	4305.359	0	4807.302	0
4005.164	1	4310.750	4	4809.636	2
4005.717	1	4311.669	5	4840.934	2
4020.194	5	4316.456	1	4845.539	0
4033.923	4	4330.060	0	4938.225	1
4040.224	4	4332.490	0	4939.311	0
4040.578	1	4351.462	1	4970.629	0
4048.782	0	4352.720	2	4999.898	2
4051.071	2	4362.289	1	5002.874	1
4051.538	0	4376.575	0	5009.323	0
4055.833	0	4377.175	3	5046.227	0
4056.620	2	4380.930	0, u	5050.001	0
4059.377	2	4392.758	3	5178.128	1
4070.067	4	4399.645	6	5239.091	1
4070.822	3	4403.952	4	5340.932	1
4072.532	2	4406.926	0	5357.081	0
4075.774	2	4411.344	2	5364.507	2
4080.737	2	4422.121	1	5449.716	4
4081.564	0	4425.936	0	5454.724	2
4082.542	1	4426.459	6	5469.648	1
4092.767	3	4449.540	0	5620.266	1
4115.957	4	4450.346	2	5625.772	3
4166.224	3	4452.987	1	5894.324	2

Bonn, November 19, 1897.

Minor Contributions and Notes.

NOTES ON THE USE OF THE GRATING IN STELLAR SPECTROSCOPIC WORK.

The results which have been obtained by Professor Poor in the direct use of the concave grating as an objective spectroscope in stellar spectroscopic work seem most promising. The writer (in common, no doubt, with many others) has long had in mind the utilization of gratings, both plane and concave, in this line of work. About two years ago (October, November 1895) I developed the general theory of the objective grating,[1] and indicated a number of methods of constructing, mounting, and using such gratings. At that time I made a number of laboratory experiments with the form of concave grating mounting now used by Professor Poor, and a little later, in conjunction with Professor Hale, made a practical test of a 12-inch wire grating, constructed under my direction in the Kenwood instrument shops, and attached to the Kenwood telescope. With this instrument the spectra of a number of stars were obtained, some as faint as the 3.6 magnitude. The details regarding some of the exposures are given in the following table. The plates used were ordinary Seed "26." In many cases, where the exposure was sufficiently long to show more than one order of spectra, the development was arrested before it was complete in order to avoid too great density in the spectra of the first order.

The results, therefore, were very satisfactory as regards the necessary times of exposure (which in many cases, *e. g.*, GG 10, 11, 17, 19, 20, 21, might have been made much shorter by making the spectra narrower). Considering the small photographic resolving power, which was between 500 and 800 units, or about the same as that of a flint prism of something less than one-fourth of an inch base, and the generally bad atmospheric conditions during the months in which the experiments were carried on (March, April), the *definition* in the best of the spectra obtained, and the resulting degree of *accuracy* with

[1] "The Modern Spectroscope. XV. Use of the Concave Grating as an Analyzing or Direct Comparison Spectroscope," this Journal, January 1896, pp. 54–62. "The Modern Spectroscope. XIX. The Objective Spectroscope," *ibid.*, June 1896, pp. 75–78.

Plate number	Date	Star	Mag.[1]	Time exposure	Width of spectrum[2]	Remarks
GG	1896			min.	mm	
5	March 11	Procyon	+0.5	1.0	0.2±	1st order, overexposed
10	March 21	β Leonis	+2.2	20.0	0.6±	1st order, somewhat overexposed 2d order, distinct but underexposed 3d order, traces
11	March 23	Sirius	—1.4	3.3	2.5	1st order, overexposed 2d order, distinct but underexposed 3d order, traces
12	March 23	Sirius	—1.4	10.0	0.3±	1st order, much overexposed 2d order, fully exposed 3d order, distinct but underexposed
16	March 30	Procyon	+0.5	8.0	0.2±	1st order, much overexposed 2d order, fully exposed 3d order, distinct but underexposed
17	April 3	γ Leonis	+2.2	3.0	0.8	1st order, fully exposed 2d order, underexposed
18	April 3	ζ Leonis	+3.6	13.0	0.15±	1st order, much overexposed 2d order, nearly fully exposed 3d order, distinct but underexposed
19	April 4	β Leonis	+2.2	4.0	1.1	1st order, little overexposed 2d order, underexposed 3d order, traces
20	April 6	δ Leonis	+2.8	8.0	0.7±	1st order, overexposed 2d order, underexposed
21	April 6	δLeonis(?)	+2.8	5.0	0.5±	1st order, fully exposed 2d order, distinct traces

which the *absolute* wave-lengths of the lines could be determined directly from the plates, were also most satisfactory.[3] The dismounting of the Kenwood telescope soon after this, temporarily put an end to these experiments. It was intended, as indicated in our paper,[4] to

[1] *H. P.*

[2] Owing to the chromatic aberration of the object-glass (the photographic objective was used) and the varying sensitiveness of the plates for different wave-lengths, the spectrum varies in width from point to point in any one order and is also different for different orders. The value given is the average width for that part of the spectrum for which the exposure is about normal.

[3] *Loc. cit.*, pp. 76, 77.

[4] *Loc. cit.*, p. 78.

undertake the construction of a larger wire grating for the 40-inch telescope, the one constructed for the 12-inch having (necessarily) too small a resolving power to be useful for anything more than a preliminary study of the method.

In comparing the efficiency of the concave and plane grating in stellar spectroscopic work, it may be noted first of all that the former has the same advantage over the latter, *when used as an objective spectroscope*, as when used in the laboratory ; *i. e.*, it avoids all the losses due to the absorption, reflection, and diffusion of light by the additional lenses or mirrors necessary in the plane grating train ; losses which become particularly serious in the case of large objectives when we are working with the ultra-violet or infra-red part of the spectrum. But although these advantages were fully recognized at the time the laboratory experiments already referred to[1] were made, it was feared that with the gratings of the linear and angular aperture then available, the time of exposure required to obtain good *stellar* spectra would be so long as to make the use of such gratings impracticable for any but the very brightest stars. Professor Poor's results with a comparatively small grating (ruled surface 1 by 2 inches, focal length about 40 inches) show, however, that the advantages attendant upon getting rid of the losses of light by the absorption and diffusion of the large telescope objective, are more considerable than was anticipated, and it now becomes of interest to determine as well as may be from his results, what may be expected from concave gratings of larger linear and angular apertures.

Let us assume, as a basis of comparison, a constant photographic purity of spectrum Q_0. From equation 26 (this JOURNAL, May 1896, p. 338) we have

$$Q = \frac{\lambda}{n\,e\,\beta}\,r \qquad (1)$$

where r is the theoretical resolving power of the spectroscope train, β the angular aperture of the camera, e the diameter of the silver grains in the plate, and n a constant which, in this case (where the slit-width is simply the angular magnitude of the star and may therefore be neglected), is about 4. We also have for the intensity of a continuous stellar spectrum formed by an objective spectroscope (this JOURNAL, June 1896, p. 58, equation 6),

[1] *Loc. cit.*, pp. 57, 58.

$$i' = \text{Const } A^2\,\beta^2.\ \frac{1}{r} \tag{2}$$

where A is the diameter of the objective. For a rectangular aperture (such as we have in the case of a grating) of height (length of ruled lines) B and aperture (length of ruled surface) A, (1) should be written

$$i' = \text{Const } A \cdot B \cdot\ \frac{A}{f} \cdot \frac{B}{f} \cdot \frac{1}{r}. \tag{3}$$

In order to maintain the photographic purity constant, it is necessary to have

$$\frac{A}{f} \cdot \frac{1}{r} = \frac{\beta_A}{r} = \text{Const.}$$

This may be done in either one of two ways: (1) by keeping f constant, in which case the resolving power will vary directly as A; the angular dispersion (and hence the closeness of ruling on the grating) remaining constant; (2) by keeping r constant, in which case the focal length increases and the angular dispersion decreases (the *linear* dispersion, therefore, remaining the same) in the same ratio as A is increased. In either case, if the spectra are linear (*i. e.*, not broadened either by aberration or by drift), we have, under the condition of constant purity

$$\frac{i'}{i'_0} = \frac{\epsilon}{\epsilon_0} \cdot \frac{A\,B^2}{A_0\,B_0^2} \cdot \frac{f_0}{f} \tag{4}$$

ϵ and ϵ_0 being the "factors of efficiency" for the two spectroscope trains.

If we maintain the focal length constant and assume that ϵ and ϵ_0 will be the same for gratings ruled with the same or similar diamond points (when f is constant the grating space remains the same for Q = const.), then the intensity of the spectra (linear), from two such gratings will vary as the ratio of the aperture times the ratio of the length of the ruled lines. A grating with a ruled surface of 2.5×5 inches (area = 12.5), and having the same focal length (40 inches) and factor of efficiency, ϵ, as the one used by Professor Poor (ruled surface 1×2 inches), would consequently give a linear star spectrum about fifteen and two-thirds times more intense. It ought therefore, under the imposed conditions of constant photographic purity, to be able to photograph the spectra of second magnitude stars, which are about one twenty-third as bright as Sirius, in about an hour and a half. If the length of the ruled lines were made five inches instead of

two and a half the intensity of the *linear* spectrum would be quadrupled and, other circumstances remaining the same, the time of exposure reduced to about 25^m for a second magnitude star or to one hour for a third magnitude star.

Practically we cannot hope to do quite as well as this, because in practice all stellar spectra are broadened to a greater or less degree by the effect of irregularities of following and the effect of atmospheric disturbances. Even could these effects be avoided it would still be necessary to mechanically broaden the spectra slightly in order to avoid false lines due to imperfections in the film. If we assume that this broadening is in all cases the same, the resulting diminution in the effective photographic intensity will vary as

$$\frac{B}{f} : \frac{B_0}{f_0} = \frac{\beta_B}{\beta_{B0}}$$

and therefore for two spectra of the same linear width the ratio of effective intensities becomes

$$\frac{i'}{i'_0} = \frac{\epsilon}{\epsilon_0} \frac{A\ B}{A_0\ B_0} = \frac{\text{Area of Grating } G}{\text{Area of Grating } G_0}. \tag{5}$$

In this case the above computed times of exposure for the 5×5-inch grating would be increased five times, or it would require about two hours' exposure to obtain the spectrum of a second magnitude star with gratings of the maximum size now obtainable. The spectra obtained with large gratings might, perhaps, be made *somewhat narrower* than those obtained with small ones, in which case the exposure times required would lie somewhere between the two values given above. It appears therefore that it is well worth while taking up this line of work with the gratings now obtainable, although we cannot hope to work with stars much below the second magnitude until larger ones are available.

It is of interest to compare these results with those obtained with the plane wire grating used in conjunction with the 12-inch photographic objective. The surface of the wire grating is 10.5×10 inches and the area used (the corners of the grating were cut off by the circular form of the aperture), is about 100 square inches. The theoretical resolving power of the grating (first order) is 1700 and that of Professor Poor's grating, about 14,400 (first order). The horizontal angular aperture $\frac{A}{f}$ is almost the same in the two cases (slightly less in

the case of the wire grating). The photographic purity obtained with the latter is therefore 1700 : 14,400, or a little more than two-seventeenths that obtained with the former. If the two gratings were equally efficient we would have for the *same* photographic purity and the same width of spectrum the ratio of intensities

$$\frac{i'}{i'_0} = \frac{2}{100} \cdot \frac{2}{17} = \frac{1}{425} \tag{6}$$

or the 10.5-inch wire grating ought to photograph the spectra of stars of a given magnitude in about one four hundred and twenty-fifth of the time required with the 2-inch concave grating. If the width of the spectra obtained in the two cases were respectively w_0 and w this ratio would be increased or diminished in the ratio $\frac{w}{w_0}$. Taking the time of exposure required with the 2-inch grating for Sirius as 40^m with a width $w = 0^{mm}.25$,[1] the time of exposure for the wire grating for a width $w_0 = 2^{mm}.5$ (the focal length was about 18 feet, or about 5.5 times that of the 2-inch concave grating), would, if the two were equally efficient, be about 1^m for spectra of the first order and about 4^m for spectra of the second order.[2] In the table already given (p. 199) we find (G G 11), that with an exposure of three and one-third minutes, the first order spectra (width $= 2^{mm}.5$) were "overexposed," the second order "distinct but underexposed." If the development had been carried farther (see remark preceding the table) the second order could have been brought up very nearly to full density. The plates used in the experiments with the wire grating were Seed "26"; the plates used by Professor Poor are the new "Gilt Edge" Seed which the makers claim are considerably more rapid than the "26."

The computed time of exposure required by the 2-inch concave grating for β Leonis (mag. 2.2), on the basis of the exposure time required for Sirius (mag. -1.4), would be

$$40^m \times 2.51^{3.6} \cong 18 + \text{hours.}$$

The time required by the 10.5-inch wire grating would be [see (6)]

[1] Measured on one of Professor Poor's original negatives kindly sent to me for examination.

[2] In this wire grating the diameter of the wire is about equal to one-half the grating interval s, and the theoretical relation between the brightness of the different orders m_1, m_2, etc., of the grating is therefore very nearly fulfilled, *i. e.*, $i'_0 \sim \frac{1}{m^2}$.

$$T_1 = \frac{18}{425}\frac{w_0}{w} = 2.6\frac{w_0}{w} \text{ minutes} \tag{7}$$
$$= 10.4 \text{ minutes}$$

when $w = 0^{mm}.25$ as before and $w_0 = 1^{mm}$.

In the case of γ Leonis (G G 17) and β Leonis (G G 19), both of magnitude 2.2, about four minutes' exposure was required to obtain a first-order spectrum of full density when $w_0 = 1^{mm}$. In these cases, then, the actual times of exposure are less than 40 per cent. of the equivalent computed time for the concave grating. The earlier result (G G 10, March 21) for β Leonis is not quite as favorable. In this case $w_0 = 0^{mm}.6$ for the second-order spectrum (somewhat greater than this for the first order, because of overexposure). Hence

$$\frac{w_0}{w} = 2.4$$

and from (7)

$$\begin{aligned} T_1 &= 6\tfrac{1}{4} \text{ minutes, for the 1st order}^1 \\ T_2 &= 25 \text{ minutes, for the 2d order} \end{aligned} \tag{8}$$

The actual time of exposure was twenty minutes, and the second order was not quite fully exposed (nor was the plate quite fully developed).

Making the comparison of the computed and actual times of exposure in the same way in the cases of δ Leonis (G G 20) and ζ Leonis (G G 18), the magnitudes of which are 2.8 and 3.6 respectively, we find

$$\text{for } \delta \text{ Leonis } \frac{\text{Comp. } T_1}{\text{Actual } T_1 *} = \frac{12.6 \text{ minutes}}{8.0 \text{ minutes}} *$$

$$\text{for } \zeta \text{ Leonis } \frac{\text{Comp. } T_2}{\text{Actual } T_2 \dagger} = \frac{22.5 \text{ minutes}}{13 \text{ minutes}} \dagger$$

The case of Procyon is not directly comparable with that of Sirius, as it has a different type of spectrum.

In view of all these results it may fairly be inferred, I think, that, in spite of the absorption of the 12-inch object-glass, the efficiency of the 12-inch wire objective grating is something like 50 per cent. higher than that of the 2-inch concave objective grating.[2] The former

[1] See footnote on p. 203.

* First order somewhat overexposed.

† Second order somewhat underexposed (but plate underdeveloped).

[2] It is quite possible that this advantage would wholly disappear in comparing the wire grating with *another* ruled concave grating, with which a larger proportion of the light is concentrated in one of the first-order spectra. Professor Poor, however,

instrument would therefore seem to offer the greater promise in the future development of this line of work, both on account of its higher efficiency and on account of the greater ease of construction and much less cost of gratings of larger aperture than those at present in use. It has, however, the disadvantage of requiring a large aperture in order to obtain moderately high resolving powers. The finest wire that can be used in the construction of large gratings must have a diameter of at least $0^{in}.0015$,[1] and in order to obtain a resolving power of 15,000 units in the first order, we would have to have an aperture of at least 50 inches (if the interval were made twice the diameter of the wire). We could, however, attain practically the same *photographic* resolving power as was obtained with the 2-inch concave grating with a wire grating of 25 inches linear aperture used in conjunction with an object-glass of about 80 feet focal length (ratio of aperture to focal length 1 : 40). In any event, gratings of considerably larger aperture than the *ruled* gratings now obtainable would be the first requisite if we are to work on stars fainter than the second magnitude.

As regards the possibilities of obtaining larger ruled gratings, it has already been stated[2] that the writer has designed a ruling machine capable of ruling gratings 15 inches in diameter, the construction of which was begun some eight months ago in the instrument shops of the Observatory, and which is now well advanced toward completion. If this machine is successful in producing good gratings of the maximum size for which it is designed (ruled surface 10×15 inches), we may hope to obtain with them spectra of fourth-magnitude stars in about two hours' exposure time — about the longest exposure that we can afford to give in stellar spectroscopic work, unless temperature conditions, etc., are unusually favorable. We may, of course, do somewhat better than this by using a smaller photographic purity than that obtained by Professor Poor with the 2-inch grating (about

informs me that the grating used by him was one having an unusually brilliant first-order spectrum, the succeeding orders being so faint that only in one or two cases were there any traces of them obtained on the plate. As shown in the table, not only the second, but in many cases the third order of spectra were obtained with the wire grating.

[1] Rubens (*Wied. Ann.*, 51, 381, 1894) constructed a small grating of gold wire only about $0^{in}.001$ in diameter. Such wire, however, would be too weak to be used in the construction of large objective gratings.

[2] "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Phil. Mag.*, May, and *Wied. Ann.*, June 1897.

$14,400 \dfrac{\lambda}{n e \beta} \cong 7000$ units), upon which the preceding comparisons have been based; but this does not seem desirable, if *ruled* gratings are to be used at all. Smaller resolving powers are best obtained by the use of prisms or of wire gratings, as already pointed out.[1]

In constructing concave gratings of larger size than 15 inches one of the most promising plans would seem to be to photograph the grating directly on the face of a concave silvered glass mirror, using as an original a grating having the requisite number of lines (7000 to 15,000), either ruled on glass (in which case the copy would be an enlarged one), or made of wire in the manner of the 10-inch grating already described (in which case the original would, as already indicated, be of 25 to 50 inches aperture, and the copy therefore in general a reduced one). We might photograph the grating directly on the silver surface by sensitizing the latter with iodine and bromine (thus converting it into a daguerreotype plate), or we might advantageously modify and extend the experiments begun by Lord Rayleigh a number of years ago,[2] and try photographing the grating on a thin

[1] The theoretical resolving power of the compound star spectrographs now in use in connection with our largest telescopes (the Pulkowa, Lick, and Yerkes refractors, and the Paris reflector) is not greater than 12,000 units (3 dense flint prisms of about 1-inch aperture), and the photographic resolving power is generally less than one-half of this. The highest resolving power yet used, so far as I am aware, in sustained spectroscopic and spectrographic research is that employed by Pickering in the four-prism objective spectroscope of the Harvard Observatory, which is about equivalent to a single 60° prism of light flint of 11 inches clear aperture, and which has therefore a theoretical resolving power of something like 30,000 units (in the visible part of the spectrum). (See Tables in this JOURNAL, **2**, 264, Nov. 1895.) But the focal length of the telescope on which this prism battery is mounted is only 153 inches $\left(\beta_A \cong \dfrac{1}{14}\right)$ and only about one-third of this resolving power is in consequence *photographically* utilized. In most of the work only one or two prisms are used, giving a practical photographic resolving power of about 2500 and 5000 units respectively (under first-class atmospheric conditions). In view of these facts it would seem desirable in taking up this line of work with the concave grating to obtain at least the photographic purity indicated above (7000 units). On the other hand, it seems extremely doubtful whether we will ever be able to fully utilize, either visually or photographically, resolving powers higher than 30,000 units in stellar spectroscopic or spectrographic research; except, perhaps, in occasional special studies of the spectra of the *very brightest* stars, or in examining and photographing *bright line* spectra.

[2] *Proc. R. Soc.*, No. 136, 1872; *Brit. Assoc. Report*, 1872; *Phil. Mag.*, **47**, 1874. The objection to the use of the lens, mentioned by Lord Rayleigh in the last of the above papers, in photographing the grating on the sensitive surface by projection, does

transparent film of bichromatized gelatine spread over the silvered surface of the mirror. If this film could be made uniform in thickness and the process properly controlled, it might be possible to produce in this way *gratings in which the central image would be nearly abolished and the brightness of the first-order spectra consequently increased nearly four times.* This highly desirable result would be brought about, for example, if the "grooves" of the grating in the developed gelatine film were approximately rectangular and had a "width" equal to half the grating space and a "depth" of

$$\frac{1}{4n}\lambda$$

n being the index of refraction of the gelatine.

It is possible, therefore, that we may in this way be able to produce large concave gratings by photography which will be considerably more efficient than either the ruled gratings or the plain wire gratings. There is, at any rate, a most promising field of experiment open in this direction.[1]

The use of gratings, both plane and concave, in place of the prism train has also been advocated for the compound spectroscope. I have myself designed and described several astronomical spectroscopes of this class,[2] but they were intended particularly for either solar work or

not hold in the case now being considered, because the *closest* ruling necessary to obtain the required resolving power will not on a 20-inch mirror *exceed* 750 lines per inch — only one-fourth the number considered by Rayleigh.

[1] In this connection it may be interesting to call attention to another line of experimentation which I have long had in mind, and which I hope will soon be taken up. The primary object of these experiments will be to determine whether the light that is brought to a focus on the photographic plate cannot be more fully utilized than it is at present, particularly in spectrographic and astrophotographic work, in which every saving of time that can be effected is of the utmost importance. One way in which it would seem that a considerable saving might be effected would be to spread the sensitive film, not on the surface of a transparent glass plate, but on the surface of a polished metallic mirror, which may be most cheaply and easily obtained by silvering, or platinizing (if silver for any reason should prove objectionable on trial) the ordinary glass plates. All the light which is now lost by passing through the film and glass would be reflected back through the film, probably nearly doubling the effective action, and thus increasing, not only the "sensitiveness" of the film, but also the "contrast" in the resulting negative. This plan would also effectually prevent all "halation."

[2] See particularly "Some New Forms of Combined Grating and Prismatic Spectroscopes of the Fixed Arm Type," this JOURNAL, **1**, 232, March 1895; and "Fixed Arm Concave Grating Spectroscopes," *ibid.*, **2**, 370, December 1895.

for the study of bright line (nebular) spectra. But I can see *no good reason* why they should be preferred to the prism train in stellar spectroscopic work, with the compound spectroscope or spectrograph, for the chief arguments for their use in the case of the objective spectroscope (*i. e.*, the avoidance of loss of light by absorption in the large object-glass, and the possibility of making absolute wave-length measurements with them without the aid of a comparison spectrum) no longer hold in this class of work. It seems to me that in this case the prism train is much to be preferred, partly on the score of greater stability, but much more because it concentrates all the light in one spectrum. We lose a great deal of light by the absorption and diffusion of the large object-glass;[1] we lose still more at the jaws of the slit by reason of the constant shifting and blurring of the image by atmospheric disturbances, and there is not so much left, even when the largest telescope and the brightest stars are at our disposal, that we can afford (as we can in the case of the Sun) to waste it recklessly in the multiplication of useless and extraneous spectra.

F. L. O. WADSWORTH.

YERKES OBSERVATORY,
January 1898.

VARIABLE STAR CLUSTERS.[1]

SINCE the announcement made in *Circular* Nos. 2 and 18, of variables discovered in clusters, a further examination of the clusters ω Centauri, Messier 3, Messier 5, and *N. G. C.* 7078 has been made by Professor Bailey. As a result, the numbers of known variables in these clusters have been increased by 62, 19, 22, and 24 respectively, making the total numbers 122, 132, 85, and 51, or 390 in all four clusters. Adding to these the 47 already announced in other clusters, makes the total number 437.

NEW VARIABLE STARS.

When a new variable star is discovered at this Observatory it is the custom to collect all the photographs of the region containing it and to derive its photographic magnitude from each of them as described in the *H. C. O. Annals*, **26**, 250. We can thus determine its bright-

[1] Much more, probably, than we subsequently lose in the prism train if the material and refracting angle (see this JOURNAL, **2**, 264, November 1895) of the elements of the latter be properly chosen.

[1] *Harvard College Observatory Circular* No. 24.

ness on from twenty to a hundred or more nights distributed over the last ten years. The approximate dates of maxima, the corresponding magnitudes, the period and the form of light curve are also determined so far as possible. Examples of such results have already been published, but every year, owing to the increasing amount of material, the work becomes more laborious although at the same time more complete and exact. Many of these stars vary irregularly so that their elements cannot be determined precisely. When the object is not a catalogue star its position, and that of each of the fainter comparison stars must also be determined from measures of their rectangular coördinates. An attempt is then made to photograph each of these variables once a month, and, if possible, to obtain corresponding observations of their visual magnitudes. As the total number of variable stars discovered here is now more than a hundred, not including those found in clusters, the labor involved in this work is very great. Accordingly, it is difficult to deduce all the required data for one star before another is found. The accompanying table gives the material that has been so far collected for the variables recently discovered here from the Draper Memorial photographs.

Constellation	Designation	R.A. 1900	Dec. 1900	Type	No. Plates	Mag. Br.	Mag. Ft.	Discoverer
		h m						
Eridanus	−16° 771	3 59.8	−16° 0′	III	35	8.3	9.4	M. Fleming
Eridanus	−25° 1766	4 7.3	−25 24	III	65	8.1	<12.5	M. Fleming
Monoceros....	− 8° 1641	6 52.5	− 8 56	III	43	8.1	10.3	M. Fleming
Puppis.......	−38° 4049	8 1.7	−38 29	IV	..	...	....	L. D. Wells
Puppis.......	−22° 2160	8 3.1	−22 38	IV	..	...	....	L. D. Wells
Hydra	− 5° 2550	8 24.7	− 5 59	III	..	...	....	M. Fleming
Carina	R	10 40.9	−58 54	...	149	9.6	10.7	L. D. Wells
Virgo........	− 5° 3424	12 2.1	− 6 12	III	..	...	...	M. Fleming
Centaurus	....	13 15.1	−61 3	III	..	...	....	M. Fleming
Apus.........	*A.G.C.* 19014	13 55.6	−76 19	III	..	...	....	———
Boötes	+14° 2700	14 1.7	+13 59	III?	..	...	....	L. D. Wells
Libra	−17° 4122	14 30.3	−17 36	II ?	38	8.3	9.6	E. F. Leland
Triang. Aust..	*A.G.C.* 20554	15 4.8	−69 42	IV	85	9.1	9.8	———
Serpens	+10° 2956	16 2.5	+10 12	III	41	9.0	<11.9	M. Fleming
Ara..........	*A.G.C.* 23005	16 54.3	−54 55	IV	..	...	....	L. D. Wells
Pavo.........	*A.G.C.* 23935	17 34.7	−57 40	IV	..	...	....	L. D. Wells
Pavo.........	....	17 41.1	−62 23	III	65	9.1	<12.8	M. Fleming
Ara..........	....	17 45.7	−51 40	III	..	...	....	M. Fleming
Cygnus	+32° 3522	19 37.1	+32 23	IV	..	...	....	L. D. Wells
Pavo.........	*A.G.C.* 27560	20 3.3	−60 14	III	..	...	....	M. Fleming
Capricornus ..	*A.G.C.* 27776	20 11.3	−21 38	IV	55	8.6	10.3	———
Microscopium .	−40° 13888	20 22.6	−40 45	III	70	8.5	<12.5	M. Fleming
Capricornus ..	−17° 6181	21 1.7	−16 49	III	79	8.1	9.3	M. Fleming
Aquarius	−14° 5960	21 7.3	−14 48	III	78	8.4	9.3	M. Fleming
Indus	*A.G.C.* 29232	21 13.6	−45 27	IV	..	...	....	L. D. Wells
Andromeda...	+47° 4318	23 50.3	+48 5	III	48	9.3	9.8	M. Fleming
Cassiopeia ...	....	23 58.2	+55 7	III	101	9.8	<13.4	M. Fleming

R. A. $3^h\ 59^m.8$. Bright hydrogen lines suspected.

R. A. $4^h\ 7^m.3$. Hydrogen lines bright.

R. A. $6^h\ 52^m.5$. Hydrogen lines bright.

R. A. $8^h\ 1^m.7$. Found to be fourth type by Mrs. Fleming.

R. A. $8^h\ 24^m.7$. Bright hydrogen lines suspected.

R. A. $10^h\ 40^m.9$. This star is No. 119 on page 627 of the *Argentine General Catalogue.*

R. A. $12^h\ 2^m.1$. Bright hydrogen lines suspected.

R. A. $13^h\ 15^m.1$. Hydrogen lines bright. The position of this star for 1875 is R.A. $= 13^h\ 13^m\ 31^s$, Dec. $= -60°\ 55'$.

R. A. $13^h\ 55^m.6$. Bright hydrogen lines suspected. In the *Uranometria Argentina*, page 243, this star, which is θ Apodis, is stated to be variable. Discovered independently by Mrs. Fleming by means of its spectrum. The photographs show a variation of about one magnitude.

R. A. $15^h\ 4^m.8$. In *Argentine General Catalogue* "var.?" Discovered independently from photographic charts by Miss L. D. Wells.

R. A. $16^h\ 2^m.5$. Hydrogen lines bright.

R. A. $17^h\ 41^m.1$. Hydrogen lines bright. The position of this star for 1875 is R.A. $= 17^h\ 38^m\ 45^s$, Dec. $= -62°\ 21'.6$.

R. A. $17^h\ 45^m.7$. Hydrogen lines bright. The position of this star for 1875 is R.A. $= 17^h\ 43^m\ 42^s$, Dec. $= -51°\ 39'.2$.

R. A. $20^h\ 3^m.3$. Bright hydrogen lines suspected.

R. A. $20^h\ 11^m.3$. Suspected of variability by Secchi and others. Found independently from the photographs by Miss L. D. Wells.

R. A. $20^h\ 22^m.6$. Hydrogen lines bright. Maxima represented by formula, $2410860 + 325\ E$.

R. A. $23^h\ 50^m.3$. Bright hydrogen lines suspected.

R. A. $23^h\ 58^m.2$. Hydrogen lines bright. The position of this star for 1855 is R.A. $= 23^h\ 55^m\ 53^s$, Dec. $= +54°\ 52'.3$.

In *Circular* No. 10 the variability of $-27°$ 15202 (erroneously printed 15203) suspected by Thome was confirmed by Miss E. F. Leland. Measures of 35 photographs give the maximum brightness 8.9, minimum < 12.3.

In *Circular* No. 17 the variability of a star in R.A. $= 0^h\ 25^m.6$, Dec. $= -46°\ 58'$ (1900) is announced. Measures of 26 photographs give the maximum brightness 9.0, minimum < 12.2.

In *Circular* No. 17 the variability of a star in R.A. $= 13^h\ 31^m.1$, Dec. $= -55°\ 58'$ (1900) is announced. Measures of 42 photographs give the maximum brightness 9.0, minimum < 12.6.

In *Circular* No. 17 the variability of a star in R.A. $= 20^h\ 8^m.5$, Dec. $= -44^\circ\ 43'$ (1900) is announced. Measures of 114 plates give the maximum brightness 9.0, minimum < 11.4.

In *Circular* No. 19 the variability of a star in R.A. $= 5^h\ 18^m.9$, Dec. $= -69^\circ\ 21'$ (1900) is announced. Measures of 51 photographs give the maximum brightness 8.2, minimum 9.4.

EDWARD C. PICKERING.

January 31, 1898.

POLARIZING PHOTOMETERS.[1]

NEARLY all of the photometric measurements obtained at the Harvard College Observatory during the last twenty years have been made with modifications of three forms of photometers which are identical in principal. The first of these is described in the *Annals*, Vol. XI, Part I, and was used for the observations contained in that publication. The second, the meridian photometer, furnished the observations contained in the *Annals*, Vols. XIV, XXIII, XXIV, and XXXIV. The third photometer is described in the ASTROPHYSICAL JOURNAL, **2**, 89. In all of these instruments the star to be measured is compared directly with another star by means of a double image prism and Nicol. In the first instrument, the images of two adjacent stars are brought together by a double image prism; in the second, images of two stars, however distant, are brought together by reflecting them by prisms or mirrors into two object-glasses; in the third photometer, images formed by a large telescope, of two stars not more than half a degree apart, are brought together by achromatic prisms.

An objection to the first form of photometer is that the emergent pencils of the images compared do not coincide. Small errors may therefore be introduced by irregularities in the cornea of the eye of the observer, or if he holds his eye in such a position that a portion of one image will be cut off by the edge of the pupil. This difficulty has recently been remedied by placing a second double image prism in the focal plane of the telescope, so that it does not affect the position of the two images but makes the emergent pencils coincide. A surprising degree of accuracy may then be obtained in the measures. Comparisons of the star o Ceti with the adjacent star $-3^\circ 355$, which follows it about 10^s, have been made by Mr. O. C. Wendell with the

[1] *Harvard College Observatory Circular* No. 25.

15-inch equatorial of this observatory, on 191 nights during the last five years. Until recently, the observations were made with the first form of photometer, the emergent pencils overlapping by about two-fifths of their diameter. Generally thirty-two settings were made each night, and the mean of the differences of each pair of sets of four settings each on the first fourteen nights of observation made during the present opposition was ±0.074 magnitudes. On the last five nights the pencils were made to coincide, as just described, and the corresponding average differences were 0.020, 0.002, 0.025, 0.022 and 0.032, mean ±0.020. On the second of these nights, January 19, 1898, the eight measures of the difference in light of the two stars, each derived from four settings, were 4.48, 4.48, 4.48, 4.48, 4.48, 4.48, 4.48 and 4.47. Owing to variations in the transparency of the air in different parts of the sky, this degree of accordance can only be expected when stars near together are compared.

The accuracy of the results attainable with the third form of photometer is shown in the ASTROPHYSICAL JOURNAL, **3**, 281, and in the observations of U Pegasi described in *Circular* No. 23. Mr. Chandler (*Ast. Jour.*, **18**, 140), while admitting the principal conclusions given in that *Circular*, denies the reality of the small difference 0.15 between the primary and secondary minima. This quantity is so small that it doubtless could only be surely determined by the most accurate photometric measurements. The individual results derived from Mr. Wendell's observations are therefore given below. Twelve observations, each consisting of sixteen settings, were made when the star was within twenty minutes of its primary minimum. Deriving from each of these, by means of the light cure, the magnitude of this minimum we obtain on October 18, 1897, 9.89, 9.94 and 9.96; on December 30, 9.90, 9.95, and 9.93; on January 1, 1898, 9.93, 9.86, and 9.85; on January 5, 9.85, and on January 7, 9.86 and 9.88. Mean of all, 9.90, greatest value, 9.96, least value, 9.85, average deviation ±0.035. Similarly fourteen observations were taken within twenty minutes of the secondary minimum with the results on October 18, 1897, 9.75 and 9.71; on October 29, 9.74, 9.69, 9.70 and 9.70; on December 28, 9.78, 9.77, 9.76 and 9.80; on January 3, 1898, 9.77, 9.77, 9.74 and 9.78. Mean of all, 9.75, greatest value, 9.80, least value, 9.69, average deviation, ±0.029. It will be noticed that the largest value of the secondary minimum is 0.05 less than the smallest value of the primary minimum. If we assume that the primary and secondary minima are really equal,

all of the first of these values give positive residuals with the mean value + 0.064, and all of the second (with one exception, + 0.01) give negative values with the mean value − 0.70. The probability that the two minima are really equal and that these deviations are due to accidental error is extremely small. It is the same as that a person should draw a red card from a pack of cards twelve times in succession, and then should draw a black card thirteen times out of fourteen. On the other hand, if these deviations are due to a sytematic error, it is very singular that this error always has one value at the time of a principal minimum, and another at the time of a secondary minimum.

EDWARD C. PICKERING.

February 8, 1898.

ON THE DEPTH OF THE REVERSING LAYER.

THE theory of the solar rotation, combined with the rotation laws empirically found for the Sun-spots, the faculæ and the reversing layer, gives a means of finding the differences in the level of these different regions of the solar atmosphere. The discussion which I have undertaken in my thesis shows these differences to be considerable, amounting to over one-tenth of the solar radius.[1] This is distinctly in contradiction to the views commonly adopted, according to which the differences in level can only be small. This latter opinion is founded upon direct observation, which seems to demonstrate that Sun-spots, photosphere and faculæ cannot differ very much in level, and that the reversing layer must be exceedingly shallow. How are these two diametrically opposite views to be reconciled? I may add that it is not only the theory of rotation which demands a deep reversing layer and great differences in level. The results of Jewell, Humphreys and Mohler are more easily explained in this way, and there is no fact which is contradictory to the hypothesis of a deep reversing layer except, apparently, the results of direct observation. But this difficulty is easily solved if we take account of refraction in the solar atmosphere. The discrepancy then entirely disappears.

While the solar atmosphere is probably inconceivably rare near the top of the faculæ, it is unreasonable to assume this to be also the case in those regions where heavy metallic vapors are certainly present according to the testimony of the Fraunhofer lines in the solar spec-

[1] *Hydrodynamische Untersuchungen mit Anwendungen auf die Theorie der Sonnenrotation*, Berlin, 1897, p. 31.

trum. There can be no reasonable doubt that, during its path among these vapors, a ray of light must suffer a sensible amount of refraction. If this is so, we shall find that the distance between two different levels of the solar atmosphere must appear very much shortened.

If μ is the index of refraction at the distance r from the center, then the observer at a great distance will see the distance r magnified in the ratio of μ to 1.[1] Now let r_1, μ_1 be the values of r and μ for the region immediately adjoining the photosphere and r_2, μ_2 for the region near the top of the faculæ. Then the distance $r_2 - r_1$ will appear to be

$$d = \mu_2 r_2 - \mu_1 r_1 \, . \tag{1}$$

According to the theory of rotation $r_2 = 1.1\ r_1$ about, so that

$$d = (1.1\ \mu_2 - \mu_1)\ r_1 \, , \tag{2}$$

and this may become very small, or even zero and negative, the actual value assumed by this expression depending upon the ratio $\frac{\mu_1}{\mu_2}$. Probably near the top of the faculæ we have sensibly $\mu_2 = 1$, so that to $\mu_1 = 1.09$ would correspond $d = 0.01 r_1$ instead of $0.1 r_1$. Such a value of μ_1, and even a larger one, does not seem improbable, considering the nature of the vapors. If our numbers for r_2 and r_1 are accepted, and d, the apparent distance, be actually measured, these simple equations offer a means of investigating the refractive power of the Sun's atmosphere. There is no good reason for neglecting it, as is usually done.

E. J. Wilczynski.

Chicago, Oct. 8, 1897.

ERRATA.

The following corrections should be made in Professor Very's articles in the December 1897 and January 1898 numbers of this Journal:

Vol. VI, p. 402, for "r' = same corrected for distortion" read "R', r' = same corrected for distortion."

P. 404, equations (5) and (6) should have R' in the denominator instead of R.

P. 405, $\frac{r}{R}$ (four times at top of page and twice in first footnote) should have been primed in every case to signify that the image must be undistorted.

Vol. VII, p. 63, last line of the computation, for "$L - l' = 59° \ 2'.4$" read "$l - l' = 59° \ 2'.4$."

[1] Wilczynski, "Schmidt's Theory of the Sun," this Journal, 1, 119.

REVIEWS.

Ueber Gesetzmässigkeiten in den Spectren fester Körper (Second Part); F. PASCHEN. *Wied. Ann.*, 60, 662, 1897.

A PRELIMINARY notice of this work was published by Paschen in this JOURNAL in 1895, and the results of the work with iron oxide (*Wied. Ann.*, 58, 455) were reviewed here last year by Professor Crew; some repetition of the material of that review can hardly be avoided if one is to consider carefully the work as a whole. From the beginning Paschen has had in mind the determination of the law of radiation of an absolutely black body, by more or less exterpolation of any laws which might be found to hold for other bodies of gradually increasing "blackness." Aside from this, however, he has found some surprisingly simple relations which hold with considerable exactness for the substances actually examined, and these also will be briefly considered.

He has studied the radiation from bright platinum, iron oxide, copper oxide, lampblack, and graphite both in air and enclosed in an exhausted glass bulb; the substances being either in the form of strips, or of coatings on platinum strips, and electrically heated.

The temperatures were determined by means of thermo-couples. His arrangement of fluorite prism and concave silvered mirrors for producing the spectrum was the same as previously used and described by him,[1] and his bolometer-galvanometer combination was of extreme sensitiveness.

He observed in two ways — first measuring the energy as a function of the wave-length, the temperature remaining constant, giving him the ordinary "energy curve"; second, the energy as a function of temperature, at a given constant wave-length, the so-called isochromatic curve. Besides reducing his observations to the normal spectrum by using his own dispersion curve for fluorite, he applied the following corrections to each observation of the energy: (1) each observation was divided by $\frac{d\lambda}{d\delta}$, (obtained from the same dispersion curve) to reduce to the condition of a constant wave-length interval falling on

[1] *Wied. Ann.*, 50, 409.

the bolometer; (2) correction for loss of energy by reflection in the prism and at the concave mirrors; (3) correction for the radiation of the shutter ($t = 15°$ to $20°$ C.), to reduce to the condition of a shutter at absolute zero — *i. e.*, to obtain the entire absolute radiation of the substance under examination; (4) a correction depending on the width of slit and width of bolometer strip, to reduce to the condition of an infinitesimal slit (pure spectrum) and infinitesimal bolometer strip.

Since this correction is a very important one, and is here used for the first time, it will be well to consider the method of deriving it. Let $f(x)$ denote the energy at any point in a pure spectrum (*i. e.*, one formed from an infinitesimal slit), x the corresponding wave-length or minimum deviation, as the case may be; then the energy falling on a bolometer strip of width a whose center is at x will be

$$\int_{x-\frac{a}{2}}^{x+\frac{a}{2}} f(x)\, dx$$

But if the slit be so widened that its monochromatic image has a width a (the condition of Paschen's experiments), the strip will be just covered by an image of intensity $f(x)$, and will also receive energy from neighboring images whose centers are at $x + v$, $x - v$, etc., up to the images whose centers are at $x + a$, $x - a$; so that the total energy falling on the strip will be

$$F(x) = \int_{v=o}^{v=a} \frac{a-v}{a} \left\{ f(x+v) + f(x-v) \right\} dv$$

It remains to express $f(x)$ in terms of $F(x)$ and a so that it can be obtained from the observed $F(x)$; this has been done by Professor C. Runge, who has developed $f(x)$ in the form of a series. If $F(x)$ is known in the form of a curve, as in the present case, the form of the series which is most convenient is

$$a\, f(x) = F(x) - \frac{1}{6} F_1(x) + \frac{2}{45} F_2(x) - \cdots$$

$$\text{where } F_1(x) = \frac{F(x+a) + F(x-a)}{2} - F(x)$$

$$F_2(x) = \frac{F_1(x+a) + F_1(x-a)}{2} - F_1(x)$$

$F_1(x)$ is obtained very simply from the curve of $F(x)$, as is made evident by drawing the chord connecting the points $F(x+a)$ and $F(x-a)$ of the curve; $F_2(x)$ is obtained in the same way from the curve expressing $F_1(x)$ as a function of x. The actual corrections were obtained from the curve of observed energy, $F(\delta)$, plotted against observed minimum deviation (δ); the first correcting term was usually all that it was necessary to use. As an example of the amount of this correction it is mentioned that in the case of one iron oxide curve at 1100° C., ordinates near the maximum are increased by from 1 to 2 per cent. of their value, while farther down on the short wave-length side, where the curvature is greatest, they are decreased by from 5 to 10 per cent.

In plotting and discussing his results Paschen prefers logarithmic expressions; that is, if λ is any wave-length and J the corresponding energy, he replaces λ by $\log\lambda$, and J by $\log J$; or he plots $\log\frac{\lambda}{\lambda_m}$ against $\log\frac{J_m}{J}$, where λ_m is the wave-length having the maximum energy J_m; this has the disadvantage, as he remarks, of exaggerating the errors at the ends of the curves and reducing them at the center. The equation which Paschen finally adopts as most nearly representing all his energy curves is

$$J = c_1\, \lambda^{-\alpha}\, \epsilon^{\frac{-c_2}{\lambda T}} \qquad (1)$$

where $J=$ energy corresponding to any wave-length λ,

$T=$ absolute temperature,

c_1, c_2, α are constants.

If J_m be the maximum energy, and λ_m the corresponding wave-length, it follows from the above equation that

$$\lambda_m \cdot T = c \qquad \text{where } c = \frac{c_2}{\alpha} \qquad (2)$$

$$J_m = c'\, T^{\alpha} \qquad \text{where } c' = c_1 \cdot c^{-\alpha} \cdot \epsilon^{-\alpha} \qquad (3)$$

$$\frac{J}{J_m} = \alpha \left\{ \log\epsilon - \frac{\lambda_m}{\lambda}\log\epsilon - \log\frac{\lambda}{\lambda_m} \right\} \qquad (4)$$

$$\lambda_m = \frac{(\log\lambda_2 - \log\lambda_1)\,\lambda_1\,\lambda_2}{(\lambda_2-\lambda_1)\log\epsilon} \qquad (5)$$

where λ_1, λ_2 are any two wave-lengths on opposite sides of the maximum corresponding to equal energies of radiation,

$$\log J = \gamma_1 - \gamma_2 \frac{1}{T} \tag{6}$$

$$\text{where } \gamma_1 = \log c_1 - a \log \lambda$$

$$\gamma_2 = \frac{c_2 \log c}{\lambda}$$

Equation (1) is of particular interest because it is of exactly the same form as that deduced by Wien[1] from purely theoretical considerations for the case of an absolutely black body, except that in Wien's case $a = 5$. The relation (2) was given by Paschen three years ago in this JOURNAL, and criticised by Very; whether this be a universal relation or not, it is true within 4 per cent. or 5 per cent. for all the substances Paschen examined, except bright platinum, for an extreme temperature range of about 900° C.— from 100° to 1000°. For bright platinum the relation is more nearly $\lambda_m T^{0.8642} = c$. Equation (3) gives a means of determining a, as does also equation (4), and the two values do not as a rule agree very well. Equation (4) expresses the property of congruency, which can also be shown by plotting log λ against log J, and then shifting the curves parallel to the coördinate axes till the maxima coincide, when they are found to quite closely overlie each other. Assuming this property of congruency to be strictly true, it furnishes a convenient means of filling out absorption bands and incomplete curves; and it is by the use of this and equation (2) that Paschen attempts to determine the energy spectrum of his shutter at a temperature of about 19° C., with which he reduces his observations to the condition of having a shutter at absolute zero. This has been criticised by Dr. H. F. Reid, who points out that since all observations of radiant energy are observations of differences— between the radiation of the given source and of the shutter used to exclude this radiation, the curves for which the property of congruence has been deduced are difference curves, and the assumed zero line is really the absolute radiation curve of the shutter. If this shutter radiation curve is very near the true zero line (*i. e.*, if the correction for the shutter is everywhere small) then we will be justified in assuming as a first approximation that the observed energy curves are the true absolute radiation curves, in exterpolating any relations which we may find to hold for high temperatures, down to the temperature of the shutter, and in applying the curve obtained by exterpolation as a

[1] *Wied. Ann.*, 58, 662.

correction to the observed curves. In doing this, then, we virtually assume at once that the radiation of the shutter is very small in comparison with that of the source; it seems possible, therefore, that even Paschen's corrected curves may fall considerably short of representing the true entire radiation of his source, at least for the lower temperatures. Equation (5) is used by Paschen to obtain the value of the wave-length of maximum energy; it having been found to give results agreeing with each other to within about 1 per cent. for pairs of wave-lengths (λ_1, λ_2) as much as 4^μ apart. Equation (6) expresses the (straight line) variation of J with T, λ being fixed (isochromatic); this straight line relation is badly fulfilled, perhaps because the correction for width of slit and bolometer strip was not applied to these observations. These isochromatics, however, plotted in another way, satisfy fairly well a condition of congruency somewhat similar to that of the energy curves.

As to the constants in the above formulæ, c is largest for one of the graphite strips in a glass bulb, and least for bright platinum; a can be obtained either from the form of the energy curves or from equation (3), and these two values always differ, sometimes by a considerable amount. For Pt, FeO, CuO, and lampblack, $a = 5.58$ about, and for carbon in bulb, about 5.09, determined by the form of energy curves; determined by equation (3) the values are quite irregular. Paschen attempts to arrange these substances in the order of their "blackness," which would probably be the same as the order in which they arrange themselves with respect to the total energy radiated, since an absolutely black body radiates more (or as much) energy of every wave-length as any other body. The total energy he obtains by integrating the above expression for J; from this comparison it would follow that for low temperatures one of the graphites in a bulb is the "blackest," while for high temperatures (900° to 1100° C.) iron oxide is the blackest. He concludes that the energy curve of an absolutely black body will be found to satisfy equation (1) with $a < 5.24$, c at least 2600, c_2 about 14,000; and there is at least no experimental objection to the view that a may become 5, as required by Wien's formula.

From the behavior of graphite in a bulb, Paschen is led to investigate analytically the radiation which would be found inside an inclosure having reflecting walls and containing a radiating body at a uniform temperature, which reflects diffusely. He finds that the result-

ing radiation would be the nearer to that of an absolutely black body the better the reflecting power of the walls; so that for such a good reflector as silver, the departure of this radiation from that of an absolutely black body would be, at $0^{\mu}.6$, only 0.17 per cent., and less for longer wave-lengths which are more perfectly reflected; and that under these conditions the position of the radiating body with respect to the walls of the inclosure becomes of secondary importance. Following up this idea, he intends to investigate the radiation from a blackened platinum strip inclosed in an internally silvered glass bulb. As a method for realizing the radiation of an absolutely black body, this seems to offer great advantages in the way of simplicity and ease of manipulation, and the results of his study will be looked for with great interest.

C. E. Mendenhall.

Johns Hopkins University,
February 14, 1898.

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PLATE IV.

SOUTH FRONT OF THE YERKES OBSERVATORY.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME VII APRIL 1898 NUMBER 4

ON THE SPECTRUM OF HYDROGEN IN THE NEBULÆ.

By J. SCHEINER.

THE spectrum of hydrogen in the nebulæ differs, as is well known, from that obtained with Geissler tubes in this respect: that while under certain conditions $H\beta$ may be well seen, the $H\alpha$ (C) line can scarcely be detected or may be invisible, whereas in the spectrum of hydrogen tubes $H\alpha$ generally appears brighter than $H\beta$ (F). In only one nebula (*G. C.* 4390) was $H\alpha$ observed by Keeler, and it was very faint. It was long ago pointed out that by reduction of intensity the hydrogen spectrum could be reduced to the line $H\beta$ in the greenish blue. Thus in 1868 Lockyer and Frankland[1] showed that under certain conditions of temperature and pressure the hydrogen spectrum reduces to the line $H\beta$, but more precise data as to the nature of these conditions are wanting. The same year Huggins[2] showed that by weakening the intensity, the nitrogen spectrum could be reduced to a single line in the green, and, what it is here important to note, that this was effected by a purely subjective weakening, produced by the absorption of neutral tinted glass or by increasing the distance of the Geissler

[1] *Proc. Roy. Soc.*, 17, 454.
[2] *Phil. Trans.*, 1868, 544.

tubes. Huggins thereby arrived at the following important conclusion: "It is obvious that if the spectrum of hydrogen were (likewise) reduced in intensity the line in the blue would remain visible after the line in the red and the lines more refrangible than F had become too feeble to affect the eye." The accuracy of Huggins' conjecture was later (1880) established by the direct observations of Fievez. It is consequently made very probable that the cause of the difficulty in seeing $H\alpha$ in the nebulæ is purely physiological, that is, it is an example of the Purkinje phenomenon. Nevertheless, this explanation does not appear to have been generally accepted, for quite recently Keeler,[1] in his beautiful investigation of the spectra of the nebulæ, expressed the opinion that because of the above mentioned peculiarity of the hydrogen spectrum the temperature of incandescence of the nebulæ must be very high. And quite recently Runge[2] has deduced important physical conclusions from this phenomenon.

It therefore seemed desirable to investigate the question of the luminosity of hydrogen in the nebulæ somewhat more closely, and to determine, by introducing circumstances approximating to those under which the nebulæ emit their light, whether objective changes of the character above referred to can be produced in the spectrum of hydrogen in the attenuated state, or whether the subjective weakening of the light is the determining factor, and if so to what extent.

The radiation of the nebulæ in space takes place under two conditions, which can be only approximately fulfilled in the laboratory. In the first place the gases are without doubt extremely attenuated, the molecular motions are exceedingly feeble, and the considerable brightness is due merely to the enormous thickness of the luminous strata (many millions of kilometers); and in the second place the emission takes place at an external temperature which can differ but little from absolute zero.

In reference to the first condition we have at hand a number

[1] *Publ. of Lick Obs.*, 1894.

[2] *A. N.*, No. 3471.

of investigations and experiments. In wide tubes excited by feeble electric discharges the hydrogen lines may be observed to vanish completely, while with thicker strata — obtained by looking lengthwise through the tubes — they again appear. Thereby, as was stated above, $H\alpha$ first disappears, and then $H\beta$. With reversed conditions the order of visibility is also reversed.

Investigations on the possible changes of the spectra of gases by lowering the external temperature were carried out by Koch[1] in 1889. He used wide Geissler tubes which were excited by means of a strong induction coil, at a temperature between —80° and —100° C. (obtained by mixing solid carbonic acid and ether), but could not detect any change whatever in the spectrum. Since it is now comparatively easy to obtain temperatures as low as —200° with liquefied air, the opportunity seemed to me to be favorable to extend the researches of Koch, and at the same time to excite the luminosity, not by the induced current itself, but by electric waves, since thereby a much smaller increase of the surrounding temperature is produced.

The considerable quantity of liquefied air necessary for the investigation was obtained by the friendly interest of Dr. Spies, of the Urania Institution in Berlin, who likewise gave me his kind assistance in carrying out the investigations themselves. As a receptacle for the liquefied air, into which the Geissler tubes were directly introduced, a glass vessel with double walls was used, the space between the two walls of which was exhausted as far as possible. This arrangement is necessary, for otherwise the exterior of the vessel will immediately be so thickly covered with frost as to make any observation impossible. Even then, when the outside glass wall is cooled merely by outward radiation, a strong deposit of frost is formed. The temperature of the outer glass envelope is, however, above the freezing point of alcohol, so that by washing with alcohol the frost can be removed. The tubes were excited in the field of

[1] *Wied. Ann.*, 38.

a Tesla high-tension transformer. Their luminosity was then quite weak, so that the hydrogen lines could just be recognized. No difference in the relative or absolute intensity of the hydrogen lines of the cold tubes, compared with the lines seen under the usual circumstances, could be detected, even when, in order to avoid raising the temperature by the radiation of the glowing gas itself, the excitation of the Tesla field was effected by a single spark, the spectrum being then observed as it momentarily flashed into view. Inasmuch as the space over the liquefied air was continually exhausted by means of an air-pump during the investigation, the temperature of the space surrounding the hydrogen could not have been higher than —200° C. in the case of a flash produced by a single spark. These investigations therefore confirm the earlier conclusions of Koch and lead to the further result that *the spectrum of hydrogen does not change when the surrounding temperature is reduced as low as —200° C.* (so far as the temperature can be ascertained without special measurements). This temperature approaches, at least, the absolute zero —273°, and it may therefore be stated as very probable that changes in the intensities of the hydrogen lines do not occur in any case when the external temperature is reduced.

This conclusion also fully agrees with the modern views concerning the luminosity of gases, according to which the radiations which produce line-spectra arise from disturbances within the individual molecules, and are therefore independent of the surrounding temperature.

I now pass to an investigation of the physiological disappearance of the $H\alpha$ line; in order, however, to obtain results free from the possibility of doubt, it was first necessary to determine whether the earlier disappearance of the $H\alpha$ line, when the intensity is objectively reduced by diminishing the electric excitement, is not at least partially caused by an actual change in the relative intensities of $H\alpha$ and $H\beta$. For this purpose I compared the intensities of $H\alpha$ and $H\beta$ with the intensities of the corresponding wave-lengths in the spectrum of a petroleum flame, by means of a spectrophotometer. With two different

Geissler tubes, I thus obtained for strong induction currents and for weak electric waves the following figures, which indicate how many times the brightness of the red in the spectrum of the petroleum flame appeared to exceed that of $H\alpha$ when the intensity of $H\beta$ was placed equal to that of the corresponding wave-length in the continuous spectrum :

Induction Current	Electric Waves
3.4	4.0
2.1	3.3
2.8	3.4
2.7	2.1
2.8	3.2

These figures are means of four settings, which, however, owing to the difficulty of such observations, are rather uncertain, so that the final values 2.8 and 3.2 are to be regarded as fairly accordant. Thus it appears that in the case of the objective weakening produced in these experiments, which was as much as one-fiftieth part of the original intensity, real changes of the relative intensities of $H\alpha$ to $H\beta$ were not produced, and hence the figures which follow are the expression of purely physiological phenomena.

For measuring the different reductions of intensity, by means of which the hydrogen lines are brought to the vanishing point, a spectrophotometer, or, in fact, any compound spectroscope, could not be used, because in such an instrument the faint lines could no longer be seen. The following simple arrangement was therefore used. The Geissler tube was set up at the distance of distinct vision (or at a distance somewhat greater), and viewed with a direct-vision system of prisms, the capillary bore of the tube serving as a slit. Between the tube and the prism-system two Nicol prisms were introduced, one of which could be turned and its angular displacement measured. By turning this prism the hydrogen lines could be made to vanish.

In all cases, even in that of feeble excitation in wide capillaries, the $H\alpha$ line appeared to me always brighter than the $H\beta$

line. Then on weakening the light, there occurred, at a certain intensity, an apparent equality of the two lines, after which $H\alpha$ disappeared and then $H\beta$.

The determination of the moment of disappearance is naturally quite uncertain; however, the figures obtained in a single series of observations agree fairly well. The marked difference between the two series observed by me may be due in part to a real difference in the sensitiveness of the eye; but they are principally to be ascribed to differences in the experimental conditions, and especially to differences in the apparent width of the lines, which gave rise to marked changes in the perception of the lines; the more accurate investigation of this latter effect is of only physiological interest. These are closely related to the recently published physiological investigations of Lummer,[1] and it is necessary to note here only this much—that for my eyes the color sensitiveness to the red remains up to the point of complete disappearance, while in the case of the $H\beta$ line previous to vanishing sensitiveness to the blue-green ceases, and the line appears merely gray.

The following table contains the ratios of the intensities of the $H\alpha$ and $H\beta$ lines, at the moment of disappearance, obtained from numerous observations. The numbers thus indicate how many times, after $H\alpha$ had vanished, the light had to be weakened in order to cause the disappearance of $H\beta$.

	Scheiner		Hartmann
	I	II	I
Wide capillary, Strong induction current	35.4	14.2	30.0
electric waves	8.1	8.1	8.0
Medium capillary, Strong	31.1	7.8	28.7
weaker induction current	15.1	19.3	....
electric waves	13.0	10.6	24.9
Narrow capillary, Electric waves	18.1	....	27.6

The above results show that, for Dr. Hartmann as well as for myself, at least an eightfold weakening of the light is

[1] "Ueber Graugluth und Rothgluth," *Wied. Ann.*, 62.

required, in order to cause $H\beta$ to disappear after $H\alpha$ has vanished. Under some circumstances, however, this amount may be as much as thirty times.

Of special interest are some observations which Professor H. C. Vogel kindly made with a tube excited by weak induction currents, because his eye is but feebly sensitive to the red, while it is remarkably sensitive far out into the ultra-violet.

The value for the relative vanishing intensities of $H\alpha$ and $H\beta$ is increased in the case of Herr Vogel to five times the maximum value obtained by myself, namely, to 150, whereas for the disappearance of the $H\gamma$ line the corresponding number is only one-fourth of mine.

As an entirely independent confirmation of the values obtained I finally made the following experiment. In the field of the spectrophotometer were cut out, by means of the ocular slit, at the position of the $H\alpha$ and $H\beta$ lines in the continuous spectrum of the petroleum flame, fine artificial lines, the appearance of which precisely resembled that of the hydrogen lines with a narrow capillary tube. When weakened, precisely the same phenomena appeared as in the case of the actual hydrogen lines. The value obtained for the above ratio was 29.2.

I may add that the apparent equality of $H\alpha$ and $H\beta$ for my eye takes place at intensities ranging in the different experiments from 2 to 10 times the intensities corresponding to the vanishing point of $H\alpha$.

Now since in all the experiments I have described the $H\beta$ line was considerably brighter at the vanishing of $H\alpha$ than I have ever observed it in the spectrum of the nebulæ, the following result is established: The absence of the $H\alpha$ line in the hydrogen spectrum of the nebulæ is due to physiological causes, and it is consequently not permissible to deduce from this peculiarity of the hydrogen spectrum in the nebulæ any conclusion whatever concerning the physical conditions under which the light-emission of these celestial bodies takes place. Whether certain nebulæ may not prove to be exceptions to this rule is to

be left an open question; it is certainly not impossible that such may be the case.

It is evident that strongly marked physiological peculiarities of our eyes, such as the one under consideration, are also of importance in many other kinds of astronomical observations. Although they have long been well known, they have seldom received due consideration. I will here mention only the systematic differences between determinations of magnitudes of the bright and faint red stars, to which Herr Wilsing[1] called attention some years ago, and to the differences in the relative intensity of the lines of the nebulæ, which have been observed in differently bright portions of the nebula in Orion by various observers. Although in the latter case the lines in question differ but slightly in wave-length, their brightness is so close to the limit of visibility that great differences in relative intensity, sufficient to cause a complete apparent reversal of the order of brightness, may be produced.

Astrophysical Observatory, Potsdam,
January 1898.

[1] *A. N.*, 112, 280.

ON THE LEVEL OF SUN-SPOTS AND THE CAUSE OF THEIR DARKNESS.

By A. L. Cortie, S. J.

In the number of the Astrophysical Journal for August 1897 Professor Riccò has published a table, formed from the drawings of Sun-spots made during the period 1881–1892 at Palermo and Catania, which supports the depression theory of the level of Sun-spots first advocated by Dr. Wilson. In view of the discussion which has been raised on this subject, it might be well to compare with Professor Riccò's table the results deduced by Father Sidgreaves from the Stonyhurst drawings made during the same period. The number of days on which the Sun was observed at the Italian stations during the eleven years was 3451, at Stonyhurst 2597. The drawings made by Professor Riccò are to a scale more than twice that of the Stonyhurst series, the diameters of the projected images of the Sun being 22.44 and 10.5 inches respectively. The smaller scale, however, is amply sufficient for the purposes of the discussion as to their bearing on the validity of the depression theory of the level of Sun-spots. Indeed, they are to be preferred to large scale drawings, as the tremors introduced into projection pictures of the Sun by magnification render accuracy of delineation somewhat difficult. This opinion based on experience is also subscribed to by Professor Frank W. Very in his paper on "Heliographic Positions" (Astrophysical Journal, 6, 256, October 1897). Moreover, comparing these drawings with those which are occasionally made here on a scale of thirty inches to the solar diameter, no discordance or contradiction has ever been detected. Nevertheless it must be borne in mind that the presumably purer Italian sky would allow of greater magnifications being more safely employed than in England. Professor Riccò has selected 185 drawings of spots from his 3451 Sun pictures as admissible for the study of the question. The same

proportion would give 139 as the number which might be expected to be chosen from the 2597 Stonyhurst pictures. The number of different spots actually chosen by Father Sidgreaves was 126, giving 163 test cases at either or both limbs. Hence we may infer that the principles which guided both observers in the exclusion of unsuitable cases were practically identical. There is, however, one particular in which they have not agreed. For in Professor Riccò's table 36 cases are quoted as neutral, or cases in which the penumbra was symmetrical and of equal width on either side of the umbra when the spots were near the limb. But such cases would have been reckoned as against the Wilsonian hypothesis by Father Sidgreaves. And rightly so; for by the hypothesis, when a spot is near the limb the penumbra ought to be wider towards the limb. If it is not wider but equal on both sides of the umbra then the case is against the Wilsonian theory. Again, Professor Riccò writes: "Greater weight must be given to the still more significant cases of spots near the Sun's limb, the penumbra of which, conforming to the appearance of a cavity seen in perspective, is invisible on the side opposite the limb. I have found twenty-three cases of this sort in eleven years, and only one contradictory case." Why such cases should be more significant than that, for instance, of the large round spot of June 1887, which is reproduced in Plate 7 of the *Memoirs R. A. S.*, 49, illustrating Father Perry's paper on "Photographs and Drawings of the Sun," and which indicates the existence rather of a mountain than of a cavity on the solar limb, it is difficult to understand. Or why more significant than those numerous cases of spots near the limb in which the penumbra in the direction of rotation has vanished altogether, leaving the umbra as a more or less thick line with a fringe of penumbra N. and S.? Such cases can be frequently observed and attention has already been called to their existence by Professor Spörer. In this connection, too, it may be well to recall to mind that Wilson demanded that the umbra of a deep spot should be lost to view altogether at 17″ from the limb, while for less degrees of depth he demands extinction of the umbra at 9″, 5″, and 3″.

Without supposing the spots to be as deeply cavernous as Wilson imagined them to be, yet the umbra might be expected to disappear when the spots are very near the limb, while exactly the contrary obtains. The umbra very rarely disappears even at positions right up to the limb, while frequently the penumbra on both sides of the umbra cannot be seen.

One fact worthy of notice, indicated by both the Italian and the Stonyhurst results, is that the number of symmetrical spots was much larger in the years 1881–1886 than in the latter half of the selected period. It will be remembered by solar observers that the maximum of 1882–1883 was of a protracted nature, the solar activity continuing unabated until the autumn of 1886, when the Sun became quiescent. The following table combines the results both for Catania and Stonyhurst, omitting the column of neutral cases as originally (*loc. cit.*) given by Professor Riccò.

PENUMBRA OF REGULAR SPOTS.

Year	Favorable to Wilson		Unfavorable to Wilson	
	Catania	Stonyhurst	Catania	Stonyhurst
1881	28	3	0	6
1882	16	5	7	12
1883	19	6	2	21
1884	22	8	5	28
1885	15	6	1	19
1886	10	6	0	16
1887	7	4	0	6
1888	0	0	1	4
1889	1	1	0	2
1890	2	0	1	0
1891	..	1	..	3
1892	11	2	1	4
Totals	131	42	18	121

From the table we deduce the proportion of cases favorable and unfavorable to the Wilsonian hypothesis :

	Favorable	Unfavorable
Catania, - - - -	7.3	1
Stonyhurst, - - -	0.3	1

If we reckon cases in which the penumbra was symmetrical about the umbra when the spots were near the limb as unfavorable, as it seems to the writer they ought to be reckoned, then the Catania results would give the proportion

favorable : unfavorable :: 2.4: 1.

But even so the discrepancy of the columns is so great that it is difficult to subscribe to the conclusion advocated by Professor Riccò; "it must, therefore, be admitted that the spots are cavities, *i. e.*, breaks or openings in the photospheric layer." Moreover, the Catania results themselves show that some 14 in every 100 spots are elevations above the photosphere. Therefore, not all spots are cavities, and at least some spots, even judging from the indications furnished by the estimated unequal breadths of the penumbra when near the limb, are situated above the photospheric level. But even allowing that the sides of the penumbra shelve downwards, no depression hypothesis has so far satisfactorily accounted for the numerous instances when the umbra, without any penumbra at all in the line of sight, or of such restricted amount that it cannot be detected even on large scale drawings, stands boldly out even up to the very limb of the Sun. On the contrary, it would be an exceptionally rare observation which could show a spot without the umbra visible at the limb. This fact is most important in framing any theory of the level of Sun-spots, and needs insisting upon. The ratio of the lateral breadths of the penumbra is relatively unimportant.

With regard to the cause of the darkness of spots, Professor Riccò seems to adopt the opinion that it is not attributable to the presence of substances which absorb on account of lower temperature. The spectra of Sun-spots, however, if we are to interpret them by the rules ordinarily applied to spectroscopic appearances, would indicate absorption as the cause of the darkness of Sun-spots. Even the continuous dark spectral band of a Sun-spot has in the region E to F been resolved into fine lines by both Young and Dunér, and in the region C to D, Young and the writer have independently detected the presence of identical

dark bands, of which the only satisfactory explanation seems to be the formation of compounds over a spot by a reduction of temperature. Again, the suggestion that Sun-spots may be particularly rich in ultra-violet light, advanced by Mr. Evershed, would seem to be negatived by the fact that, at least as far as selective absorption is concerned, the richest field for observing the characteristic phenomena of Sun-spot spectra is to be found at the opposite end of the spectrum. Beyond the reversal of H and K, photographs of Sun-spot spectra in the violet and ultra-violet regions are comparatively, if not entirely, wanting in details.

Another point worthy of notice in this discussion as to the level of Sun-spots, is that the results derived from a study of spots when near the limbs concern only regular spots, which are a very small proportion of all the spots observed, in Professor Riccò's results 185 in 3324 spots. Even if the phenomena of unequal penumbral width should indicate depression in every single case studied, which is far from being the case, yet it is not a logical sequence to draw the conclusion that all spots, at all times of their life histories, even when they are forming as scattered groups and separating into two main spots, as they generally do, are below the level of the photosphere. The only conclusion we are warranted in drawing is that Sun-spots, at the time in their life histories when they are round and quiescent, are below the level of the photosphere. Presumably the thermal researches of Frost, Langley, and Wilson were not made on this particular class of quiet, round spots, but on spots in general, whatever might be their form and structure. As a means of reconciling discordant results, I would suggest the possibility of a difference of level in Sun-spots at different periods of their life history.

In a note in the number of the JOURNAL for November 1897 Professor Hale discusses the subject of the appearance of spots when seen in transit across the Sun's limb, and also presents a very suggestive scheme of an ideal spot. In the whole course of the continuous observations of the Sun of the past seventeen

years, made at Stonyhurst, a transit of a spot across the limb has been seen but twice, and drawn on one of these occasions. The phenomenon is therefore very rarely observed, and deserves full discussion. The first case occurred on May 8, 1884, at 10:00 A.M., G. M. T. I quote the exact words of the observer, Mr. W. McKeon's notebook. "The new group of April 25 was now just appearing at the following limb as a large black spot. The spot was exactly on the limb, and when observed casually appeared to cause a notch in it. Upon close examination, however, with excellent definition the finest possible thread of light was seen behind the spot. Near the north of the spot I thought I could see a facula project out from the limb. The sky becoming hazy, I was unable to be absolutely certain of what I saw," *i. e.*, in regard to the projecting facula. The second case occurred on September 17 of the same year, when the penumbra on the following side of a large black spot was drawn when exactly on the limb. The umbra of the same spot was photographed on the limb some ten hours earlier in the famous photograph secured at Delhra Dun. The notched appearance presented in such photographs is attributed by the Rev. F. Howlett (*Monthly Notices*, **55**, No. 2, 1894) to "the lack of sufficient power in the already degraded light near the limb to depict the yet more degraded dusky spot." Father Sidgreaves (*Monthly Notices*, **55**, No. 5, 1895) fully accepts the evidence of the Indian plate—"the notching umbra is perfectly natural, of a strong silver deposit"—and contends that it "admits of no other explanation than that of a high elevation of the umbra." In this second case also, when drawn at Stonyhurst September 17, 3:34 P.M., G. M. T., "a very fine thread of light was discernible behind the spot" (observer's note). A careful study of the life histories of these two spots from the Stonyhurst drawings brings to light the following facts: On April 20 there was no sign of the first group, but it is shown on the drawing of the 22d. The faculæ round this new group were very scarce. In character it consisted of two main spots, the leader being the larger, with a few small spots between them, a common type of outburst. On

April 23 and 24 the observer again notes that faculæ were remarkably scarce and not bright about the group, which was now near the P. limb. It then transited the invisible hemisphere and was next seen as a notch on the F. limb on May 8. The drawing of May 9 shows it to be a single round black spot, and the note occurs that the faculæ were "rather faint near the large spot at the F. limb, and radiated from the spot, but not to any great distance." On May 10 an area of bright faculæ is drawn surrounding the spot. Between this and the 17th it continues as a round spot, with its encircling ring of faculæ. On the 17th there was a remarkable outburst of "veiled spots" in the region immediately following the spot. On May 21 it reaches the preceding limb, reappears again on June 6, still as a round spot, but much reduced in area, and on June 17 is drawn for a third time near the preceding limb. This is its last appearance. It was therefore a large, long-lived, and dense black spot.

If the cavernous hypothesis be correct we might reasonably have expected that a spot which caused a notch on the limb on May 8, would, on appearing on the visible hemisphere, have shown the penumbra of very unequal extent on either side of the umbra. When the spot was drawn on May 9 it was distant about $1'$ from the limb, and yet the breadth of the penumbra was almost equal on either side of the spot. It is difficult to judge to which side the preponderance should be given. It is catalogued as against the depression hypothesis. On May 20, when distant about $49''$ from the P. limb, it is decidedly against Wilson's cavernous theory. On June 6, at its next return, when distant $1'30''$ from the F. limb, the penumbra is again almost equal on either side of the umbra; allowing for foreshortening a rough measure gives an excess of only $0^\circ.2$ in favor of a depression. On the 16th it is near the opposite limb, $1'19''$ distant, and now it is unmistakably a depression. On the 17th, when distant $7'$ from the limb, it is a very small hazy line. It is a depression, therefore, when it is disappearing.

The life history of the second case, that of September 17, was of the same character as its predecessor. On September 6

a single minute umbral dot is seen; on the 7th there are three dots amidst not very dense faculæ; on the 10th there is a scattered group of small spots. Next day the resolution into two main spots with the intervening group of small spots has commenced. For the three days, September 12–14, the two main spots grow greatly in area, amidst faculæ which cling to the spots and are not extensive. By the 15th the process of resolution is completed, and only the leader and rear spot of the group remain. The faculæ are few and tolerably compact, as is usual in newly formed groups. On September 16, as in the former case, we find it noted "faculæ scarce near the large new group approaching the P. limb," nor was there any great increase in the faculæ when on the 17th the leader of the group was in transit across the limb. The fact is noticeable that the penumbra on the following side of the umbra should have been seen at all in such a position. On October 2, when the group reappeared on the E. limb of the Sun, the faculæ radiating from the group were of considerable extent. The leading spot was the larger, and on October 3 the smaller began to break up and form into small spots, which by October 10 were a considerable number. This development was at the expense of the leader, which simultaneously became smaller, though remaining regular in outline. On October 13 and 14 the whole group was dying rapidly. At the next rotation the place of the group was occupied by a great area of faculæ. The first umbral dots, their rapid increase in area and formation into two spots with a companion group of small spots between them, the growth of the leader, the breaking up of the rearmost spot, the final dying out of either the isolated leader or the greatly reduced group, as mere penumbral patches, the compact and clinging faculæ in the early stages of the life of the group, its gradual extension and radiation in branched forms as the group grows older, and its final occupation of the total region of the spot outburst, is the usual sequence of phenomena in this type of spots. It was fully illustrated in the two groups under discussion.

With regard to the second example, on September 16, when

45″ from the limb and about to cause a notch, the penumbra, as in the first instance discussed, was by no means so unequal on either side of the umbra as it ought to have been had the spot been a cavity. However, the exact measure is difficult to give as the spot was broken. But on October 2, at its second appearance, when 20″ from the limb, if a rough outline drawing be correct it is not a cavity, the difference of the measures being about 0°.7 against the depression theory.

On referring to my notes of spectroscopic observations of Sun-spots, I find that I observed the spectrum of the first group on May 11, May 18, and June 12. I may state that I had hitherto not been aware that these observations, and those of the second group to be detailed below, were observations of the groups at different returns on the visible hemisphere. On May 11 the general absorption of the spot was so intense in the red as to almost mask the selective absorption, which was observed with difficulty. The lines, too, were more than usually widened in the penumbra. On May 18 the masking effect still continued, but the lines were not affected in the penumbra as on the 11th. On June 12 the spot was still very dark on the slit, and some iron lines were noted as being particularly affected in the spot. The spectrum of the second group was observed on September 11 and 12, and October 9. On September 11 some prominent calcium lines were observed as very much darkened in the umbra. On the 12th reversals and distortions of the C line were observed, and on October 9 the iron line at 6024.1 was not widened in the usual spindle-shaped manner, but was of the same width in both umbra and penumbra.

These two spots, then, which caused notches in the solar limb, by both visual and spectroscopic evidence, were long-lived dense black spots. Yet they were undoubtedly not deeply cavernous, and if not actually raised above the photospheric level during the greater portions of their lives, were the shallowest of depressions. To Lalande's objections against the cavernous theory of spots, drawn from Cassini's and de la Hire's observations of notches on the limb made by transit-

ing spots, Dr. Wilson replied (*Phil. Trans.*, **73**, 1783) that "a large shallow excavation, with the sloping sides or umbra (*i. e.*, penumbra) darker than common, may be more or less perceptible at the limb." The observations detailed above of the two spots observed on the limb in 1884 would certainly not negative, but would rather justify his contention, and in studying their life history I was independently, before consulting Wilson's paper, led to the same conclusion.

It would seem, then, that while many spots are above the photospheric level, and many below it, it is possible that individual spots are at different levels at different periods of their life histories, while notches at the limb are due to spots which are large black shallow depressions. The weight of spectroscopic evidence, too, favors the *opinion that the darkness is due to absorption.*

STONYHURST COLLEGE OBSERVATORY,
March 4, 1898.

SOURCES OF ERROR IN INVESTIGATIONS ON THE MOTION OF STARS IN THE LINE OF SIGHT.

By H. C. Vogel.

The *Bulletin Astronomique* for February 1898 (Vol. XV) contains an article by M. Deslandres entitled "Causes d'erreur dans la recherche des vitesses radiales des astres. Importance de l'erreur due aux variations de température. Méthodes de correction." In this paper there are comments on certain of the methods of observation employed at Potsdam in determinations of the motion of stars in the line of sight, and M. Deslandres allows himself to make charges to which I must emphatically reply, since his suppositions are incorrect and are to be ascribed to the haste, which is wholly inexcusable in view of the gravity of the charges, with which M. Deslandres has read Vol. VII of the *Publicationen des Astrophysikalischen Observatoriums*. For the benefit of those who have read M. Deslandres' article, but may not have at hand my "Untersuchungen ueber die Eigenbewegung der Sterne im Visionsradius auf spectrographischen Wege," I will here consider somewhat more closely certain of the details referred to.

After a long introduction occupying a special section of his paper, M. Deslandres turns to a consideration of the methods of applying the comparison spectrum. He finds the plan employed by me of placing the Geissler tube at right angles to the slit, "very simple," but adds (p. 53) "but the pencils of the two sources to be compared are very different, and on account of this fact there may result a systematic error, which Vogel nevertheless has not examined in his great memoir." Now the simplicity of the method can hardly be regarded as a disadvantage; further, I am perfectly willing to admit the correctness of M. Deslandres' contention that the pencils from the two light-sources which are to be compared are different. But a simple consideration shows that the path of the light from the artificial light-source in the plane which passes at right angles to the slit through the

optical axis of the instrument, in which both the length of the spectrum and the expected displacement lie, is exactly the same as for the star. From this it appears that systematic errors are not to be expected if only the optical parts of the apparatus, when tested by themselves, are found to be free from defects. As a matter of fact, no error could be found in the previously undertaken test of the complete apparatus. (Vol. VII, p. 24.) The method, which I published over twenty-six years ago in the *Astronomische Nachrichten* and in the Bothkamp *Beobachtungen*, and gave in complete detail, with illustrations, in 1892, in Part 1 of Vol. VII of our Publications (beginning on p. 17), I cannot too highly commend, since it excludes the large accidental errors which easily enter in other methods, and which without careful attention may remain constant through long periods. It is, moreover, a simple matter for any one to convince himself by direct observation whether in my method systematic errors do or do not appear. I must, however, take decided exception to vague assertions made concerning it.

On p. 54 M. Deslandres remarks: "However, the photographic method contains a serious source of error which Vogel in fact mentions, but without according it due weight and without giving its value for the apparatus employed.[1] This error is the displacement of the spectra caused by changes of temperature during the exposure." I now continue my comments In his paper in the *Annuaire* for 1891 M. Cornu could have referred only to the preliminary paper which I published on the Potsdam spectrographic observations, since Part 1 of Vol. VII of the *Publicationen des Astrophysikalischen Observatoriums* did not appear until 1892. But even in this paper, which was published in 1889 in the *Astronomische Nachrichten*, No. 2896, the influence of temperature was investigated (pp. 250–251), and M. Cornu, who at all events favored this article with a thorough examination, could not criticise a shortcoming which did not exist.

In the later publication I have devoted several pages to the

[1] "This error is not mentioned in M. Cornu's memoir on the subject (*Annuaire du Bureau des Longitudes*, 1891)."

great influence which temperature exercises on spectrographic observations. The special method of measurement employed for stars of Class II is the direct result of a consideration of the spectra as varied by temperature. But its importance can hardly have been fully appreciated by M. Deslandres, since in his own investigations he has dealt with only the larger and more palpable errors.

With reference to this last question I take the liberty of quoting from my own investigations (Vol. VII, Pt. 1, p. 24), at the same time remarking that the most important points raised here, together with the concluding sentence, may be found given in almost exactly the same words in the paper published in 1889, in No. 2896 of the *Astronomische Nachrichten*.

The influence that a variation of temperature during "the exposure exercises on the photograph is essentially caused by a change in the index of refraction of the prisms and by changes in the separate parts of the apparatus. As is well known, both the refracting angle and the dispersion of prisms are very decidedly affected by temperature, and in consequence of this change there occurs a displacement of the spectrum on the plate and a simultaneous lengthening or shortening of the spectrum. From this broadening of lines may result, which, however, if the variation of temperature is not very irregular, may exercise no injurious effect on the measures. Experience has shown that even in cases when the thermometer attached to the apparatus (not the temperature of the prisms) has changed 2° during an exposure of an hour, no noticeable effect could be observed. In the case of greater changes of temperature and longer exposures the effect becomes evident through the poor definition of the photograph. Certain experiments in which the apparatus was artificially heated have shown this appearance in a very marked way, but at the same time have demonstrated that well-enclosed prisms very slowly follow the temperature changes of the outer air. Expansion or contraction of the metal parts of the apparatus during the exposure may also cause a displacement of the plate; but this displacement will be extremely small, and it may

be assumed that neither this nor the changes in the optical parts of the apparatus can have any appreciable effect on the position of the lines of the comparison spectrum with reference to those of the star spectrum, if only, *as has always been the case in the observations*, the artificial light-source is employed during *the whole time of exposure*, or at *intervals which are symmetrical with reference to the middle of the exposure time*." This last sentence disposes of the criticism made by M. Deslandres (p. 58, line 13 from the bottom). He gives further clear evidence of the hasty manner in which he has read my article when he says: "Vogel does not give the ordinary time of exposure of the hydrogen tube employed." Further below he remarks, "the exposure of the comparison spectrum is *naturally* always made in the middle of the exposure of the star," and M. Deslandres is of the opinion that this procedure gives only a first approximation in allowing for the effect of temperature. In my own opinion it is not natural, but wholly wrong to expose the comparison spectrum only at the middle of the exposure time for the star, as follows from the concluding sentence of the long extract from my article which has been already quoted. Regarding the few observations with the iron spectrum, which are really to be considered only as investigations on the utility of a spectrum other than that of hydrogen, I remark in Vol. VII, p. 19 (below): "The exposure for the iron spectrum, which, as has already been mentioned, is very short, is given either at the beginning and the end of the exposure of the star spectrum or at the middle of the exposure. In the latter case the sharpness of the lines of the comparison spectrum affords *no evidence* regarding the unchanged condition of the apparatus during the much longer exposure on the star; on account of the simplicity of this procedure I have made the exposure on the artificial spectrum at the middle of the exposure of the star spectrum, but as in the case of the other photographs, I have employed as a check the hydrogen spectrum, which was exposed *during nearly the entire exposure time for the star*."

The method which M. Deslandres employs of photographing

on the plate several iron spectra at various distances from the star spectrum at the beginning, the middle, and the end of the exposure of the star spectrum, has also been practically tested by Mr. Newall (*Monthly Notices*, Vol. LXVII, No. 8). He employs not only the three iron spectra used by M. Deslandres, but four, two of which are on each side of the star spectrum.

I am unable to see any great advantage in this method; for there is little use in causing the changes which take place during the exposure of the star to be directly visible, if only one so arranges the observations that the changes can have no influence on the measures. This is the case when one employs my method, photographing the comparison spectrum immediately before and immediately after the spectrum of the star, and in such a way that in both instances, and naturally on both sides of the spectrum of the star, the lines lie as close as possible to this spectrum, and when no variation has occurred, are exactly superposed. Even if the comparison spectra are very narrow, in four exposures the outer ones lie so far from the spectrum of the star that the measures are rendered very difficult on account of the curvature of the spectral lines.

I believe that all the criticisms which M. Deslandres has directed against the Potsdam observations may now be regarded as completely disposed of.

I may remark in passing that I consider little advantage will follow from M. Deslandres' idea of surrounding the spectrograph with water for the purpose of protecting it from temperature changes, on account of the experience derived from the entirely similar experiments which I made ten years ago with Professor Scheiner, when in our preliminary investigations we were almost ready to doubt the possibility of obtaining valuable results on account of the marked effect of temperature on our apparatus. These experiments led to no important results, but they gave us a better knowledge of the peculiarities of the apparatus and led us to give them such consideration that finally observations of great precision resulted. What degree of precision was attained, not only for the small portion of the spec-

trum employed in determinations of motion, but also for the whole extent of the region included on the spectrogram, may be seen in Part II, Vol. VII, of the *Publicationen des Astrophysikalischen Observatoriums*, which was published in 1895, and with which M. Deslandres seems to be unfamiliar.

I have no occasion to deal further with M. Deslandres' article, as it contains, following the custom of the younger astrophysicists, nothing more than a number of suggestions regarding various forms of construction which, as they have not been tried in practice, are for the most part of very little value.

Astrophysical Observatory,
Potsdam, February 28, 1898.

THE VARIATION OF SOLAR RADIATION.

By Frank W. Very.

The measurement of the absolute value of solar radiation is so difficult, that for a long time an estimate of the solar constant prevailed, which was but little more than half of that now accepted; and there is, as yet, no definitely established doctrine as to the amount, or even as to the fact, of any change in this fundamental quantity, although presumably all astronomers would concede the probability of some small fluctuation in the so-called "constant," to say nothing of a progressive variation in the course of ages, which is not considered here.

Every direct measurement of solar radiation requires correction for instrumental errors and atmospheric absorption. The correction at sea-level, even under favorable circumstances, nearly equals the quantity directly indicated by the actinometer, and the absorptive quality of the terrestrial atmosphere fluctuates fortuitously, not only from day to day, but from minute to minute. Under these circumstances, refinements in actinometry are of small avail. Methods of measurement are already far in advance of other conditions of the problem which are beyond our control. It is evident that the possible fluctuations in the Sun's absolute radiative energy most intimately concern the science of meteorology, and it is possible that the problem will have to be solved entirely by meteorological methods.

There is at present but little unanimity of opinion as to the value of such methods. The fact that simultaneous weather conditions in different parts of the Earth are diverse, has appeared to some sufficient to make the determination of a connection between solar and terrestrial changes hopeless, while others have denied that there can be any connection under such circumstances. I cannot agree with these positions. The same cause produces opposite effects under varying concomitant conditions. Gravity which causes the fall of a stone, makes the balloon rise. The

prevailing methods of indiscriminate averaging in meteorology eliminate some of the most important factors. On the other hand, there is undoubtedly a tendency to coax results by the application of sorting methods which need to be warily handled, and should be, as far as possible, founded on rational considerations.

The aurora is the terrestrial phenomenon which exhibits the most intimate dependence upon solar changes. Tacchini[1] traces a connection between its appearance and chromospheric outbursts, and coincidences between remarkable auroras and extraordinary Sun-spots are frequent. Tacchini's opinion that "terrestrial auroras are more closely related to the phenomena of the chromosphere than to those of the spots" is perhaps rather conjectural, since we only know of the existence of prominences if they happen to occur when their locality is passing through a very limited zone at the Sun's limb. I should prefer to base estimates of solar activity on spectroheliographic studies of the entire visible surface, or upon the integrated area of Sun-spots, with the proviso that the appearance of large or rapidly changing prominences, together with marked distortions of the hydrogen lines in the spectrum of Sun-spots, or the breaking out of brilliant plumes and bridges in spot-groups with rapid changes of form, are presumably indices of much greater solar activities than those connected with mere magnitude of disturbed area. The latter is, nevertheless, more amenable to consistent and comparable numerical estimate.

It seems to have been fully established that auroras are more numerous and more brilliant in the latitude of the northern United States, or of northern Europe, at the time of maximum Sun-spot activity; but there is also evidence that still farther north the reverse of this is true. The fact, first established by Loomis, that auroras are not distributed uniformly over the polar regions, but occur most frequently in an oval belt, 10° to 20° wide, surrounding the magnetic pole, the longest diameter of the oval (some 55°) extending in the direction of the eastern parts of North America and Asia, is to be supplemented by the further

[1] This JOURNAL, 1, 211, March 1895.

fact that this belt fluctuates, attaining its greatest expansion, and contributing auroral displays to the lowest latitudes, in years of many Sun-spots, but contracting towards the magnetic pole when Sun-spots are few.

In the *Monthly Weather Review* for November 1896, Professor Abbe has given isobars for the level of 3000 meters altitude, and points out that at this altitude there is a permanent low of an oval form over the polar regions, having its longer axis directed to eastern North America and Asia. The diminished friction of air masses moving over the ocean, permits a more rapid replenishing of this permanent low from the side of the oceanic basins, which narrows the limits of the low from these sides, while the general eastward revolution of the atmosphere along the outer boundary of the high-level permanent low deflects its longer axis from the continental centers to their eastern margins. Professor Abbe points out that the storms of the temperate zone follow the margin of this high-level permanent low, with allowance for some fluctuation in its limits or in the position of its longer axis. A glance at Chart VII. in the publication mentioned, will show that the high-level isobar of sixteen inches defines the average winter storm-track somewhat closely.

Now I would suggest that the sixteen-inch isobaric boundary of this high-level permanent low is not far from the outer limit of Loomis' auroral zone, and that it is desirable to inquire whether the limits of the high-level low may not, like the auroral zone, vary with the Sun-spot cycle. Certain facts make me suspect that there is such a concomitant variation, but it must be established by more complete evidence. If there is such a variation, I believe we may be permitted to anticipate that the cause of this connection is somewhat as follows:

The Sun-spots are an evidence of exceptional overturnings in the solar atmosphere, and while themselves radiating less powerfully than the neighboring photosphere, the more active replenishing of the outer layers of the photosphere by intensely heated matter from below, probably increases the total radiation of the

Sun at the greatest Sun-spot manifestation. If so, the Earth's torrid and warm temperate zones, which occupy eight-tenths of the entire area of the spheroid, experience higher temperature and increased evaporation from the tropical oceans at this time. Large bodies of moist air are then brought into the higher latitudes, disturbing the equilibrium there and intensifying storms. Anti-cyclones will also be intensified at the same time as cyclones, so that while some regions are experiencing exceptional storms, others are enjoying uncommonly fine weather. Since, owing to the shallowness of the Earth's atmosphere, the interchange of warm and cold air in middle latitudes is mainly by alternating hot and cold waves, and since the warm air rises and proceeds on its journey to the poles as an upper current, while cold air from the poles presses down beneath it as a surface current, it follows that any increase of storm-intensity in middle latitudes lowers the general surface temperature there. We find that maximum Sun-spot years are about 2° F. colder, and have a barometric pressure 0.035 inch higher than minimum years in the Mississippi valley.[1] In central Europe, it appears from curves published in the *Meteorologische Zeitschrift* for November 1896 (Taf. XII), that the temperatures of maximum Sun-spot years are lower by from 1°.5 to 2°, although the turning points of the temperature curves are apt to lag a year or two behind the corresponding points of the solar curve.

As to the cause of the aurora, I suggest that it comes from polar air, electrified either by friction or by insolation, and kept from rapid discharge by its dryness, until it comes in contact with the moist air carried over it from the mid-latitude storm-belt. Over a wide zone on the poleward side of the storm-belt there is a quiescent brush-discharge of electricity in an upward direction, under the control of the magnetic lines of force as regards the general direction of the discharge, but only sufficiently intense to produce the typical argon fluorescence,[2] along

[1] *Monthly Weather Review*, January 1887, Hazen's Curve.

[2] Or perhaps it should be said "fluorescence of some argon compound" as suggested by BERTHELOT, *C. R.*, April 16 and June 24, 1895.

atmospheric strata, or through elevated air-streams, where water-vapor is relatively abundant.

Moisture-laden streaks, thrown up by great storms, may extend to higher levels than is commonly supposed, like the solar prominences projecting above the average chromospheric level; although after having been conducted along such moist filaments to the upper conductive regions of greater rarefaction, farther discharge may be easy through air entirely deprived of moisture; at any rate, there is good evidence that the aurora sometimes extends to altitudes of at least 500 miles, where there can be no assistance to the conductive process by water-vapor. The enlargement of the area of cold polar winds, if such enlargement can be proven to exist at Sun-spot maxima, would naturally force the belts of auroral discharge farther from the poles, and would largely explain the greater frequency of the aurora in middle latitudes at that time. It is possible, also, that the aurora may then be more brilliant on account of an intensifying of storms, but to prove this we need numerical estimates of storm-activity similar to those which have been attempted by the United States Weather Bureau for cold waves.

A recent aurora will serve very well to illustrate the hypothesis put forth here. At 7:15 P.M. (E. M. T.) Tuesday, March 15, 1898, an irregular luminous band, 20° to 30° wide, first visible about three-quarters of an hour earlier as a bright spot in the east, stretched entirely across the sky from E. N. E. to W. N. W., passing through the zenith at Providence. On the north side the light faded away indefinitely. Southward the band was sharply defined, reaching out huge tentacles, which at times stretched farther south than Orion. These great luminous projections, resembling swollen-tipped fingers of a gigantic hand, were somewhat equally spaced, and alternated with deep bays, cutting into the belt, with nearly semicircular margins, which, passing by continuous curvature into the edges of the tentacles, were occasionally bright and sharp on the north side, and pushed up straight columns converging by perspective towards the magnetic zenith; but, as a rule, the diffuse luminosity was quiescent.

Not so the individual members of the great series of slightly curved tentacular projections, of which there were eight or ten visible from horizon to horizon, becoming smaller in the distance, and both smaller and brighter in the east than in the west. These moved slowly from east to west, occupying about an hour in passing across the sky, new forms appearing in the east as the western ones vanished. Lower down, in the north, the appearance of folds of a hanging curtain, with momentaneous scintillation of vertical striae, appeared and disappeared several times; but the broad, quiescent, tentacular band, through which the brighter stars were visible, persisted for nearly an hour, receding northward very slowly. The air, which had been calm at first, began to move feebly from the south, and seemed to be pushing back the auroral arch. About 8 : 15 P.M., the south wind having become brisk, the arch receded more rapidly, and the northern sky suddenly changed, developing four or five irregular, broken, luminous belts, the breaks being possibly due to opaque clouds. A very brilliant display broke out in the northeast. For a short time the procession of moving columns appeared, passing from west to east. The whole sky was covered with patches of faint luminous haze, but without any quivering. Then the arches faded and receded towards the northern horizon. Other observers reported a final very bright column in the southeast at 9 : 15 P.M.

A less brilliant display had been seen on the previous evening, culminating at 9 : 45 P.M. in a double undulating arch of 10° to 20° altitude, sending up streamers as high as the North Star, and progressively brightening from east to west.

A group of Sun-spots was approaching the Sun's western limb on the 15th. The largest spot showed unmistakable signs of great activity and rapid change. Its four-cornered umbra indicated inrushes from as many sides.

Anti-cyclonic conditions and a cloudless sky prevailed on the 15th, but on the following morning altostratus, passing into stratus, covered the sky from the south. It seems a fair interpretation of these appearances to assume that on the evening of the 15th, the center of the anti-cyclone being northeast of my

station, the incipient south wind brought enough moisture into the lower atmosphere to start the upward brush-discharge from fresh volumes of electrified air successively farther and farther north, the vigor of the discharge being related to the velocity of the moisture-laden wind and to the previous strength of electrification. The slow westerly movement of the great luminous projections was perhaps due to an east wind in the upper air, preceding the cyclone then advancing from the west. The general and gradual movements of the aurora, occupying hours in their consummation, are presumably connected with atmospheric movements. The processional, darting, and quivering motions, being almost instantaneous, must, of course, be attributed to variations in the magnetic field, whether these variations are to be referred to irregular distribution of weather conditions, permitting electric brush-discharge at particular points, the local, partial discharge disturbing an otherwise uniform magnetic field, or whether the magnetic irregularity is of immediate cosmic origin. If the difficulties in the way of the cosmic theory could be successfully met, it would explain more satisfactorily the obvious connection between solar and auroral changes, and it must be admitted that the meteorological theory is also open to serious objections, inasmuch as conditions, apparently equally favorable for auroral display, do not develop any aurora in the absence of the peculiar solar conditions.

On the equatorial side of the mid-latitude storm-belt, it is quite probable that thunderstorms arise under conditions which equally give auroras on the poleward side. At any rate, Dr. Veeder's contention in favor of a simultaneous appearance of auroras in high latitudes, and thunderstorms in low latitudes at the approach of strong faculæ or Sun-spots to the Sun's eastern limb[1] is supported by a good deal of evidence, although I should say that not merely the advent of spots on the Sun's limb, but also their attainment of the central meridian, and, in general, any exceptionally sudden increments of solar activity, are conditions favorable to auroras and thunderstorms. The

[1] *Monthly Weather Review*, July 1887, p. 206.

point can be best examined by comparing terrestrial conditions before and after an exceptionally great Sun-spot which appears near the minimum epoch, when such occurrences are rare, with the corresponding conditions during the appearance of the Sun-spot.

Such occurrences were the following: In June 1889 (in the midst of a spot-minimum) we learn from the *Monthly Weather Review* that *no* Sun-spots were seen *until the 15th*, when a very large spot, attended by many small ones, appeared on the east limb. "Thunderstorms were reported in the greatest number of states and territories (thirty-six) on the 15th; in thirty-one on the 14th; in thirty on the 17th; in twenty-eight on the 16th; 20th, and 28th; in twenty-seven on the 29th; in twenty-six on the 21st," etc. Thunderstorms were therefore most numerous in this month at or near the time of the appearance of this spot, and they covered more than the average territory during its continuance. Auroras also were seen on the morning of the 15th (date of appearance of spot), and on the 20th, with a magnetic storm on the 21st (date of spot-passage over central solar meridian).

The same spot reappeared by solar rotation on the 12th of July, was central on the 18th, and passed off on the 24th. "Thunderstorms were reported in the greatest number of states and territories (thirty-nine) on the 13th and 14th," or again immediately after the reappearance of the spot, and they again covered more than the average territory during its visibility. Auroras were twice as frequent during the continuance of the spot, and the principal one (on the 20th) was two days after the spot's meridian passage.

Professor Bigelow[1] seems to think that solar magnetic influences are sufficient by themselves to produce not only terrestrial auroras, but also cold waves. The theory of Mr. E. B. Elliot[2] that the greater frequency of auroras near the equinoxes is due to the larger number of electric lines of force cut by the Earth

[1] *Monthly Weather Review*, March 1895.

[2] *Bull. Phil. Soc.*, 1, 45.

in its orbital motion at those seasons, and the more plausible magnetic influence of the neighboring Moon detected by Clayton[1] may also be mentioned, as well as Lord Kelvin's contention that immediate magnetic influence from a Sun-spot involves an enormous expenditure of energy, and in so far is improbable. I am not prepared to discuss the question from this side, but it seems to me possible that the Sun's radiant energy fluctuates more widely than is now generally admitted, and that it is capable of producing both the thermal and the electro-magnetic terrestrial changes, without invoking any direct magnetic effect, or at least that this explanation ought to be more thoroughly tested before it is abandoned.[2] If such an alternative hypothesis is warranted, it appears certain that variations of the intensity of solar radiation must occur in a single day sufficient to greatly increase terrestrial meteorological activities, and such fluctuations of the solar "constant" ought to be sought. I doubt whether they have yet been sought in the right way. The difficulties in the way of direct measurement have heretofore proved insuperable, but possibly they may be overcome. The meteorological method, however, has not been exhausted, and modifications in its application deserve trial. Most of our material comes from the temperate zones and is difficult to analyze. The fact that so large a part of the Earth is in lower latitudes than the average storm-belts, and that the greater part of this surface is oceanic, warns us that our mid-latitude storms are a mere fringe on the grander field of tropical atmospheric activities.

[1] *Am. Jour. Sci.*, (4) 5, 81, February 1898.

[2] Hirn (*Constitution de l'espace céleste*, p. 247, Paris, 1889) has suggested that the solenoidal earth-currents, which are assumed to be the source of terrestrial magnetism, may originate thermo-electrically in the successive unequal warming of the periphery of the Earth by solar radiation. Any sudden variation in the solar constant must disturb the regularity of the flow, and give strong induction effects. M. Hirn, however, preferred the hypothesis of a solar electric surface-charge "whose intensity depends every moment on the phenomena which transpire within the star," and whose inductive action on the faces of the Earth, presented towards, and turned away from the Sun, combined with the shifting of these areas of induced electrification by the Earth's rotation, might produce earth-currents, whose exceptional variations are, in this case, attributable to sudden changes in the solar electric charge.

If temperature and humidity observations could be collated from the logs of vessels crossing the torrid zone, estimates of oceanic evaporation from day to day, combined with rainfall measures, might lead to the detection of the variation of solar radiation. We have the summaries of Buchan, but lack details. What is needed is some method of eliminating the superabundant variations in all meteorological elements, originating in telluric conditions, without also getting rid of the special solar term at the same time. For this purpose some parts of the torrid zone offer certain advantages. Comparing the barometer-curves of Port Darwin (lat. 12° 28′ S.) in North Australia, and Adelaide (lat. 34° 57′ S.) in South Australia, the tropical station exhibits an almost entire freedom from the barometric changes which, at the southern station, register the passage of every storm-center even at too great a distance to otherwise affect the weather. The temperatures at the tropical station are also remarkably steady, but are influenced by the direction of the wind. Specially favorable stations would seem to be those small islands in the tropical regions of the Pacific Ocean, famous for their steady trade winds, the islands themselves being too small to affect the result by their land conditions. The region of the permanent area of high barometer in the north Atlantic might answer, but the favorably situated Azores are rather larger and higher than we should wish, since the varying land conditions ought to be kept subordinate. A high mountain near the equator, such as the Boyden station on El Misti, might seem to be sufficiently isolated, but here we have a position lying on the boundary between arid and excessively moist regions, where fluctuations are to be anticipated according as the air is derived from one or the other source.

In the temperate zones the case is all but hopeless, the local variations are so large. By vast labor in the summation of a great many series, some slight residual may finally emerge which is really of solar significance, but the uncertainty of the result gives it little weight. It does not appear to be necessary that the sign of such temperate residual should agree with that

from the tropics. While higher tropical temperature produced by greater solar radiation must increase convection which may transfer extra heat to the temperate and polar zones, it is quite possible that certain regions may get more than their share of the returning polar winds concerned in the convection, and may have their temperatures lowered thereby. This, in fact, appears to be the case in the Upper Mississippi valley, and in Europe, as has been already stated.

The elimination of minor telluric fluctuations in the case of deeply buried earth-thermometers is to be noted, since it has given, in the case of the Edinburgh thermometers, an apparently authentic residual which coincides with the eleven-year Sun-spot period.

Great efforts have been made to detect some influence on the weather from the varying presentation of the Sun in its rotation. Clayton's storm-period of $3^d\ 2^h$, differs slightly from an even submultiple of the Sun's synodic rotation, derived from Sun-spot observations, and his storm-period of $7^d\ 6^h.43$ is no better in this respect, while both are quite as likely to be connected with the physical properties of the Earth's atmosphere and the dimensions of the Earth, as with any direct solar influence. Moreover, since different elements of the solar atmosphere have angular velocities which vary by at least 50 per cent., the choice of any period of solar rotation, as affecting solar radiation, unless governed by some marked property separable on independent grounds, is too conjectural to serve as a safe mathematical basis of computation.

Other suggested coincidences can hardly be regarded as better than complete failures. It seems rather ludicrous, when the best meteorologists can scarcely foretell the weather for more than a day in advance, to find scientists advocating periodic terms in weather changes, whose time they are prepared to state to the fourth or fifth decimal of a day, and attributed to the influence of solar rotation, in regard to which there might be selected values differing by many days without doing violence to any established principle.

While the possibility that there is some little-understood magnetic control of the weather, whose elucidation may lead to another "triumph of the fifth decimal place," cannot be denied, it seems at present more probable that solar radiation is the controlling influence in weather changes, and I would urge the collection of meteorological records from small islands in the trade-wind zones in mid-ocean, where local influences are small, as a possible means of determining variations of solar radiation. If such variations are found to synchronize at widely separated stations, and above all if increased oceanic tropical temperature immediately antedates sudden and otherwise inexplicable increments of storm-activity in temperate regions, a strong point will have been made in favor of solar thermal variability; while if the evidence is inconclusive, it may fix the limits within which it is permissible to assume a variation of solar radiation.

The hypothesis of diverse radiation from northern and southern solar hemispheres will account for one annual maximum and one minimum of solar radiation received by the Earth, or by any other planet which does not move in the plane of the Sun's equator. If the polar regions of the Sun radiate differently from its equatorial zone, or if there are zones of especially strong radiative power in Sun-spot latitudes, two annual maxima and two minima of solar radiation are possible; but since the Sun's equator is only inclined $7\frac{1}{4}°$ to the ecliptic, no further multiplication of zones of unequal radiative power, capable of detection in the rounds of a body moving in the ecliptic, is probable.

The solar radiation is purely superficial. It is true that infra-red rays penetrate terrestrial clouds more readily than shorter waves, and doubtless emanate from greater depths in the photospheric cloudlike material; also, since the solar surface is very irregular, a large part of the photospheric rays are interstitial; but the radiation from the intensely heated layers below the photosphere is presumably discontinuous and of short wave-length, and thus incapable of penetrating the

luminous mist. Wilson and Fitzgerald[1] also suggest that refraction and reflection from irregular convection currents may play the part of an opaque mist. We have no means of determining the thickness of the cloudy shells of condensed vapor which sheath the streams of hot viscid gas from the solar depths, but since superimposed penumbral filaments exhibit no intensification at points of crossing, the cloud-thickness is of the same order as that of terrestrial clouds, and the precipitated mist sufficiently dense to protect the enclosed transparent gas from further cooling by radiation, as happens beneath terrestrial cloud-canopies. If there are terrestrial thermal fluctuations due to the direct influence of solar radiation, they probably originate either in the differential surface temperature of particular photospheric regions, which can only be maintained by differences in the circulatory mechanism by which heat is supplied to those areas, or else in the varying thickness or absorptive power of the Sun's atmosphere.

Sun-spots often recur in the same vicinity, and the viscosity of the deeper and hotter layers of solar substance is no doubt great enough to preserve local differences of temperature through long intervals.

Tacchini[2] states that during the years 1893 and 1894, and for the last three-quarters of 1892, prominences, faculæ, and spots were most frequent in the solar zones south of the equator. In the first quarter of 1895, the same observer[3] noted signs of change. Spots and faculæ continued to be more numerous in the southern zones, but prominences had become more frequent north of the equator, and in the second quarter of that year, the spots followed suit. In the last half of 1895,[4] faculæ and spots had their greatest development in the northern zones; but in the first half of 1896,[5] the preponderance of spots and faculæ was again south of the equator. In the second half of 1896,[6] with the exception of the great group of September 10 to 22, all

[1] This JOURNAL **5**, 101, February 1897.

[2] *Ibid.*, **2**, 26, June 1895.

[3] *Ibid.*, **2**, 225, October 1895.

[4] *Ibid.*, **3**, 252, April 1896.

[5] *Ibid.*, **4**, 182, October 1896.

[6] *Ibid.*, **5**, 159, March 1897.

activities were greater in the southern zones, and continued so during 1897.[1]

Now the south pole of the Sun inclines most towards the Earth on March 6, and the southern zones are more favorably presented during the half year from December 6 to June 4. Since this period includes that portion of the northern winter in which storms are most frequent, while the south-temperate zone is enjoying summer, we might anticipate that if regions where Sun-spots are numerous radiate more powerfully than others, the tropical and south-temperate zones should be warmer during the presentation of the Sun's south pole when solar activity predominates in the southern zones, while at the same time the north-temperate zone should be cooler than usual, owing to increased activity of terrestrial storms, and greater prevalence of cold waves. The hypothesis can only be fairly tested near the Sun-spot maximum, and the one which has just passed is exceptionally favorable for this purpose, both on account of a considerable discrepancy in the solar activity in northern and southern hemispheres, and because the solar phenomena have been carefully analyzed on a common plan in the summaries of Tacchini.

For comparison, I have tabulated the accumulated departures from the usual monthly mean temperatures for the twenty-one districts into which the Weather Bureau has divided the United States, taking the data from the *Monthly Weather Review*. The observations for each year have been divided into two groups: one for southern solar presentation from December to May inclusive; the other from June to November inclusive, corresponding to northern solar presentation. The year from December 1892 to November 1893, which was near the maximum of the Sun-spot period, is particularly noteworthy. For the entire country the departures were below normal in both halves of the year, but the six months of southern solar presentation were the colder by nearly two degrees Fahrenheit, in mean monthly departure. Only three regions out of the twenty-one

[1] This JOURNAL, 6, 244, October 1897, and 7, 170, March 1898.

showed a contrary sign from the general mean, and these were on the extreme southern border, namely, the South Pacific, South Plateau, and South Slope regions. The South Atlantic and Gulf States agreed with the rest of the country in showing lower temperature at the time of southern solar exposure, but by diminished amounts. The greatest negative departures were in North Dakota and the Upper Mississippi valley ; and, in general, the negative departures increased northward. There is every reason to suspect that south of the Tropic of Cancer, the departures may have been above normal, as hypothesis would indicate. No body of assorted information exists for the great tropical region in any way comparable with the admirable series of meteorological digests which Professor Abbe and his assistants have been placing at our disposal for central North America. It would be unfair to compare the readings from only a few tropical stations with so extensive a series, but as far as I have been able to find out, it appears that the tropical regions were experiencing exceptionally high temperature during this period of southern solar presentation, and the south-temperate zone shared in this influence to the extent of showing a very much smaller negative departure than during the time of northern solar presentation. In 1894 there was a smaller departure in the opposite direction in the United States, although the solar conditions remained about the same. This suggests that the very large negative departure of 1893 may have been partly due to an eccentric displacement of the pole of cold towards the American continent, and that this, by reaction, produced an opposite displacement in the following year. In this case a mean of the two years would more nearly express the true relation ; but it may be safer to include the entire series of six years summarized in the following table, from which it appears that with a mean excess of 37.6 per cent. in southern solar activity, the six half years of south solar presentation gave a mean accumulated temperature departure from normal of $-0^{\circ}.683$ F. in the United States, the corresponding half years of north solar presentation giving $+0^{\circ}.045$ F.

	Accumulated semi-annual temperature-departures from normal											
	Dec. 1891 to May 1892	June to Nov. 1892	Dec. 1892 to May 1893	June to Nov. 1893	Dec. 1893 to May 1894	June to Nov. 1894	Dec. 1894 to May 1895	June to Nov. 1895	Dec. 1895 to May 1896	June to Nov. 1896	Dec. 1896 to May 1897	June to Nov. 1897
New England	+ 9°.5	+ 2°.5	− 16°.6	− 1°.2	+ 4 .5	+ 2°.4	− 3°.4	+ 2°.9	+ 1°.3	+ 2°.8	+ 2°.8	− 0°.9
Mid. Atlantic States	+ 1 .1	− 1 .1	−16 .1	− 2 .0	+11 .7	+ 1 .9	−11 .8	+ 3 .3	+ 4 .6	+ 2 .8	+ 0 .2	+ 0 .7
South Atlantic States	− 5 .0	− 7 .4	− 6 .9	− 1 .3	+10 .1	− 3 .5	−16 .6	+ 0 .7	+ 1 .2	+ 6 .3	− 3 .6	+ 5 .0
Key West	− 7 .2	−10 .0	− 3 .0	− 1 .5	+ 1 .7	− 7 .7	−12 .2	− 2 .8	−13 .6	+ 0 .9	− 0 .4	− 3 .2
Eastern Gulf	− 7 .7	− 5 .5	− 4 .5	− 1 .7	+ 6 .5	− 5 .3	−17 .2	+ 0 .2	− 3 .3	+ 6 .4	− 2 .3	+ 8 .8
Western Gulf	− 2 .1	− 1 .7	− 2 .4	− 1 .4	+ 7 .7	− 4 .0	−13 .9	− 4 .3	+ 5 .8	+ 6 .5	+ 6 .0	+ 9 .7
Ohio Valley	− 1 .4	− 2 .0	−13 .5	+ 0 .7	+ 8 .7	+ 3 .1	−15 .4	+ 2 .7	+ 9 .9	+ 1 .6	− 0 .6	+10 .5
Lower Lakes	+ 2 .5	+ 1 .8	−17 .2	+ 3 .6	+12 .3	+ 5 .8	− 6 .7	+ 1 .1	+10 .0	− 0 .8	+ 3 .0	+ 4 .7
Upper Lakes	+ 8 .9	+ 5 .2	−18 .3	+ 5 .8	+12 .2	+ 8 .5	+ 2 .8	+ 0 .6	+19 .2	− 1 .5	+10 .4	+10 .4
North Dakota	+12 .9	+ 4 .8	−25 .1	+ 2 .3	+ 3 .9	+13 .1	+19 .9	−11 .5	+12 .2	−25 .2	+ 1 .1	+ 6 .4
Upper Mississippi Val.	+ 2 .4	− 0 .3	−25 .1	+ 5 .0	+10 .1	+ 8 .5	+ 0 .4	+ 1 .1	+21 .2	− 7 .7	+ 6 .0	+13 .2
Missouri Valley	+ 1 .4	+ 2 .9	−16 .0	+ 2 .9	+ 8 .3	+10 .2	+ 8 .9	− 2 .9	+22 .1	−12 .6	+ 7 .0	+12 .8
Northern Slope	+ 1 .8	+ 3 .1	−14 .8	− 3 .3	+ 4 .4	+ 9 .5	− 0 .3	−10 .5	+ 8 .5	−15 .0	+10 .2	+ 1 .4
Middle Slope	− 4 .4	+ 5 .3	− 4 .6	+ 1 .9	+ 7 .7	+ 6 .0	− 1 .7	− 3 .9	+20 .6	− 2 .3	+ 8 .5	+10 .5
Southern Slope	+ 0 .1	+ 4 .3	+ 5 .3	+ 6 .0	+10 .1	+ 1 .2	−13 .5	− 7 .2	+15 .5	+ 4 .5	+ 4 .0	+ 4 .4
Southern Plateau	− 9 .2	+ 1 .1	− 5 .5	− 5 .6	− 8 .8	− 0 .6	− 1 .9	− 5 .4	+ 0 .2	+ 4 .3	− 1 .5	− 0 .9
Middle Plateau	−15 .8	+ 4 .9	−20 .8	− 1 .9	− 2 .5	+ 0 .5	− 9 .5	− 7 .4	− 5 .6	+ 2 .5	− 0 .7	− 0 .5
Northern Plateau	+ 7 .4	+ 3 .9	−22 .7	−13 .9	− 1 .1	+ 4 .2	+ 7 .3	− 6 .9	+ 9 .2	± 0 .0	+13 .7	± 0 .0
North Pacific Coast	+ 3 .6	− 1 .0	−15 .4	−12 .3	− 7 .0	+ 0 .6	− 2 .4	− 4 .1	− 3 .2	− 3 .9	+ 2 .4	− 1 .2
Middle Pacific Coast	− 5 .4	+ 3 .7	−13 .8	− 7 .2	− 9 .5	+ 3 .1	− 3 .5	− 4 .3	− 3 .9	− 0 .1	− 0 .9	− 2 .2
South Pacific Coast	− 4 .5	− 9 .5	− 4 .4	−11 .8	− 11 .3	− 8 .5	− 2 .4	− 7 .2	+ 1 .8	+ 0 .1	+ 0 .6	− 5 .7
Mean accum. departures	−0 .53	+0 .24	−12 .45	− 1 .76	+ 3 .80	+ 2 .33	− 4 .43	− 3 .09	+ 6 .37	− 1 .45	+ 3 .14	+ 4 .00
Solar presentation	S	N	S	N	S	N	S	N	S	N	S	N
Relative size of spots	1 .99	(2 .33)	(2 .65)	2 .94	2 .19	1 .51	1 .29	1 .20	0 .75	0 .96	0 .89	0 .53
Excess[1] N.: S. prom'nces	+0 .016	−0 .324	−0 .925	−0 .443	−0 .952	−0. 302	+0 .204	+0 .151	−0 .008	−0 .433	−0 .214	−0 .398
Excess N.; S. faculæ	−0 .099	−0 .156	−0 .464	−0 .211	−0 .228	−0 .588	−0 .273	+0 .079	−0 .672	−1 .727	−0 .460	−0 .304
Excess N.: S. spots	−0 .028	−0 .230	−0 .298	−0 .438	−0 .421	−0 .270	−0 .066	+0 .163	−0 .568	−1 .950	−0 .787	−0 .302
Mean excess N.: S.	−0 .037	−0 .237	−0 .562	−0 .364	−0 .534	−0 .387	−0 .045	+0 .131	−0 .416	−1 .237	−0 .487	−0 .335

[1] Computed by the formulæ $\frac{N-S}{S}$ for north excess, $\frac{N-S}{N}$ for south excess, from quarterly means.

Sun-spots	Port Darwin Lat. 12° 28′ S.				Alice Springs Lat. 23° 38′ S.				Cape Borda Lat. 35° 45′ S.				C. Northumberland Lat. 38° 5′ S.			
	Solar				Solar				Solar				Solar			
	Max.		Min.		Max.		Min.		Max.		Min.		Max.		Min.	
Extreme monthly temperature	Max. 1883	Min. 1883	Max. 1888	Min. 1888	Max. 1883	Min. 1883	Max. 1888	Min. 1888	Max. 1883	Min. 1883	Max. 1888	Min. 1888	Max. 1883	Min. 1883	Max. 1888	Min. 1888
January	98 .0	71°.0	93 .8	72°.2	116°.0	54°.0	113°.5	61°.5	91°.0	47°.5	90°.0	48°.5	97°.0	50°.0	95°.0	46°.7
February	97 .0	71 .0	97 .4	71 .8	112 .0	58 .0	109 .0	56 .0	85 .7	48 .5	81 .5	50 .0	91 .0	52 .0	91 .0	44 .0
March	102 .0	68 .0	98 .0	73 .4	98 .0	45 .0	101 .8	49 .0	84 .0	46 .0	80 .0	48 .0	93 .0	44 .0	94 .0	48 .0
April	104 .0	70 .0	98 .0	73 .5	98 .0	40 .0	102 .0	46 .0	78 .0	49 .0	78 .9	44 .8	79 .0	46 .0	88 .0	42 .2
May	100 .0	63 .0	95 .3	69 .0	86 .0	33 .0	86 .0	33 .0	67 .0	43 .0	72 .4	45 .0	67 .0	38 .0	78 .0	39 .0
June	98 .0	66 .0	93 .0	65 .0	87 .0	32 .0	83 .5	28 .0	61 .0	51 .5	61 .9	41 .3	60 .0	42 .0	64 .9	40 .2
July	95 .0	63 .0	98 .0	62 .9	86 .0	26 .0	76 .0	24 .5	61 .0	41 .0	60 .9	39 .7	58 .0	36 .0	65 .4	38 .0
August	95 .0	65 .0	94 .3	64 .5	96 .0	29 .0	90 .5	31 .0	60 .0	44 .7	63 .9	36 .7	62 .0	37 .0	65 .5	34 .0
September	101 .0	63 .0	98 .8	69 .7	97 .0	32 .0	97 .2	31 .0	67 .5	43 .0	76 .7	39 .7	67 .0	41 .0	80 .6	42 .2
October	102 .0	72 .0	100 .0	71 .2	104 .0	44 .0	106 .4	45 .5	69 .0	44 .5	76 .1	43 .3	79 .0	41 .0	78 .7	38 .3
November	102 .0	70 .0	99 .2	72 .1	106 .0	49 .0	110 .9	57 .4	78 .0	45 .0	94 .7	46 .1	84 .0	45 .0	98 .7	46 .2
December	102 .0	70 .0	97 .2	73 .7	116 .0	54 .0	109 .8	65 .8	82 .0	49 .0	92 .8	48 .2	91 .0	44 .0	95 .7	45 .0
Year	104 .0	63 .0	100 .0	62 .9	116 .0	26 .0	113 .5	24 .5	91 .0	41 .0	94 .7	36 .7	97 .0	36 .0	98 .7	34 .0

The classical paper by Mr. C. A. Schott on "Atmospheric Temperature in the United States"[1] gives an instance of a very marked and wide-spread negative mean annual temperature departure of from two to three degrees Fahrenheit, coinciding with the Sun-spot maximum of 1837 to 1838; but there are other instances showing disagreement between the several stations, or an opposite influence on temperature at the Sun-spot maxima. To eliminate the causes of such disagreement in values tabulated from observations in temperate latitudes, it may be necessary to know:

1. The relative activity of the northern and southern solar hemispheres.

2. The temperature departure along a terrestrial latitude parallel, or at least at points differing in longitude by about 180°, in order to allow for any eccentricity in the pole of cold.

Better still would it be to gather the thermal data of the tropical oceans.

To illustrate the results which may be anticipated from a comparison of tropical and temperate temperatures, a table[2] of monthly extreme maximum and minimum temperatures in Australia is given herewith.

Not only for the years, but also largely for the separate months, the maximum temperatures are higher for a time of many Sun-spots in the torrid zone, and lower in the temperate zone. But no single example which could be chosen, would be entirely free from objection; and in citing these comparisons, my object is mainly to direct attention to the desirability of the accumulation and tabulation of special kinds of meteorological data as a means of solving the question of solar variability.

[1] *Smithsonian Contributions to Knowledge*, 21, Washington, 1876. Curves of secular change, opposite p. 310.

[2] Taken from *Meteorological Observations made at the Adelaide Observatory and other places*, under the direction of Charles Todd, F. R. A. S.

ARC-SPECTRUM OF VANADIUM.

HENRY A. ROWLAND and CALEB N. HARRISON.

THIS paper is a preliminary notice of a series of investigations which we have undertaken on the arc-spectra of certain metals that have hitherto not been carefully studied by modern methods.

The work will be confined to the visible and ultra-violet part of the spectrum. The plates used were in large part selected from a series of plates made some years ago by one of us in the study of the solar spectrum. Others have, however, been taken recently for the purpose of this investigation. They are nineteen inches in length. The grating used was a six-inch Rowland concave of twenty-one and a half feet focal length and ruled with 20,000 lines to the inch. It has the usual mounting, described by Professor Ames in the *Johns Hopkins University Circular* of May 1889.

The arc-spectra and solar spectrum are on each plate and permit comparison according to the method of Rowland.

In order to eliminate all lines due to impurities in the study of an element, a comparison is made with the spectra of the carbon poles and of all elements likely to be associated with the element. This is accomplished by the superposition of plates which are on the scale, thus insuring coincidence of corresponding lines. The intensity of these lines are an element in their determination.

The measuring engine employed has a nearly perfect screw and a pitch so as to measure wave-lengths directly in ten-millionths of a millimeter.

The basis of all measurements is the Table of Standard Wave-Lengths.[1]

A correction for each line is obtained by measuring numerous solar standard lines in addition to those of the element studied. The difference in wave-lengths taken from the table and the corresponding lines measured give when platted a correction curve for each plate. The character of line and its posi-

[1] ROWLAND, *A New Table of Standard Wave-lengths. Astron. and A. P.*, **12**, 1893.

tion on the plate is considered in assigning weights preparatory to drawing the correction curves. The wave-length of each line is the mean of at least two readings, direct and reversed. By "direct" reading is meant that which is obtained when the plate is being moved so that the successive lines coming in the field of the microscope are of increasing wave-length, and by "reversed" reading that obtained when the plate is moving so that successive lines seen through the microscope are of decreasing wave-length.

In the following tables the intensities are given on a scale from 1, n (very faint and nebulous on the plate) to 15 (strongest line). The column headed wave-length (uncorrected) are readings taken from the engine. Corrections marked 1 and 2 are obtained respectively from the correction curve and values calculated for the Earth's motion for latitude 39° 18′. The symbol * after a corrected wave-length means the average of several measurements on different plates.

Hesselberg's readings are taken from his article in this JOURNAL, "Note on the Chemical Composition of the Mineral Rutile," Vol. VI., p. 22, June 1897.

The wave-lengths from 4200.000 to 5786.000, inclusive, were measured from plates recently taken, and their time of exposure was duly recorded. This permitted us to make the correction for the Earth's motion. We were unable to apply a similar correction to other wave-lengths on account of not having a record of their date of exposure.

In measuring the lines in the arc-spectra numerous standard solar lines were also measured for the purpose of drawing a correction curve. The plates marked "Solar Lines for Standards" have in the first column micrometer readings; the second column, headed "Standard," are taken from the Table of Standard Wave-lengths; the third column, marked "Difference," is the difference between the standard and the micrometer reading, with the proper sign annexed. The remaining columns are values calculated as due to the Earth's motion. The symbols, †, ‡, *, , represent the relative weights assigned to standards, and are in ascending scale of magnitude.

SOLAR LINES FOR STANDARDS.

PLATE 32

Micrometer readings	Standard	Difference	Micrometer readings	Standard	Difference
3079.715	.724	—.009	3232.414	.404	+.010
3080.850	.863‡	—.013	3246.133	.124‡	+.009
3086.879	.891‡	—.012	3247.694	.680†	+.014
3094.723	.739	—.016	3260.389	.384*	+.005
3094.999	95.003	—.004	3267.846	.839	+.007
3115.141	.160	—.019	3274.093	.092†	+.001
3121.265	.275‡	—.010	3287.792	.791‡	+.001
3137.443	.441	+.002	3295.955	.957	—.002
3140.868	.869	—.001	3302.510	.501‡	+.009
3167.299	.290*	+.009	3303.648	.648‡	.000
3172.179	.175‡	+.004	3308.928	.928*	.000
3176.117	.104*	+.013	3318.162	.163*	—.001
3188.140	.164‡	—.024	3331.748	.741*	+.007
3200.030	.032‡	—.002	3348.015	.011*	+.004
3218.400	.390	+.010	3351.888	.877	+.011
3224.378	.368	+.010	3356.230	.222	+.008
3231.450	.421†	+.029	3377.669	.667‡	+.002

PLATE 36.

Micrometer readings	Standard	Difference	Micrometer readings	Standard	Difference
3405.200	.272‡	—.072	3558.670	.670†	.000
3406.510	.581	—.071	3564.686	.680‡	+.006
3406.886	.955	—.069	3570.236	.225†	+.011
3425.672	.721	—.049	3581.337	.344†	—.007
3440.720	.759†	—.039	3583.490	.483	+.007
3441.095	.135†	—.040	3597.198	.192	+.006
3455.338	.384	—.046	3600.875	.880‡	—.005
3464.571	.609	—.038	3612.234	.217	+.017
3486.010	.036‡	—.036	3618.946	.924†	+.022
3491.446	.464	—.018	3622.170	.147‡	+.023
3497.990	.991‡	—.001	3623.345	.332	+.013
3500.698	.721	—.023	3631.632	.619†	+.013
3510.987	.987	.000	3640.555	.536‡	+.019
3518.471	.487	—.016	3652.614	.692	+.022
3521.392	.404†	—.012	3658.719	.688	+.031
3540.257	.266‡	—.009	3667.424	.397	+.027
3545.339	.333*	+.006	3680.077	.064†	+.013
3550.000	.006	—.006	3683.228	.202‡	+.026

PLATE 36—*continued.*

Micrometer readings	Standard	Difference	Micrometer readings	Standard	Difference
3687.630	.607†	+.023	3770.156	.130*	+.026
3695.220	.194‡	+.026	3780.870	.846‖	+.024
3707.223	.186‡	+.037	3781.347	.330‖	+.017
3709.385	.397†	—.012	3783.700	.674‡	+.026
3716.595	.585‡	+.010	3794.039	.014‖	+.025
3720.105	.086†	+.019	3795.176	.150†	+.026
3732.566	.542‡	+.024	3804.198	.153‖	+.045
3737.308	.282†	+.026	3805.521	.487*	+.024
3743.529	.502†	+.027	3820.590	.567†	+.023
3745.718	.701‡	+.017	3821.348	.318‡	+.030
3756.240	.211	+.029	3823.679	.651‡	+.028
3763.968	.942†	+.026	3826.039	.024†	+.015

PLATE 40.

Micrometer readings	Standard	Difference	Micrometer readings	Standard	Difference
3804.105	.153	—.048	3961.682	.676†	+.006
3805.441	.487*	—.046	3971.482	.478‖	+.004
3815.940	.985†	—.045	3977.898	.891‖	+.007
3821.278	.318†	—.040	3981.927	.914‖	+.013
3823.617	.651‡	—.034	3984.086	.078†	+.008
3826.007	.024†	—.017	3987.243	.216†	+.017
3832.418	.446†	—.028	4003.927	.916*	+.011
3836.192	.226‖	—.034	4016.584	.578‖	+.006
3856.509	.517‡	—.008	4029.805	.796‖	+.009
3860.032	.048†	—.016	4030.918	.914†	+.004
3864.425	.441	—.016	4033.221	.225†	—.004
3871.519	.528*	—.009	4045.970	.975†	—.005
3875.203	.224	—.021	4048.895	.893‡	+.002
3886.423	.427†	—.004	4055.708	.701‖	+.007
3897.588	.599	—.011	4062.599	.602‖	—.003
3905.662	.666†	—.004	4071.920	.904†	+.016
3916.877	.875	+.002	4073.928	.920‖	+.008
3924.680	.669	+.021	4077.879	.883†	—.004
3925.355	.345	+.010	4083.765	.767‡	—.002
3925.796	.792	+.004	4088.712	.716‖	—.004
3937.487	.474	+.013	4103.100	.101‡	—.001
3944.160	.159†	+.001	4107.651	.646‡	+.005
3950.109	.101	+.008	4114.612	.600‖	+.012
3950.498	.497	+.001	4121.967	.968‖	—.001
3954.012	.001	+.011	4157.938	.948‖	—.010
3957.189	.180†	+.009	4185.055	.063‖	—.008
3960.439	.429	+.010	4197.254	.251‡	+.003

PLATE 44[1] (JAN. 24, 12:11 P. M.).

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
4185.098	.063	+.035	+.003	4260.683	.638‡	+.045	+.003
4199.311	.263*	+.048	+.003	4267.980	.958‡	+.022	+.003
4202.241	.188*	+.053	+.003	4271.967	.924†	+.033	+.003
4215.714	.687*	+.027	+.003	4283.209	.170*	+.039	+.003
4222.415	.381*	+.034	+.003	4289.568	.523*	+.045	+.003
4226.932	.892†	+.040	+.003	4289.929	.881*	+.038	+.003
4250.998	.956‡	+.042	+.003	4293.293	.249*	+.044	+.003
4254.540	.502‡	+.038	+.003				

NEW PLATE 44.

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
4274.994	.958*	+.036	+.003	4462.594	.621#	—.027	+.003
4289.553	.523	+.030	+.003	4468.635	.663#	.028	+.003
4293.268	.249‡	+.019	+.003	4489.871	.911#	—.040	+.003
4299.170	.152*	+.018	+.003	4494.700	.735*	—.035	+.003
4306.097	.071‡	+.026	+.003	4501.404	.444*	—.040	+.003
4318.840	.818	+.022	+.003	4508.412	.456*	—.044	+.003
4325.957	.940‡	+.017	+.003	4517.656	.702#	—.046	+.003
4343.409	.387*	+.022	+.003	4533.357	.419#	—.062	+.003
4369.958	.943*	+.015	+.003	4541.630	.690#	—.060	+.003
4376.119	.103*	+.016	+.003	4544.780	.864#	—.084	+.003
4383.725	.721†	+.004	+.003	4546.065	.129#	—.064	+.003
4404.925	.927‡	—.002	+.003	4549.735	.808#	—.073	+.003
4407.840	.850‡	—.010	+.003	4563.854	.939*	—.085	+.003
4415.291	.299‡	—.008	+.003	4571.186	.277*	—.091	+.003
4425.598	.609*	—.011	+.003	4578.639	.731	—.092	+.003
4435.099	.132†	—.033	+.003	4590.029	.129	.100	+.003
4454.925	.950‡	—.025	+.003	4603.023	03.126#	—.103	+.003
4460.439	.462#	—.023	+.003				

PLATE 48 (JANUARY 24, 12:37 P.M.).

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
4602.231	.183	+.048	+.004	4643.640	.645†	—.005	+.004
4607.559	.509‡	+.050	+.004	4648.880	.835†	+.045	+.004
4611.500	.453†	+.047	+.004	4668.344	.303‡	+.041	+.004
4637.732	.683	+.049	+.004	4678.394	.353‡	+.041	+.004
4638.238	.194	+.044	+.004	4679.071	.028‡	+.043	+.004

[1] The symbol # indicates solar wave-length, not standard.

PLATE 48 (JANUARY 24, 12:37 P.M.).—*Continued.*

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
4683.785	.743	+.042	+.004	4823.700	.697*	+.003	+.004
4686.440	.395	+.045	+.004	4859.924	.934*	—.010	+.004
4690.360	.324	+.036	+.004	4903.470	.488†	—.018	+.004
4691.610	.581†	+.029	+.004	4919.147	.183‡	—.036	+.004
4703.218	.180‡	+.038	+.004	4920.665	.682†	—.017	+.004
4714.630	.599†	+.031	+.004	4934.215	.247‡	—.032	+.004
4722.370	.349‡	—.021	+.004	4957.763	.786†	—.023	+.004
4727.654	.628‡	—.021	+.004	4973.244	.274	—.030	+.004
4754.245	.226‡	+.019	+.004	4978.722	.782†	—.060	+.004
4783.618	.601†	+.017	+.004	4981.866	.915	—.049	+.004
4805.275	.253‡	+.022	+.004	4994.260	.316	—.059	+.004
4810.725	.723†	+.002	+.004	4999.628	.693*	—.065	+.004

PLATE 51 (JAN. 24, 10:8 P. M.).

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
4903.479	.488‡	—.009	+.005	5127.592	.530*	+.062	+.005
4920.697	.682‡	+.015	.005	5133.870	.871‡	—.001	+.005
4924.111	.109*	+.002	.005	5139.642	.645†	—.003	+.005
4934.240	.247†	—.007	.005	5141.910	.916	—.006	+.005
4957.479	.482†	—.003	.005	5146.658	.664	—.006	+.005
4957.793	.786†	+.007	.005	5151.055	.026†	+.029	+.005
4973.287	.274	+.013	.005	5155.938	.937	+.001	+.005
4978.774	.782‡	—.008	.005	5159.230	.240	—.010	+.005
4980.348	.362‡	—.014	.005	5162.452	.448	+.004	+.005
4981.915	.915	.000	.005	5165.581	.588	—.007	+.005
4994.320	.316	+.004	.005	5171.770	.783*	—.013	+.005
4999.686	.693‡	—.007	005	5172.855	.871†	—.016	+.005
5005.896	.904	—.008	.005	5173.912	.912	.000	+.005
5007.425	.431‡	—.006	.005	5193.128	.139	—.011	+.005
5014.424	.422‡	+.002	+.005	5204.697	.708‡	—.011	+.005
5020.210	.210*	.000	+.005	5202.464	.483‡	—.019	+.005
5050.009	.008	+.001	+.005	5210.543	.556*	—.013	+.005
5060.254	.252*	+.002	+.005	5215.335	.352*	—.017	+.005
5064.836	.833	+.003	+.005	5217.541	.559	—.018	+.005
5068.950	.946	+.004	+.005	5225.669	.690	—.021	+.005
5083.519	.525	—.006	+.005	5233.095	.124‡	—.029	+.005
5090.956	.959	—.003	+.005	5242.632	.662*	—.030	+.005
5097.183	.176*	+.007	+.005	5250.787	.825	—.038	+.005
5105.718	.719	—.001	+.005	5253.600	.649	—.049	+.005
5110.585	.570‡	+.015	+.005	5273.286	.344‡	—.058	+.005
5115.574	.558	+.016	+.005	5281.909	.968*	—.059	+.005
5121.804	.797‡	+.007	+.005	5288.640	.708	—.068	+.005

PLATE 54 (JANUARY 18, 1:02 P.M.).

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
5154.295	.237	+.058	+.005	5288.742	.708*	+.034	+.005
5159.280	.240	+.040	+.005	5296.956	.873*	+.081	+.005
5262.496	.448	+.048	+.005	5307.586	.546*	+.040	+.005
5165.625	.588	+.037	+.005	5316.835	.870†	—.035	+.005
5171.834	.783*	+.051	+.005	5324.404	.373†	+.031	+.005
5173.983	.912	+.071	+.005	5333.111	.092*	+.019	+.005
5183.824	.792†	+.032	+.005	5349.680	.623†	+.057	+.005
5188.900	.948‡	+.048	+.005	5353.586	.592*	—.006	+.005
5189.075	.020‡	+.055	+.005	5370.176	.165*	+.011	+.005
5193.189	.139‡	+.050	+.005	5393.385	.378*	+.007	+.005
5198.938	.885*	+.053	+.005	5397.344	.346	—.002	+.005
5202.523	.483‡	+.040	+.005	5405.989	.987	+.002	+.005
5210.611	.556	+.055	+.005	5415.418	.421*	—.003	+.005
5215.401	.352‡	+.049	+.005	5424.276	.284†	—.008	+.005
5217.608	.559*	+.049	+.005	5434.724	.742†	—.018	+.005
5225.752	.690*	+.062	+.005	5447.112	.130‡	—.018	+.005
5230.075	.014*	+.061	+.005	5455.818	.826†	—.008	+.005
5233.163	.124‡	+.039	+.005	5462.683	.732*	—.049	+.005
5242.710	.662*	+.048	+.005	5463.147	.174*	—.027	+.005
5250.435	.391	+.044	+.005	5466.585	.608*	—.023	+.005
5253.690	.649	+.041	+.005	5528.575	.636‡	—.061	+.005
5264.409	.371†	+.038	+.005	5534.995	5535.073*	—.078	+.005
5265.941	.884†	+.057	+.005	5543.344	.418	—.074	+.005
5270.536	.495‡	+.041	+.005	5544.092	.158*	—.066	+.005
5273.600	.554‡	+.046	+.005	5555.030	.113	—.083	+.005
5283.844	.803‡	+.041	+.005				

PLATE 58 (JANUARY 18, 2:07 P.M.).

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
5555.130	.113	+.017	+.007	5658.102	.096*	+.006	+.007
5569.858	.848‡	+.010	+.007	5662.750	.745‡	+.005	+.007
5576.326	.319	+.007	.007	5679.264	.249	+.016	+.007
5582.205	.195*	+.010	+.007	5682.880	.861*	+.019	+.007
5588.990	.980‡	+.010	+.007	5688.445	.434‡	+.011	+.007
5598.731	.715‡	+.016	+.007	5708.635	.620‡	+.015	+.007
5601.512	.501‡	+.011	+.007	5709.609	.616†	—.007	+.007
5615.530	.526	+.004	+.007	5711.323	.318‡	+.005	+.007
5615.882	.879‡	+.003	+.007	5715.325	.309‡	+.016	+.007
5624.263	.253	+.010	+.007	5742.076	.066*	+.010	+.007
5624.778	.768*	+.010	+.007	5753.354	.342‡	+.012	+.007
5634.182	.169‡	+.015	+.007	5763.220	.215‡	+.005	+.007

PLATE 58—*continued*. (JANUARY 18, 2:07 P.M.)

Micrometer readings	Standard	Difference	Correction 2	Micrometer readings	Standard	Difference	Correction 2
5782.350	.346‡	+.004	+.007	5805.445	.448‡	—.003	+.007
5788.142	.136‡	+.006	+.007	5806.950	.954‡	—.004	+.007
5791.207	.207†	.000	+.007	5809.440	.437‡	+.003	+.007
5798.080	.087*	—.007	+.007	5816.602	.594‡	+.008	+.007
5798.403	.400‡	+.003	+.007				

THE ARC-SPECTRUM OF VANADIUM.

Wave-length (uncorrected)	Correction [1]	Correction	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
3094.779	+.014		1, n	3094.793	
3101.024	+.014		1	3101.038	
3102.401	+.014		10	3102.415	
3109.269	+.014		1	3109.283	
3109.367	+.014		1	3109.381	
3110.812	+.014		1	3110.826	
3113.024	+.014		1	3113.038	
3118.482	+.014		8	3118.496	
3120.836	+.013		1	3120.849	
3121.248	+.013		1	3121.261	
3123.008	+.012		1	3123.020	
3125.390	+.012		5	3125.402	
3126.327	+.011		5	3126.338	
3130.398	+.010		5	3130.408	
3133.446	+.009		5	3133.455	
3135.052	+.008		1	3135.060	
3137.298	+.007		1	3137.305	
3139.856	+.006		1	3139.862	
3142.591	+.005		2	3142.596	
3146.084	—.002		1	3146.086	
3164.950	—.005		1	3164.945	
3168.250	—.006		1	3168.244	
3183.534	—.009		9	3183.525	
3184.106	—.009		10	3184.097	
3185.516	—.009		10	3185.507	
3187.830	—.010		4	3187.820	
3188.634	—.010		2	3188.624	
3190.808	—.010		5	3190.798	
3194.040	—.010		1	3194.030	
3198.131	—.010		1	3198.121	
3199.944	—.010		1	3199.934	
3202.505	—.010		6	3202.495	
3205.388	—.010		1	3205.378	
3205.699	—.010		3	3205.689	
3207.531	—.010		4	3207.521	
3208.474	—.010		1	3208.464	
3210.263	—.010		1	3210.253	
3210.556	—.010		1	3210.546	
3212.560	—.010		1	3212.550	
3215.497	—.010		1	3215.487	
3217.250	—.010		1	3217.240	
3218.995	—.010		1	3218.985	
3226.233	—.010		1	3226.223	

[1] Obtained from correction curve.

VANADIUM — *continued*,

Wave-length (uncorrected)	Correction 1	Correction	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
3227.530	—.010		1	3227.520	
3229.734	—.010		1	3229.724	
3230.775	—.010		1	3230.765	
3232.074	—.010		1	3232.064	
3233.310	—.010		2	3233.300	
3233.888	—.010		1	3233.878	
3238.000	—.010		2	3237.990	
3249.700	—.010		1	3249.690	
3250.904	—.010		1	3250.894	
3251.995	—.009		1	3251.886	
3254.845	—.009		2	3254.836	
3255.778	—.009		1	3255.769	
3256.900	—.008		1	3256.892	
3259.665	—.007		1	3259.658	
3261.205	—.007		1	3261.198	
3262.429	—.007		1	3262.422	
3262.187	—.007		1	3262.180	
3266.033	—.006		1	3266.027	
3267.828	—.005		8	3267.823	
3271.247	—.004		8	3271.243	
3271.763	—.004		2	3271.759	
3273.141	—.004		1	3273.137	
3276.255	—.003		8	3276.252	
3277.884	—.003		1, n	3277.881	
3278.055	—.002		1, n	3278.053	
3279.978	—.002		1	3279.976	
3281.240	—.002		1	3281.238	
3282.661	—.002		1	3282.659	
3284.492	—.002		1	3284.489	
3285.134	—.001		1	3285.133	
3288.560	—.001		1	3288.559	
3288.438	—.001		1	3288.437	
3289.516	—.001		2	3289.515	
3290.363	—.001		2	3290.362	
3291.806	—.001		3	3291.805	
3298.277	—.001		1	3298.276	
3299.224	—.001		2	3299.223	
3309.306	—.001		2	3309.305	
3313.142	—.001		1	3313.141	
3314.144	—.001		1	3314.143	
3314.980	.000		1	3314.980	
3322.085	—.001		1	3322.084	
3324.515	—.001		1	3324.514	
3329.985	—.002		3	3329.983	
3333.695	—.002		1	3333.693	
3356.480	—.009		2	3356.471	
3365.683	—.013		3	3365.670	
3405.936	+.076		1, n	3406.012	
3406.914	+.075		1, n	3406.989	
3414.300	+.070		1, n	3414.370	

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
3418.608	+.068		1, n	3418.676	
3425.145	+.059		1, n	3425.204	
3457.010	+.038		1, n	3457.048	
3489.625	+.023		1, n	3489.648	
3497.062	+.019		1, n	3497.081	
3501.597	+.017		1	3501.614	
3517.425	+.011		1	3517.436	
3529.870	+.006		1	3529.876	
3533.816	+.004		1	3533.820	
3543.629	+.002		1	3543.631	
3545.329	+.001		1, n	3545.330	
3545.418	+.001		1	3545.419	
3551.670	−.001		1, n	3551.669	
3553.413	−.001		1	3553.412	
3573.659	−.007		1	3573.652	
3574.922	−.007		1	3574.915	
3578.015	−.008		1	3578.007	
3582.962	−.009		1	3582.953	
3583.854	−.010		1	3583.840	
3589.900	−.011		1	3589.889	
3592.170	−.011		1	3592.159	
3593.530	−.011		1	3593.519	
3600.179	−.013		1	3600.166	
3639.180	−.020		1	3639.160	
3639.740	−.020		1	3639.720	
3644.013	−.021		1	3644.023	
3644.859	−.021		1	3644.038	
3649.078	−.021		1	3649.057	
3663.716	−.022		1, n	3663.694	
3665.278	−.022		1, n	3665.256	
3667.863	−.022		1, n	3667.841	
3671.363	−.023		1	3671.840	
3672.542	−.023		1, n	3672.519	
3675.858	−.023		2	3675.835	
3676.830	−.023		1, n	3676.807	
3680.078	−.023		1, n	3680.055	
3680.237	−.023		1	3680.214	
3683.266	−.023		3	3683.243	
3683.626	−.023		1	3683.603	
3686.415	−.023		3	3686.392	
3688.230	−.023		3	3688.207	
3690.431	−.023		3	3690.407	
3692.380	−.023		3	3692.357	
3695.472	−.023		1, n	3695.449	
3696.018	−.023		4	3695.995	
3704.687	−.023		1	3704.664	
3704.854	−.023		7	3704.831	
3705.190	−.023		5	3705.167	
3706.190	−.023			3706.167	
3708.875	−.023		1	3708.852	

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
3719.074	—.023		1	3719.051	
3719.147	—.023		1	3719.124	
3722.159	—.023		1	3722.136	
3722.357	—.023		1	3722.334	
3738.158	—.023		1	3738.129	
3738.923	—.023		1	3738.901*	
3740.397	—.023		1	3740.374*	
3741.653	—.023		1	3741.630	
3778.835	—.027		3	3778.808	
3790.475	—.027		2	3790.448	
3790.620	—.027		1	3790.593	
3800.019	—.027		3	3799.992	
3803.640	—.027		3	3803.613	
3807.378	+.047		2	3807.425	
3807.579	+.047		3	3807.626	
3808.090	+.046		4	3808.136	
3813.567	+.045		4	3813.612	
3818.327	+.043		5	3818.370	
3820.044	+.043		4	3820.087	
3820.616	—.027		2	3820.589	
3821.566	+.042		4	3821.607	
3823.035	—.027		2	3823.008*	
3828.640	+.040		7	3828.680	
3840.833	+.033		6	3840.866	
3844.533	+.032		4	3844.565	
3847.423	+.030		3	3847.453	
3849.404	+.029		2	3849.433	
3855.460	+.026		4	3855.486	
3855.939	+.026		7	3855.965	
3864.959	+.021		5	3864.980	
3875.179	+.016		5	3875.195	
3886.681	+.010		2	3886.691	
3890.290	+.008		4	3890.298	
3892.465	+.006		4	3892.471	
3896.254	+.005		2	3896.259	
3898.079	+.003		1	3898.082	
3902.369	+.002		7	3902.371	
3909.997	—.002		5	3909.995	
3914.441	—.004		1	3914.437	
3919.605	—.005		1	3919.600	
3922.029	—.006		1	3922.023	
3922.554	—.006		3	3922.548	
3924.775	—.007		3	3924.768	
3925.357	—.007		3	3925.350	
3933.784	—.009		3	3933.775	
3944.143	—.010		3	3944.133	
3952.083	—.010		1	3952.073	
3961.662	—.010		5	3961.652	
3968.597	—.009		1	3968.588	
3979.549	—.009		1	3979.540	

*Average of four or more measurements.

VANADIUM—*continued.*

Wave-length (uncorrected)	Correction [1]	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
3990.702	—.009		5	3990.693	
3992.925	—.009		3	3992.916	
3998.856	—.009		3	3998.847	
4005.847	—.009		1, n	4005.838	
4022.046	—.008		1, n	4022.038	
4023.516	—.008		1, n	4023.508	
4031.968	—.007		1	4031.961	
4033.199	—.007		3	4033.192	
4034.626	—.007		2	4034.619	
4042.765	—.006		1	4042.759	
4051.490	—.005		4	4051.485	
4057.211	—.005		2	4057.206	
4057.961	—.005		1	4057.956	
4064.065	—.005		2	4064.061	
4071.668	—.004		2	4071.664	
4077.853	—.004		1, n	4077.849	
4090.707	—.004		5	4090.703	
4092.536	—.004		2	4092.532	
4095.611	—.004		5	4095.607	
4098.514	—.004		1, n	4098.510	
4099.925	—.004		7	4199.921	
4102.289	—.004		3	4102.285	
4104.520	—.004		2	4104.516	
4107.603	—.004		1	4107.599	
4109.910	—.004		7	4109.906	
4111.920	—.004		5, R	4111.916	
4113.641	—.004		3	4113.637	
4115.316	—.005		7	4115.311	
4116.636	—.005		9	4116.631	
4118.325	—.005		1, n	4118.320	
4119.580	—.005		3	4119.575	
4120.660	—.005		2	4120.655	
4124.200	—.004		1	4124.196	
4128.156	—.004		7	4128.152	
4131.301	—.c04		1, n	4131.297	
4132.127	—.004		6	4132.123	
4134.620	—.003		7	4134.617	
4159.819	+.003		2	4159.822	
4174.145	+.010		1	4174.155	
4182.769	—.042	+.003	1	4182.733	
4183.110	—.042	+.003	4	4183.071*	
4189.988	+.020	+.003	2	4190.011	
4202.545	—.042	+,003	2	4202.506	
4205.240	—.042	+.003	2	4205.201	
4210.041	—.042	+.003	5	4210.002	
4225.408	—.042	+.003	1	4225.369	
4226.910	—.042	+.003	4, R	4226.871	
4232.643	—.042	+.003	7	4232.604	
4233.146	—.042	+.003	7	4233.007	
4234.188	—.042	+.003	7	4234.149	

[1] Due to Earth's motion. * Average of four or more measurements.

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4234.710	—.042	+.003	7	4234.671	
4235.948	—.042	+.003	4	4235.909	
4257.556	—.042	+.003	4	4257.517	
4259.493	—.042	+.003	4	4259.454	
4262.350	—.042	+.003	4	4262.311	
4268.826	—.042	+.003	0	4268.787	
4271.745	—.042	+.003	17	4271.706	4268.85
4277.140	—.042	+.003	7	4277.101	4271.80
4284.247	—.042	+.003	5	4284.208	
4291.997	—.022	+.003	1	4291.978	
4296.285	—.022	+.003	7	4296.266	
4297.859	—.022	+.003	7	4297.840	
4299.259	—.022	+.003	1	4299.240	
4303.716	—.022	+.003	2	4303.697	
4309.968	—.022	+.003	7	4309.949	
4318.822	—.022	+.003	2	4318.803	
4330.209	—.021	+.003	0	4330.181	
4333.003	—.021	+.003	10	4332.985	4330.15
4341.178	—.019	+.003	10	4341.162	4333.00
4353.054	—.017	+.003	18	4353.040	4341.15
4355.151	—.016	+.003	4	4355.138	4353.05
4356.117	—.016	+.003	4	4356.104	
4363.700	—.013	+.003	4	4363.690	
4364.387	—.013	+.003	4	4364.377	
4368.765	—.012	+.003	4	4368.756	
4373.390	—.010	+.003	6	4373.383	
4373.991	—.010	+.003	3	4373.984	
4379.389	.000	+.003	1	4379.392	
4380.715	—.007	+.003	4	4380.719	4379.42
4381.191	—.007	+.003	1	4381.187	
4384.877	—.005	+.003	1	4384.875	4384.95
4390.142	—.003	+.003	7, R	4390.142	4390.15
4392.233	—.002	+.003	4	4392.234	
4393.256	—.001	+.003	3	4393.258	
4393.998	—.001	+.003	4	4394.000	
4395.379	.000	+.003	· 10, R	4395.382	4395.40
4397.389	.000	+.003	1	4397.392	
4400.733	+.002	+.003	10	4400.738	4400.75
4403.825	+.003	+.093	4	4403.831	
4406.271	+.003	+.003	8	4406.277	
4406.798	+.004	+.003	8, R	4406.805	4406.85
4407.793	+.005	+.003	8, R	4407.801	4407.90
4408.360	+.005	+.003	5, R	4408.368	4408.40
4408.657	+.005	+.003	5, R	4408.665	4408.65
4412.290	+.006	+.003	4	4412.299	
4416.615	+.008	+.003	5	4416.626	4416.65
4421.726	+.010	+.003	10	4421.739	
4423.361	+.011	+.003	8	4423.375	
4424.068	+.011	+.003	2	4424.082	
4424.729	+.011	+.003	4	4424.743	

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4425.579	+.012	+.003	1	4425.594	
4428.660	+.013	+.003	5	4428.676	
4436.290	+.016	+.003	7	4436.309	
4437.985	+.016	+.003	7	4438.004	4438.03
4441.826	+.018	+.003	2	4441.847	4441.90
4443.486	+.019	+.003	4	4443.508	
4444.358	+.019	+.003	3	4444.380	4444.40
4449.718	+.020	+.003	4	4449.741	
4451.046	+.021	+.003	4	4451.070	
4452.156	+.021	+.003	8	4452.180	4452.12
4454.913	+.023	+.003	1	4454.939	
4456.047	+.023	+.003	1	4456.073	
4456.642	+.023	+.003	3	4456.668	
4457.605	+.024	+.003	3	4457.632	
4458.888	+.024	+.003	1, n	4458.915	
4459.891	+.024	+.003	8	4459.918	4459.95
4460.434	+.025	+.003	10, R	4460.462	4460.45
4460.821	+.025	+.003	4	4460.849	
4462.504	+.026	+.003	10	4462.533	4462.55
4465.645	+.027	+.003	3	4465.675	
4468.143	+.028	+.003	3	4468.174	
4468.900	+.028	+.003	3	4468.931	
4469.840	+.028	+.003	7	4469.871	4469.90
4470.950	+.029	+.003	1	4470.872	
4474.174	+.030	+.003	7	4474.207	
4474.866	+.030	+.003	7	4474.899	
4480.170	+.033	+.003	3	4480.206	
4489.056	+.037	+.003	7	4489.096	
4490.940	+.038	+.003	4	4490.981	
4491.298	+.040	+.003	2	4491.343	
4491.607	+.038	+.003	1	4491.648	
4496.190	+.040	+.003	5	4496.233	
4497.531	+.040	+.003	5	4497.574	
4500.955	+.043	+.003	2	4501.001	
4501.366	+.043	+.003	1	4501.412	
4502.075	+.043	+.003	4	4502.121	
4506.696	+.045	+.003	1	4506.744	
4509.413	+.047	+.003	2	4509.463	
4511.554	+.048	+.003	2	4511.605	
4513.740	+.049	+.003	2	4513.792	
4514.305	+.049	+.003	4	4514.357	
4515.676	+.050	+.003	1	4515.729	
4517.683	+.052	+.003	3	4517.738	
4520.275	+.053	+.003	2	4520.331	
4520.629	+.053	+.003	2	4520.685	
4524.320	+.055	+.003	5	4524.378	
4525.279	+.055	+.003	2	4525.337	
4528.108	+.057	+.003	3	4528.168	
4529.415	+.058	+.003	2	4529.476	
4530.910	+.059	+.003	3	4530.972	

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4534.044	+.060	+.003	3	4534.107	
4537.768	+.063	+.003	4	4537.834	
4540.112	+.064	+.003	4	4540.179	
4545.496	+.067	+.003	10	4545.566	4545.62
4549.751	+.070	+.003	8	4549.824	4549.85
4551.941	+.072	+.003	2	4552.016	
4552.660	+.072	+.003	5	4552.735	
4560.813	+.077	+.003	7	4560.893	
4564.673	+.080	+.003	1	4564.756	
4571.870	+.086	+.003	5	4571.959	
4577.255	+.090	+.003	7	4577.348	4577.40
4578.813	+.092	+.003	5	4578.908	
4579.278	+.092	+.003	2	4579.373	
4580.466	+.093	+.003	8	4580.562	4580.55
4581.313	+.093	+.003	1	4581.409	
4583.868	+.096	+.003	2	4583.967	
4586.454	+.097	+.003	8	4586.554	4586.55
4591.303	+.100	+.003	5	4591.406	4594.30
4594.197	+.016	.003	10, R	4594.216	
4606.366	—.049	+.004	4	4606.321	
4607.435	—.049	+.004	1	4607.390	
4608.680	—.049	+.004	1, n	4608.635	
4609.866	—.049	+.004	4	4609.821	
4611.148	—.049	+.004	1	4611.103	
4614.020	—.048	+.004	1, n	4613.976	
4614.138	—.048	+.004	1, n	4614.094	
4616.234	—.048	+.004	11, n	4616.190	
4619.940	—.048	+.004	0	4619.896	
4621.470	—.048	+.004	1, n	4621.426	
4624.625	—.048	+.004	4	4624.581	
4626.710	—.048	+.004	4	4626.666	
4630.280	—.048	+.004	1, n	4630.236	
4635.389	—.047	+.004	7	4635.346	
4636.386	—.047	+.004	1, n	4636.343	
4640.275	—.047	+.004	5	4640.232	
4640.959	—.047	+.004	5	4640.916	
4644.281	—.046	+.004	1	4644.239	
4644.666	—.046	+.004	2	4644.624	
4646.198	—.046	+.004	1	4646.156	
4646.613	—.046	+.004	8	4646.571	
4648.088	—.046	+.004	1, n	4648.046	
4649.110	—.046	+.004	2	4649.068	
4653.147	—.045	+.004	1	4653.106	
4655.451	—.045	+.004	1, n	4655.410	
4657.179	—.045	+.004	1, n	4657.138	
4662.645	—.044	+.004	1, n	4662.605	
4663.354	—.044	+.004	3	4663.314	
4669.527	—.043	+.004	1	4669.487	
4670.705	—.043	+.004	8	4670.666	
4673.874	—.042	+.004	1	4673.836	

VANADIUM—*continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4679.998	—.041	+.004	1	4679.961	
4681.110	—.041	+.004	1	4681.073	
4684.670	—.040	+.004	3	4684.634	
4687.135	—.039	+.004	5	4687.100	
4690.472	—.038	+.004	1, n	4690.438	
4699.537	—.036	+.004	2	4699.505	
4702.720	—.035	+.004	1, n	4702.689	
4705.308	—.035	+.004	3	4705.278	
4706.387	—.034	+.004	5	4706.357	
4706.790	—.033	+.004	5	4706.761	
4707.658	—.033	+.004	3	4707.629	
4708.426	—.033	+.004	1, n	4708.397	
4709.159	—.033	+.004	1, n	4709.130	
4710.774	—.032	+.004	5	4710.746	
4713.666	—.031	+.004	1	4713.639	
4715.514	—.030	+.004	1	4715.488	
4715.676	—.030	+.004	1	4715.650	
4716.105	—.030	+.004	4	4716.079	
4716.403	—.030	+.004	1	4716.377	
4717.900	—.030	+.004	5	4717.874	
4721.469	—.029	+.004	1	4721.444	
4721.729	—.029	+.004	4	4721.704	
4723.079	—.028	+.004	4	4723.055	
4723.650	—.028	+.004	1, n	4723.626	
4724.099	—.028	+.004	1, n	4724.075	
4728.862	—.026	+.004	1, n	4728.840	
4729.746	—.026	+.904	5	4729.724	
4730.596	—.026	+.004	2	4730.574	
4731.465	—.026	+.004	1	4731.443	
4731.767	—.026	+.004	1	4731.745	
4732.130	—.026	+.004	1	4732.108	
4737.944	—.024	+.004	1	4737.924	
4738.525	—.024	+.004	1	4738.505	
4739.869	—.024	+.004	1, n	4739.849	
4742.838	—.023	+.004	5	4742.819	
4746.845	—.022	+.004	5	4746.827	
4747.331	—.022	+.004	1, n	4747.313	
4748.741	—.022	+.004	5	4748.723	
4751.208	—.021	+.004	5	4751.211	
4751.480	—.021	+.004	1	4751.463	
4751.776	—.021	+.004	5	4751.759	
4752.053	—.021	+.004	1, n	4752.036	
4757.702	—.020	+.004	4	4757.686	
4758.953	—.019	+.004	1	4758.938	
4759.225	—.019	+.004	1. n	4759.210	
4764.238	—.018	+.004	1, n	4764.224	
4765.873	—.018	+.004	1	4765.859	
4766.851	—.017	+.004	7	4766.838	
4769.221	—.017	+.004	1, n	4769.208	
4772.793	—.016	+.004	1	4772.781	

VANADIUM—*continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4773.275	—.016	+.004	1	4773.263	
4776.655	—.015	+.004	5	4776.644	
4781.524	—.014	+.004	1, n	4781.514	
4784.672	—.013	+.004	5	4784.663	
4786.715	—.013	+.004	7	4786.706	
4789.111	—.012	+.004	1	4789.103	
4793.142	—.011	+.004	2	4793.135	
4794.737	—.011	+.004	1, n	4794.730	
4795.300	—.011	+.004	2	4795.293	
4797.125	—.010	+.004	8	4797.119	
4798.157	—.010	+.004	1	4798.151	
4799.216	—.010	+.004	1	4799.210	
4799.978	—.010	+.004	4	4799.972	
4802.378	—.009	+.004	1, n	4802.373	
4803.245	—.009	+.004	1, n	4803.240	
4807.740	—.008	+.004	10	4807.736	
4808.845	—.007	+.004	1, n	4808.842	
4819.225	—.004	+.004	2	4819.225	
4823.030	—.003	+.004	1, n	4823.031	
4827.636	—.002	+.004	10	4827.638	
4829.005	—.001	+.004	1	4829.008	
4829.424	—.001	+.004	1, n	4829.427	
4830.876	—.001	+.004	1, n	4830.879	
4831.832	.000	+.004	8	4831.836	
4832.613	.000	+.004	8	4832.617	
4833.209	.000	+.004	3	4833.213	
4834.001	.000	+.004	1, n	4834.005	
4834.260	.000	+.004	1, n	4834.264	
4835.035	+.001	+.004	1, n	4835.040	
4843.188	+.003	+.004	2	4843.195	
4846.791	+.004	+.004	1, n	4846.799	
4848.995	+.005	+.004	1	4849.004	
4849.253	+.005	+.004	1. n	4849.262	
4849.449	+.005	+.004	1, n	4849.458	
4851.676	+.006	+.004	10	4851.686	
4852.145	+.006	+.004	1, n	4852.155	
4854.104	+.006	+.004	1, n	4854.114	
4855.543	+.007	+.004	1, n	4855.554	
4857.230	+.007	+.004	1, n	4857.241	
4858.798	+.007	+.004	2	4858.809	
4862.789	+.008	+.004	4	4862.801	
4864.930	+.009	+.004	10	4864.943	
4870.320	+.010	+.004	1, n	4870.334	
4871.438	+.011	+.004	3	4871.453	
4873.155	+.011	+.004	1, n	4873.170	
4875.658	+.012	+.004	10	4875.674	
4880.728	+.014	+.004	6	4880.746	
4881.727	+.014	+.004	10	4881.745	
4882.341	+.014	+.004	2	4882.359	
4885.808	+.015	+.004	2	4885.827	

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
4886.971	+.015	+.004	2	4886.990	
4890.245	+.016	+.004	1	4890.265	
4891.393	+.017	+.004	2	4891.414	
4891.746	+.017	+.004	3	4891.767	
4894.374	+.018	—.004	3	4894.396	
4900.796	+.020	+.004	3	4900.820	
4904.550	+.021	+.004	5	4904.575	
4905.025	+.021	+.004	3	4905.050	
4907.020	+.022	+.004	1, n	4907.046	
4908.856	+.022	+.004	1	4908.882	
4913.249	+.024	+.004	1, n	4913.277	
4916.413	+.024	+.604	1	4916.436*	
4919.141	+.026	+.004	1, n	4919.171	
4922.514	+.027	+.004	1	4922.543*	
4925.810	+.027	+.004	7	4925.837*	
4932.181	+.030	+.004	3	4932.212	
4933.760	+.031	+.004	1	4933.786*	
5002.502	—.002	+.005	2	5002.505	
5005.790	—.002	+.005	1, n	5005.793	
5014.808	—.002	+.005	4	5014.811	
5047.481	—.002	+.005	1, n	5047.484	
5051.778	—.002	+.005		5051.781	
5060.828	—.002	+.005	1, n	5060.831	
5064.293	—.002	+.005	1	5064.296	
5105.321	—.002	+.005	2	5105.324	
5128.700	.000	+.005	7	5128.705	
5137.765	+.002	+.005	1, n	5137.772	
5138.590	+.002	+.005	4	5138.597	
5139.697	+.002	+.005	2	5139.704	
5148.885	+.003	+.005	4	5148.893	
5159.582	—.049	+.005	2	5159.438	
5159.510	+.005	+.005	2	5159.520	
5165.124	—.049	+.005	1	5165.072*	
5167.013	—.049	+.005	1	5166.961*	
5169.170	—.049	+.005	1	5169.126	
5170.102	—.049	+.005	1	5170.114	
5172.328	—.049	+.005	1, n	5172.284	
5174.702	—.049	+.005	1, n	5174.714	
5176.731	—.049	+.005	1, n	5176.683*	
5177.004	—.049	+.005	1, n	5176.956*	
5178.782	—.049	+.005	1, n	5178.733*	
5179.325	—.049	+.005	1. n	5179.275*	
5180.975	—.049	+.005	1. n	5180.926*	
5182.979	—.049	+.005	1	5182.993	
5183.077	—.049	+.005	1, n	5183.033	
5192.241	—.049	+.005	1	5192.193*	
5193.232	—.049	+.005	5	5193.184*	
5193.843	—.049	+.005	1	5193.795*	
5195.070	—.049	+.005	6	5195.021*	
5195.615	—.049	+.005	2	5195.564*	

*Average of four or more measurements.

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
5197.197	+.013	+.005	1, n	5197.215	
	—.049	+.005	1, n	5197.215	
5200.564	—.049	+.005	1, n	5200.520	
5206.839	—.049	+.005	1, n	5206.790*	
5207.892	—.049	+.005	1, n	5207.844*	
5212.377	—.049	+.005	1, n	5212.399	
5213.887	—.048	+.005	1, n	5213.837	
5216.821	—.048	+.005	1	5216.772*	
5225.881	+.022	+.005	3	5225.920	
5233.851	+.026	+.005	2	5233.895	
5234.205	+.026	+.005	7	5234.249	
5240.315	+.029	+.005	2	5240.364	
5241.097	—.047	+.005	3	5241.055	
5258.260	+.043	+.005	1, n	5258.308	
5260.473	+.045	+.005	1, n	5260.527	
5261.188	—.044	+.005	1, n	5261.149	
5271.156	—.042	+.005	1	5271.119	
5317.092	—.030	+.005	1, n	5317.067	
5319.280	—.030	+.005	1, n	5319.255	
5329.511	—.030	+.005	1, n	5329.486	
5330.640	—.029	+.005	1, n	5330.616	
5338.831	—.024	+.005	1, n	5338.812	
5353.633	—.019	+.005	3	5353.619	
5383.654	—.008	+.005	1	5383.651	
5388.537	—.008	+.005	1, n	5388.534	
5402.144	—.001	+.005	5	5402.148	
5415.469	+.005	+.005	5	5415.479	
5418.307	+.006	+.005	1	5418.318	
5424.267	+.009	+.005	2	5424.281	
5434.390	+.015	+.005	4	5434.410	
5437.863	+.017	+.005	1	5437.885	
5443.443	+.018	+.005	1	5443.466	
5445.003	+.023	+.005	1, n	5455.031	
5467.998	+.029	+.005	1, n	5468.032	
5471.528	+.030	+.005	1, n	5471.563	
5487.413	+.037	+.005	1	5487.455	
5488.269	+.038	+.005	4	5488.312	
5490.137	+.039	+.005	1	5490.181	
5505.045	+.047	+.005	1	5505.097	
5506.045	+.047	+.005	1	5506.097	
5507.691	+.048	+.005	4	5507.744	
5508.812	+.048	+.005	1	5508.865	
5511.358	+.050	+.005	1	5511.413	
5515.245	+.051	+.005	1, n	5515.301	
5517.380	+.052	+.005	1. n	5517.437	
5533.988	+.063	+.005	1, n	5534.056	
5535.013	+.064	+.005	1, n	5535.082	
5535.589	+.065	+.005	1, n	5535.659	
5542.880	+.069	+.005	1, n	5542.954	
5545.025	+.071	+.005	1, n	5545.101	

*Average of four or more measurements.

VANADIUM—*continued.*

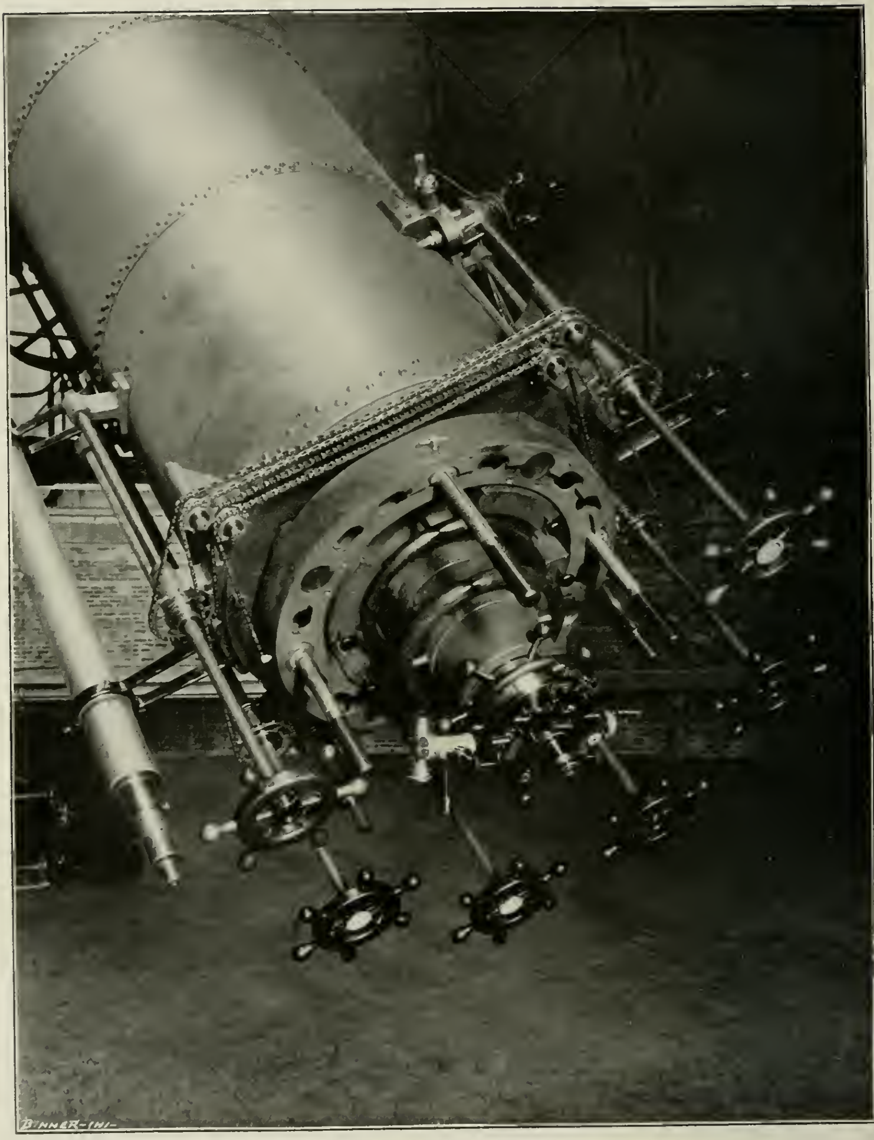
Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
5546.088	+.072	+.005	1	5546.165	
5547.229	+.072	+.005	4	5547.306	
5548.323	+.073	+.005	1, n	5548.401	
5558.908	+.081	+.005	1	5558.995	
5561.898	—.010	+.007	1, n	5561.897*	
5566.065	+.086	+.005	1, n	5566.156	
5567.610	+.087	+.005	1	5567.702	
5576.755	—.010	+.007	1	5576.752	
5584.605	—.010	+.007	1, n	5584.602	
5584.748	—.010	+.007	5	5584.745	
5584.982	—.010	+.007	1, n	5584.979	
5586.235	—.010	+.007	1, n	5586.232	
5588.716	—.010	+.007	1, n	5588.713	
5592.673	—.010	+.007	4	5592.670	
5593.211	—.010	+.007	1, n	5593.208	
5594.734	—.010	+.007	1, n	5594.731	
5598.050	—.010	+.007	1, n	5598.047	
5601.630	—.010	+.007	1	5601.627	
5604.446	—.010	+.007	1	5604.443	
5604.878	—.010	+.007	1	5604.875	
5605.190	—.010	+.007	4	5605.187	
5622.322	—.010	+.007	1	5622.319	
5624.449	—.010	+.007	1	5624.446	
5624.856	—.010	+.007	7	5624.853	
5625.124	—.010	+.007	4	5625.121	
5626.270	—.010	+.007	7	5626.267	
5627.889	—.010	+.007	7	5627.886	
5632.705	—.010	+.007	1	5632.702	
5634.148	—.010	+.007	1	5634.145	
5635.745	—.010	+.007	1	5635.742	
5646.356	—.011	+.007	5	5646.352	
5657.123	—.011	+.007	1	5657.119	
5657.695	—.011	+.007	5	5657.689	
5668.612	—.011	+.007	5	5668.608	
5671.095	—.011	+.007	10	5671.091	
5683.456	—.012	+.007	1, n	5683.451	
5688.003	—.012	+.007	1, n	5687.998	
5698.770	—.012	+.007	1	5698.765	
5703.830	—.012	+.007	10	5703.825	
5707.241	—.012	+.007	10	5707.236	
5709.203	—.012	+.007	1, n	5709.198	
5716.466	—.012	+.007	1, n	5716.461	
5725.886	—.012	+.007	3	5725.881	
5727.294	—.012	+.007	10	5727.289	
5727.905	—.012	+.007	5	5727.900	
5733.340	—.011	+.007	1	5733.336	
5734.258	—.011	+.007	2	5734.254	
5737.314	—.011	+.007	5	5737.310	
5743.678	—.010	+.007	5	5743.675	
5752.988	—.010	+.007	1	5752.985	

*Average of four or more measurements.

VANADIUM — *continued.*

Wave-length (uncorrected)	Correction 1	Correction 2	Intensity and Character	Wave-length (corrected)	Wave-length (Hasselberg)
5761.676	—.009	+.007	1, n	5761.674	
5772.657	—.007	+.007	2	5772.657	
5776.929	—.006	+.007	1, n	5776.930	
5782.846	—.005	+.007	1, n	5782.848	
5783.762	—.005	+.007	1, n	5783.764	
5784.643	—.004	+.007	1	5784.646	
5786.410	—.004	+.007	1	5786.413	

PLATE V.



EYE-END OF THE 40-INCH YERKES TELESCOPE.

Minor Contributions and Notes.

THE NORTHERN DURCHMUSTERUNG.

The *Durchmusterung* charts of the northern sky are indispensable to every active astronomical observatory, and to every astronomer who wishes to study the fainter stars. Unfortunately, the original edition of this work is exhausted, so that copies can no longer be supplied. A new edition is being prepared by the Bonn Observatory and will be published shortly, provided that subscriptions for a hundred copies, at seventy marks each, are promised before May 1, 1898. The price is very low considering the amount of material furnished. After that date, the price will be raised to one hundred and twenty marks. The Astronomical Conference, held at the dedication of the Yerkes Observatory, appointed the undersigned a committee to aid this project. Orders for copies may be sent to the publishers, Messrs. A. Marcus and E. Weber, Bonn, Germany, or will be transmitted to them by any member of the committee. It is proposed to publish a list of American subscribers, and it is hoped that at least fifty copies will be taken by American astronomers. Since charts deteriorate rapidly by constant use several copies should be taken by each of the larger observatories. The members of the committee have shown their appreciation of the value of this work by ordering twelve copies for use in the institutions under their direction. It is of the greatest importance that the subscription list should be filled as it is probable that in the future many similar enterprises may be undertaken, whose success will depend upon that now attained.

Edward C. Pickering.
J. G. Hagen, S. J.
M. B. Snyder.
} Committee.

NOTE ON PROFESSOR CAMPBELL'S OBSERVATIONS OF VARIATIONS IN THE INTENSITIES OF THE LINES IN THE ORION NEBULA.

In the November number of the Astrophysical Journal, Professor Campbell attacks, with much indignation, some remarks of mine

criticising his discoveries, contained in an article on Professor Keeler's work on the spectra of nebulæ. Such sensitiveness is somewhat surprising on the part of one who is himself given to severely taking others to task. Further, an astronomer who frequently observes phenomena which others cannot see, and fails to see those which others can, must be prepared to have his opinions contested. If, as Professor Campbell complains, I have only supported my views by a single example, I was only withheld by courteous motives from adding another, namely, the fact that Professor Campbell cannot perceive the lines of aqueous vapor in the spectrum of Mars which were seen by Huggins and Vogel in the first place, and, after Mr. Campbell had called their existence in question, were again seen and identified with certainty by Professor Wilsing and myself. Whether such instances will be multiplied in the future remains to be seen.

That Professor Campbell should call in the testimony of other astronomers to support his results is quite natural, but I find it strange that he should select for this purpose two gentlemen whose names are as yet quite unknown as skilled observers. In fact, Messrs. Aitken and Wright, in their excess of youthful zeal, betray their inexperience by remarking that they found it "easy," and "very easy," to verify the variation in the relative brightness of the nebular lines—a feat which a Huggins found almost beyond his powers. Professor Schaeberle also used a similar expression. I regret, however, that the tone adopted by this gentleman (who is in no way concerned in the discussion) renders it impossible for me to take any further notice of his remarks.

Professor Campbell compels me now to detail the grounds upon which the conclusions given in my brief criticism were based. To begin with, I must designate the method of observation adopted by Mr. Campbell as unfavorable for the attainment of good results. This is due to the fact that he used the great refractor, an instrument not very well designed for the object in view, since, owing to the great linear extent of the focal image, only a small portion of the nebula can be seen through the slit at once, and therefore estimates of variations, if such existed, in the relative brightness of the nebular lines, could only be made by a series of successive observations involving a difficult memory exercise.

In view of this I must adhere to my former statement, that my observations were made "under very favorable circumstances." The instrument I used was the photographic refractor, whose ratio of aper-

ture to focal length is as 1 to 10, so that the surface intensity of illumination of the image is three or four times as strong as in the case of the great Lick refractor. The focal length being 3.4 meters, the diameter of the focal image, while quite sufficient to enable the details of the nebula to be recognized, is yet less than the length of the slit of the spectroscope. Thus different parts of the nebula can be examined at once. The brightest regions of the nebula are to be recognized by the corresponding apparent widenings or knots upon the spectrum lines, and if the relative brightness be constant, these knots must appear in the same proportion to the fainter portions for each of the three nebular lines. The determination of relative brightness can thus be made by direct comparison, and I think that this gives the method a great advantage over Professor Campbell's. By moving the slit all parts of the nebula can, of course, be brought into the field of view and seen in conjunction with other parts. I have with profit used a rather high-power eyepiece in these experiments, the light-grasping power rendering this possible; and thus I had the same advantage as regards working with a large image that would have been given by the use of a more powerful telescope.

Professor Campbell's result practically amounts to this: That the line appeared to him, with reference to the two other nebular lines, brighter in the fainter regions than in the brighter regions of the nebula. I do not doubt for a moment that Professor Campbell honestly thinks he saw this as he says, but I emphatically dispute his interpretation, namely, that the appearance was based upon a real variation of relative intensity. On the contrary, I maintain that "so far as a matter of this kind can be established," the relative intensity of the three nebular lines is constant over the Orion nebula.

The whole appearance recorded by Professor Campbell is nothing more nor less than the "Purkinje phenomenon." The human eye possesses the physiological peculiarity that its maximum sensibility to light, which in the case of strong illumination lies in the yellow, shifts, as the illumination decreases, towards the more refrangible end of the spectrum, and finally, as the limit of visibility is approached, lies very near the F or $H\beta$ line.

I have gone more fully into this subject in another paper, which will be published at the same time with these remarks.[1] The results given there for an extreme case can easily be applied to the present question.

[1] See p. 231.

May I be allowed to add a few words? Professor Campbell challenges me to publish, "without delay," my estimations of the relative intensities of the nebular lines in the region of the star Bond No. 734. I regret that I am unable to comply with this severe demand, as Nature has not endowed me with the power of measuring relative intensities by use of the eye alone, without any photometric apparatus. I could arrange the intensities according to an arbitrary scale, but I cannot say how many times one line is brighter than another. Until now I have always believed, on the ground of physiological investigations, that this was a task beyond the power of any human being, but after the wonderful agreement achieved by the Lick observers, I am better informed.

May I be permitted to suggest that Professor Campbell might derive some advantage from the study of Physiological Optics?[1]

J. Scheiner.

Astrophysical Observatory,
Potsdam, January 1898.

A PROVISIONAL LIST OF PHOTOMETRIC UNITS.[2]

The insuperable difficulty of measuring photometric quantities in mechanical units renders more or less unsatisfactory any system of units we may adopt for dealing with luminous energy.

For the sake, however, of intelligent communication with each other, some such system is indispensable.

We have accordingly adopted the following as representing the best scientific usage.

In practice we shall seldom, if ever, have occasion to deal with the total radiation which any source of energy emits.

For the present we are engaged in transmitting only that portion of the total radiant energy which is capable of affecting the retina of the normal eye.

[1] The editors would suggest that further discussion of this question be postponed until an appeal to purely photographic methods shall have removed it from the province of physiological optics. For while Professor Campbell does not appear to have taken the Purkinje phenomenon into consideration, Professor Scheiner seems to have overlooked the variations in the apparent relative intensities of the lines which he himself insists must arise from this source. The publication of a photograph of the spectrum, taken with a slit extending from the Trapezium to the star Bond No. 734, should set all doubts at rest.

[2] Prepared for use in the Laboratory of the American Luxfer Prism Company.

To this fraction of the total radiant energy we shall give the name

LUMINOUS ENERGY.

Concerning this term the following two points are to be borne in mind: (1) that while it is a practical impossibility to go into the laboratory and measure just what fraction of the total radiant energy exists in the form of luminous energy, yet this fraction is a perfectly definite quantity; (2) "Luminous energy" is equivalent to "light" only when the latter is used in the narrow sense so as not to include actinic and thermal effects.

These conventions fixed, we are ready to consider the following photometric quantities.

I. *Intensity* of a point-source (or of a source which is sufficiently small compared with its distance to be treated as a point-source) is defined as *the amount of luminous energy emitted per second.*

The word "intensity" is used in a great variety of senses in scientific terminology. If, therefore, any ambiguity should at any time arise as to its exact meaning, it may be modified to read "luminous intensity," which is never employed in any sense except as above defined.

Concerning the nature of intensity in general, it need only be added that it does not represent a quantity of energy such as that contained in a storage cell or in a coiled spring. It is a rate of flow of energy, a ratio between a quantity of energy and a time. The product, intensity by time, is luminous energy; and this product determines the amount of one's gas or electric-light bill.

Since it is impracticable to determine intensity in mechanical measure the following unit is suggested:

Unit of intensity is defined as *the intensity of a Hefner lamp in a horizontal direction, the dimensions*[1] *of the Hefner lamp being those prescribed by the Reichsanstalt at Berlin.*

Name: This unit is called "one candle."

Symbol: for $\begin{cases} \text{intensity, } J. \\ \text{candle power, } c.\,p. \end{cases}$

Much has been said both *pro* and *con* concerning the Hefner lamp. That the flame is red, that it is not perfectly steady and that its intensity varies with the composition of the atmosphere in which it burns must be freely admitted. Since, however, it burns a fuel of definite

[1] For these dimensions, see Palaz, *Industrial Photometry*, pp. 136–143.

chemical composition, since the purity of this fuel is easily tested, since there is no charring of the wick, since the dimensions of the lamp are so chosen that slight deviations from the prescribed size produce a minimum disturbance, and since this standard is highly recommended by the *Reichsanstalt* and the *Committee of the American Institute of Electric Engineers*,[1] its adoption is here suggested.

The quantity which we shall next consider, viz., *Luminous Current*, is the only one out of the entire list for which there is no practical use in the laboratory. Its introduction, however, leads to great simplification in the definitions of the three succeeding quantities, enabling us to avoid the use of π or any of its multiples, concerning which so many pages have been written in the case of the electrical units.

II. *Luminous Current* is defined as *the rate at which luminous energy is emitted by a point-source through a solid angle of one steradian.*

Unit: The luminous current of one candle, *i. e.*, of one Hefner lamp.

Name: "Lumen." Proposed by L. Weber.

Symbol: for $\begin{cases} \text{luminous current, } \phi \\ \text{lumen, } lm \end{cases}$

It is evident that if a point-source radiated uniformly in all directions its intensity would be 4π times its luminous current, *i. e.*,

$$J = 4\pi\phi.$$

But for an element of surface which radiates from one side only as, for instance, a diffusing screen,

$$J = 2\pi\phi.$$

III. *Illumination* is defined as *the ratio of the luminous current to the area upon which it falls.*

This is the same as saying that the illumination is measured by the number of lumens per square centimeter at the point in question. The numerical value of the illumination at any point in a room measures, in general, the success with which that part of the room is lighted. It must not be forgotten, however, that of two equal illuminations, one produced by rays from one direction only, the other by rays from many directions, the latter is as a rule much more effective.

Illumination is a property of a surface at a point; and is determined only by the area and the light immediately incident upon it, independently of the source.

It is evident, however, that in case of a point-source the illumi-

[1] *Transactions Amer. Inst. Elec. Engineers*, 13, 1896.

nation varies inversely as the square of the distance between the point and surface; in case of a linear source the illumination varies inversely as the distance; in case of a plane source, of practically infinite extent, such as the sky, the illumination is entirely independent of the distance separating the source and the illuminated surface.

Concerning the illumination produced by the sky, two facts are always to be borne in mind, viz., (1) that, for all photometric purposes, the sky is a surface at an infinite distance; and (2) that in practice the sky is nearly always diaphragmed: it may be by the cell of a lens, it may be by a window frame, it may be by an ordinary diaphragm. Consequently the natural unit in which to measure the amount of the sky producing illumination at any point is one steradian subtended by that point.

Unit of illumination is one lumen per square centimeter.

Name: "Lux."

Symbol: for $\begin{cases} \text{illumination, } E \\ \text{Lux, } lx \end{cases}$

IV. *Brightness* is defined as *the luminous current leaving unit area of apparent surface.*

The fundamental distinction between brightness and illumination is that, in the former, the surface is considered as the origin of a luminous current; while in the latter, the surface is considered as the recipient of the luminous current.

The Violle standard is essentially a standard of brightness, becoming a standard of intensity only when used with a diaphragm of measured area. We can assign to any portion of the sky a definite brightness only when we imagine the sky to be a surface at a definite distance. The illumination which any portion of the sky produces at a point, however, is quite independent of the imaginary distance of the sky, being a function of the solid angle subtended by the portion of the sky in question.

Unit of brightness is that brightness which yields one lumen per square centimeter of apparent surface.

Name: Lumen per square centimeter.

Symbol: for $\begin{cases} \text{brightness, } B \\ \text{unit of brightness, } lm \text{ } per \text{ } cm^2 \end{cases}$

Not infrequently one is called upon to consider both the intensity of a source and the length of time for which it is available. An incandescent lamp during the first and second halves of its life does not

furnish equal quantities of light to the user. The quantity of sky light available varies tremendously with the weather, with the time of day, and with the season of the year. In comparing the sky light available in January with that at our disposal in July we shall need, therefore, the following unit:

V. *Quantity of light* is defined as *the product of the luminous current by the time it flows.*

Unit quantity of light is one lumen for one second.

Name: . . . .

Symbol: . . . .

DIFFUSION.

Mascart has shown how we may, with a high degree of approximation, compute the increase of illumination produced within any closed space by the light which is "diffusely returned" from the enclosing walls. For this purpose Mascart denotes by Q the amount of light emitted by the illuminating source per second, and by N, a constant, which ranges from, say, 0.04 for black velvet to 0.82 for very white paper.[1] Then the amount of light incident upon the walls of the space per second is W, where

$$W = Q \frac{1}{1 - N} \cdot$$

I venture to think that this factor $\frac{1}{1 - N}$ is of more importance than is generally admitted, and suggest the adoption of the following definition for N:

VI. *Diffusion constant* is defined as the *ratio of the brightness to the illumination at any point on a surface.*

The numerical value of this constant represents the fraction of the incident light at any point which is diffusely reflected by the surface through unit solid angle.

If we denote the diffusion constant by N, then in the system of units which we have employed above

$$N = \frac{\text{brightness}}{\text{illumination}} \cdot$$

Sometimes, however, brightness is defined differently from the manner in which it has been defined above, viz., to denote the

[1] For an excellent discussion of this whole subject, as well as a good series of experimental determinations of N, see SUMPNER: *Phil. Mag.*, February 1893.

intensity (instead of the luminous current) of unit area. In this case the diffusion constant becomes the ratio between 2π brightness and the illumination. That is, its defining equation becomes

$$N = 2\frac{\pi \text{ brightness}}{\text{illumination}}.$$

For, maintaining our definitions of illuminations and diffusion constant, as above given, it is evident that the numerical value of N will vary inversely as the numerical value of the unit of brightness; and, farther, that the unit of brightness depends upon the defining equation of brightness, if consistency is to be preserved.

VII. *Luminosity.*

In all that has been said above, it has been tacitly assumed that the quantities under consideration (brightness, illumination, etc.) refer to luminous energy of the same quality, *i. e.*, to lights of the same composition or colors of the same hue. But in practice it becomes very frequently necessary to compare lights of different color.

In general, indeed, the brightness of the wall of a room has a different hue from that of the illumination which produces this brightness.

Accordingly it becomes necessary for us to define just what we mean when we say that a certain room illuminated by blue light is just as brilliantly lighted as a certain other room which is illuminated by red light. The ease with which one can read a newspaper depends not only upon the intensity of the light with which it is illuminated, but very largely upon the quality of this light. That particular property of any color which determines its value as an illuminant is called its "luminosity."

Thus, for the normal eye, yellow light is much more useful than red of the same intensity: and red light, in turn, is a more powerful aid to distinct vision than blue of the same intensity.

It only remains now to give to "luminosity" a quantitative definition. This is done by the use of a principle discovered by Rood (*Amer. Jour. Sci.* 46 (3), 173, 1893), viz., that when a normal eye is allowed to perceive a colored surface for a short interval of time, say a fraction of a second, the intensity of the sensation is independent of the hue, and depends only upon the luminosity. If a circular cardboard disk be covered, one-half with gray, the other half with a colored pigment, the two halves will have equal luminosities when on rotation all sense of flickering disappears. It is thus found that each color

in the spectrum requires a definite gray to "match" it. The amount of white in the gray semicircle which matches any given color is a measure of the luminosity of this color. In this connection it may be added that any color is completely defined only when we know the following three things about it, viz.:

1. Its "hue," *i. e.*, the wave-length of the light in the solar spectrum which most nearly matches it.

2. Its "luminosity."

3. Its dilution, sometimes called "purity," sometimes called "saturation," *i. e.*, the amount of white light which is mixed with the pure color producing it.

HENRY CREW.

THE TOTAL SOLAR ECLIPSE.

FROM the preliminary reports in *Nature*, the *Observatory* and elsewhere it appears that the total eclipse of January 22 was observed throughout India with remarkable success. An exceptional number of skilled observers, supplied with the most complete instrumental equipments hitherto employed for such a purpose, were scattered along the line of totality. In pleasing contrast with the general disappointment which attended the observations of the last eclipse, come the uniform reports of perfect weather and valuable results. Encouraged by Mr. Shackleton's signal success in photographing the spectrum of the "flash" in 1896, several parties have given this work the principal place in their programme of observations. At Viziadrug Mr. Fowler and Dr. W. J. S. Lockyer, using prismatic cameras having in the first instance two 45° prisms of 6 inches aperture and in the second a single 45° prism of 9 inches aperture, secured some sixty photographs of the spectrum of the Sun's limb. Many of these were taken in such a way as to furnish a complete spectroscopic history covering a period of ten seconds. Sir Norman Lockyer, who had charge of this party, estimates that some of the photographs show as many as a thousand lines. In addition to the lines of the reversing layer the photographs show several monochromatic images of the corona. In the spectrum of the flash the two-prism camera gave about double the number of lines obtained by Mr. Shackleton at Novaya Zemla.

Of perhaps even greater importance are the photographs of the spectrum of the "flash" made at Pulgaon with slit spectroscopes by Captain Hills. As the spectroscopes employed for this purpose are of

high dispersion, the photographs should serve admirably for wave-length determinations. Captain Hills also succeeded in photographing the spectrum of the corona to 4′ from the Sun's limb. Beyond this point, however, nothing was shown, and it is therefore not surprising, though greatly to be regretted, that Mr. Newall did not succeed in his attempt to determine the motion of the corona in the line of sight at a distance of 8′ from the limb. Professor Campbell had also intended to photograph the spectrum of the corona with a view to measuring its rotation, and as the brief report cabled by him to the Lick Observatory expresses satisfaction with his results it is to be hoped that he has been as successful in this direction as in others. In any event he seems to have photographed the spectrum of the reversing layer, for which purpose he had provided a slit spectroscope. As the "flash" spectrum was also successfully photographed by Professor Naegamvala and Mr. Evershed with objective spectroscopes, material has been secured for an extensive study of the spectrum of the Sun's limb.

The spectrum of the corona received less attention, but photographs were obtained by Mr. Newall and others. The green coronal ring was observed visually by Mr. Newall with a grating spectroscope, and Mr. Maunder endeavored to trace out the regions giving 1474 light, using for this purpose an opera-glass with a direct vision prism on one eyepiece. Mr. Maunder states in an article in *Knowledge* that the corona was too faint as seen in this way to render possible a detailed examination of the distribution of the light of this wave-length. As no rifts were seen, however, the observations as far as they go confirm the result obtained many years ago by Tennant.

The form of the corona was photographed by scores of observers, provided with the greatest variety of instruments. These ranged from the kinematographs of the Rev. J. M. Bacon and Lord Graham, through hand cameras giving a solar image about a millimeter in diameter up to objectives of great focal length, giving images four or five inches in diameter. Professor Campbell employed in this work the apparatus so successfully used by Professor Schaeberle in South America in 1893. Mr. Burckhalter's device for simultaneously giving suitable exposures to both the inner and outer corona is said to have produced good results, though the region near the Sun was over-exposed. The form of the corona, a cut of which is given in the *Observatory* for March, is similar to that of 1896. It is important to

add that for the first time in some years Professor Turner has obtained photographs which clearly show the polarization of the corona in at least one streamer.

From this brief outline of some of the principal results obtained it is evident that we may confidently expect the reduction of the photographs to add in no small degree to our knowledge of the Sun.

G. E. H.

OCCULTATION OF 26 ARIETIS OBSERVED PHOTOGRAPHICALLY.[1]

THE disappearance of a bright star when occulted by the Moon is always a striking phenomenon. There is no celestial event whose time is susceptible of more precise determination. For many years various plans have been suggested, both here and elsewhere, by which this time could be determined with greater accuracy than by ordinary visual observation. In fact, the apparatus for photographing the eclipses of Jupiter's satellites, used here for several years, was devised in part for this purpose.

On February 25, 1898, Mr. Edward S. King for the first time succeeded in satisfactorily photographing the occultation of a star. The apparatus used was an improved form of that constructed for photographing the eclipses of Jupiter's satellites, and described in the ASTROPHYSICAL JOURNAL 1, 146. The plate was moved automatically every second by means of an electro-magnet. A motion of about $0^{cm}.03$ was given to the plate whenever the circuit was closed, and of an equal amount when it was opened. Connecting the apparatus with the standard clock, Frodsham 1327, two images alternately faint and bright were obtained every second. As the faint images are three magnitudes fainter than the bright images, the ratio of the durations was about one to sixteen, so that the absolute durations were $0^{s}.06$ and $0^{s}.94$. It is here assumed that, as the times of exposure were very short, the chemical action was proportional to the time. This assumption is verified by actual measurement.

Considering only the images taken during the minute following $6^{h}\ 35^{m}\ 0^{s}$, the bright images of 26 Arietis, as shown below, are equally intense including that having an exposure lasting from $50^{s}.06$ to $51^{s}.00$. Since this image appears to be as bright as the others, the light of the

[1] *Harvard College Observatory Circular*, No. 26.

star could not have begun to diminish much before the time $51^s.00$. If the star had disappeared suddenly at $50^s.9$ the last image would be at least 0.12 of a magnitude fainter than the others, an amount readily measurable. The next image is apparently invisible. Had the disappearance taken place at $51^s.06$ the image would appear, and would

40^s 50^s

Occultation of 26 Arietis.

be as bright as the other faint images. A slight darkening of the film is perceptible near the position the next image would have had, with an intensity nearly equal to that of the fainter images. If this were due to the star, it would denote that the latter suddenly disappeared at about $51^s.12$. The absence of the preceding image would indicate a more gradual disappearance. In any case, the time is fixed at $51^s.1$, to within one-tenth of a second. As the clock was $2^m\ 19^s.4$ fast, not including armature time, the corresponding Greenwich Mean Time is $12^h\ 54^m\ 26^s.5$. By using shorter exposures the uncertainty in the time of disappearance can doubtless be greatly reduced, especially in the case of the brighter stars. Since satisfactory images of 26 Arietis, magnitude 6.1, were obtained in $0^s.06$, it is probable that occultations of stars as faint as the ninth magnitude can be observed photographically.

Measures were next made of the intensity of the last five images of 26 Arietis, to see if there was any diminution in light due to the absorption of a lunar atmosphere. The distances of these images from the Moon's limb were $1''.8$, $1''.4$, $1''.0$, $0''.6$, and $0''.2$, respectively. The corresponding changes in light, expressed in magnitudes as compared with ten more distant images, were $+0.03$, $+0.03$, -0.02, $+0.03$, and -0.02. A positive sign denotes that an image was fainter than those at a greater distance from the Moon. From this it appears that

no diminution in light was perceptible. No correction need be applied to any of the above calculations for the diameter of the star's disc, since, assuming its intrinsic brightness equal to that of the Sun, its time of disappearance would be only $0^s.002$ (*Proc. Amer. Acad.*, 16, 1).

In this connection it is interesting to note that the determination photographically of the position of the Moon, by means of a star about to be occulted, was one of the subjects investigated by Professor G. P. Bond forty years ago. He obtained a number of photographs of the Moon and α Virginis shortly before the occultation of the latter on June 2, 1857. EDWARD C. PICKERING.

March 3, 1898.

COMPARISON STARS FOR VARIABLES.[1]

APPLICATIONS have recently been received from several observers of variable stars for photometric magnitudes of the comparison stars used by them. The measurements described below are available and will be furnished in advance of publication to any astronomer who desires to use them. Sequences of comparison stars have been selected for about one hundred variable stars of long period. Examples of sixteen of these sequences will be found in the pamphlet entitled *Variable Stars of Long Period*, 4°, Cambridge, 1891. Each of the comparison stars brighter than the tenth magnitude has been measured on at least three nights with the meridian photometer. About five stars of each sequence, from the eleventh to the thirteenth magnitude, have been measured on two nights with the photometer having achromatic prisms. (ASTROPHYSICAL JOURNAL, 2, p. 89.) The intervals between the adjacent stars in the sequences have been estimated in grades on three or more nights by Mr. Wendell with the fifteen-inch telescope, and by Mr. Reed with the six-inch telescope. From these estimates and measures a system of magnitudes has been derived in which the accidental errors are very small, and in which the scale for faint as well as for bright stars is nearly the same in all parts of the sky. This scale, which is that of the meridian photometer, is substantially the same as that of the Potsdam Observatory, and of the *Uranometria Oxoniensis*, the differences not exceeding one or two-tenths of a magnitude. Observations are completed of the variable stars T

[1] *Harvard College Observatory Circular*, No. 27.

Andromedae, T Cassiopeiae, R Andromedae, S Ceti, S Cassiopeiae, R Piscium, R Arietis, T Persei, *o* Ceti, S Persei, R Ceti, U Ceti, R Tauri, S Tauri, R Aurigae, U Orionis, R Lyncis, R Geminorum, S Canis Minoris, R Cancri, S Hydrae, T Hydrae, R Ursae Majoris, X Virginis, R Comae, T Virginis, Y Virginis, T Ursae Majoris, R Virginis, S Ursae Majoris, U Virginis, R Hydrae, S Bootis, R Camelopardali, U Herculis, W Herculis, R Ursae Minoris, R Draconis, χ Cygni, S Cygni, R Delphini, U Cygni, V Cygni, T Aquarii, T Cephei, S Cephei, SS Cygni, S Aquarii, R Pegasi, S Pegasi, R Aquarii, and R Cassiopeiae, and it is expected that the others will be finished in a few months. An attempt is made at this Observatory to compare, by Argelander's method, the brightness of each of these variable stars once a month. If astronomers elsewhere would reduce their observations to the same scale of magnitudes, a uniformity in results would be obtained which is now unfortunately lacking. These measures are now being extended to other variable stars of long period, and it is hoped that later all stars of this class may be observed regularly and according to a uniform system.

Photometric magnitudes, determined with the meridian photometer, can also be furnished of many other stars brighter than the tenth magnitude, besides those mentioned above. Volumes XIV, XXIII, XXIV, and XXXIV of the *Annals* give the magnitudes of all stars from the North to the South Pole, brighter than the sixth magnitude, besides many fainter stars generally distributed in zones at regular intervals of five degrees in declination. Later observations have been made of all stars of the magnitude 7.5 and brighter, north of the declination $-40°$. A redetermination of the brightness of all the stars in the *Harvard Photometry* is included in this work.

MISCELLANEOUS NOTES.

The variability of a star in the constellation Aquila, whose position is in R. A. $19^h\ 33^m.3$, Dec. $+ 11° 29'$ (1900), has recently been announced by the Rev. T. D. Anderson (*A. N.*, **145**, 79). Measures of fifty-seven photographs give the maximum brightness 9.2, minimum < 12.9. The variations can be closely represented by the formula, *J. D.* $2,411,550 + 330\ E$.

The Rev. T. E. Espin, in *Wolsingham Observatory Circular*, No. 45, calls attention to a red star of the eighth magnitude in R. A. $8^h\ 12^m\ 16^s$, Dec. $+ 32° 19'$ (1855), not contained in the *Durchmusterung*.

This star appears on thirty photographs taken from December 3, 1889, to March 5, 1898, and no variation in light exceeding two or three tenths of a magnitude is indicated. The individual results differ from their mean by $\pm$ 0.13 mag.

EDWARD C. PICKERING.

MARCH 7, 1898.

THE NEW DIRECTOR OF THE LICK OBSERVATORY.

IT is a great pleasure to be able to announce, just as the JOURNAL is going to press, that Professor Keeler has decided to accept his recent appointment as Director of the Lick Observatory. The writer takes the liberty of severing his editorial relations for a moment, in order that he may offer his heartiest congratulations to both Professor Keeler and the Lick Observatory on this important event. In spite of a meager equipment, used with difficulty in the smokiest region in the United States, Professor Keeler has obtained results of the highest order since leaving Mt. Hamilton seven years ago. He will now have at his disposal the perfect instruments employed in his well-known researches on the motion of the nebulæ in the line of sight. It may be permitted one whose acquaintance with Professor Keeler is not confined to a knowledge of his scientific work, to say that the Regents of the University of California have chosen wisely in selecting a new director. The future will show that the interests of the Lick Observatory have been placed in competent hands.

GEORGE E. HALE.

Reviews.

Die Photometrie der Gestirne, von Professor Dr. G. Müller, Observator am Königlichen Astrophysikalischen Observatorium zu Potsdam. (Pp. x+556, 81 figures. Leipzig: Engelmann, 1897).

The oldest branch of astrophysics, celestial photometry, although prolific in periodical literature, has been the subject of singularly few books. The works of Bouguer, Lambert, Seidel, and Zöllner have successively recorded progress, but until now no complete manual of this important branch has been published. The present work, thus practically the first in the field, deals with the subject in so comprehensive, thorough, and satisfactory a manner that it is difficult to speak of it without enthusiasm.

The book is divided into three parts: I. The elements of theoretical celestial photometry (pp. 1–144); II. Photometric apparatus (pp. 145–304); III. The results of photometric observations of the heavens (pp. 305–510).

The first chapter gives a clear mathematical exposition of the fundamental laws of photometry, with an important section on physiological intensity and Fechner's (or Weber's) psycho-physical law, which expresses the fact that the subjective impression is proportional to the logarithm of the objective brightness. Other sections take up the illumination of surfaces from luminous point sources and from surface sources; and Lambert's theorem that the quantity of light is proportional to the cosine of the angle at which it emanates, for which a proof by Lommel, rigorous for self-luminous, opaque objects, is given. Diffuse reflection is considered, and it is shown that for substances so reflecting Lambert's theorem has no validity. Bouguer's theory of reflection is taken up, and its extension, with limitations, by Seeliger is explained; and finally the development is given of the "Lommel-Seeliger" law of illumination, which shows the dependence of the amount of light reflected upon the absorptive and diffusive power in the interior of the substance and upon the angles of incidence and emergence in the form

$$\frac{\cos i \cos \epsilon}{\cos i + \lambda \cos \epsilon},$$

(λ being the ratio between the absorptive power on entering and on leaving the reflecting substance). The difference between the two laws of reflection is illustrated by taking the simple case of vertical incidence ($i = 0$) upon a plane surface. According to Lambert's theory the apparent brightness would be the same when viewed from all angles (ϵ), while the theory of Lommel and Seeliger indicates that when viewed from a point (nearly) in its own plane the apparent brightness would be but half of that when viewed perpendicularly. Seeliger's somewhat complicated mathematical developments are followed until the final expression is obtained for the light received by the eye from the surface element *ds*, namely,

$$Q = \frac{1}{4\pi k} \mu L \, ds \frac{\cos i \cos \epsilon}{\cos i + \cos \epsilon} \left[1 + \frac{\mu}{2k} \cos \epsilon \, lg \left(\frac{1 + \cos \epsilon}{\cos \epsilon} \right) + \frac{\mu}{2k} \cos i \, lg \left(\frac{1 + \cos i}{\cos i} \right) \right]$$

where k is the coefficient of absorption of the substance, μ its diffusive power, and L is the quantity of light from the source received at the unit volume of surface. From the symmetry of the equation with respect to i and ϵ, it would follow that the brightness should be independent of the azimuth—the same whether observer and source were on the same side or on opposite sides of the normal to the surface. But observations by Seeliger and by Messerschmitt show that the brightness is greatest when eye and source are opposite each other, *i. e.*, differ in azimuth by 180°. Hence even this formula is not accurate, and it can be regarded as only approximate.

The definition of *albedo* is next taken up in the light of these recent researches. The common use of the term is only correct where Lambert's cosine law obtains, for there the percentage of light reflected back to the eye (relative brightness) is the same for all angles of incidence; but for a law of illumination that involves both the angle of incidence and emergence, the albedo differs for each incidence and the term loses its significance. Seeliger has therefore given an expression to define the albedo, which after integration is independent of the angle of incidence, viz.,

$$A' = 2\pi C \int_0^{\frac{\pi}{2}} \tan i\, di \int_0^{\frac{\pi}{2}} f(i, \epsilon) \sin \epsilon\, d\epsilon,$$

wherein for $f(i, \epsilon)$ may be substituted the expression for any law of reflection.

For $f(i, \epsilon) = \dfrac{\cos i \cos \epsilon}{\cos i + \lambda \cos \epsilon}$ (Lommel-Seeliger),

the albedo after integration becomes

$$A' = \frac{\pi C}{\lambda} \left\{ 1 - \lambda\, ln\, \lambda + \frac{\lambda^2 - 1}{\lambda}\, ln\, (1 + \lambda) \right\}$$

in which C and λ are constants peculiar to each substance and depending upon its power of absorption and reflection, and ln signifies the natural logarithm.

In chapter II these fundamental principles are applied to the most important problems of celestial photometry: the illumination of planets and satellites; the illumination of a system of small bodies; the eclipses of Jupiter's satellites. Parallel developments of the necessary formulæ are given according to the three laws of illumination upon which they may be based, namely,

$$\text{Lambert's: } dq_1 = \Gamma_1\, ds \cos i \cos \epsilon$$

$$\text{Lommel-Seeliger: } dq_2 = \Gamma_2\, ds\, \frac{\cos i \cos \epsilon}{\cos i + \lambda \cos \epsilon}$$

$$\text{Euler's: } dq_3 = \Gamma_3\, ds \cos i,$$

in which dq is the quantity of light emitted at angle ϵ from the surface element ds, which receives its light under an angle i; the constants depending upon the intensity of the incident light, and upon the power of reflection, scattering (or "diffusion"), and absorption of the substance concerned.

The first section thus treats of the calculation of the amount of light we receive at different phases of planets, and of the determination of albedo; of the distribution of light on a planet's disk; of the illumination of satellites; of the calculation of the "Earth-shine" on the Moon. It is a decided merit of this work that it reproduces in so full a manner the important theoretical investigations of Seeliger, which are rather inaccessible to most observatories because of their publication in the Proceedings of the Bavarian Academy.

The second section of this chapter gives at considerable length Seeliger's theory of the illumination of Saturn's rings, by means of which the brightness of the system can be reduced to that for vanished rings. The eclipses of Jupiter's satellites occupy ten pages, and it is shown that the advantage of reducing all measures to the time of half brightness, as originally suggested by Cornu, holds good for any law of illumination. The third chapter of Part I is devoted to the extinction of light in the Earth's atmosphere, taking up in successive sections the theories of Lambert, Bouguer, Laplace, and Maurer, and then comparing the theories with observations, with an outcome favorable to the theory of Laplace. As a practical average value of the transmission coefficient Müller recommends 0.835.

Part II of the work gives in 160 pages a full description of the instruments employed in celestial photometry, classified according to their principle and illustrated by some forty admirable engravings. Both the theory and the practical working of the various instruments are clearly set forth, with valuable comments suggested by the large experience of the author. Spectral-photometers occupy a chapter, and actinometers, the bolometer, and other objective modes of measuring radiations are more briefly treated.

In Part III the photometric observations of the Sun, Moon, planets and satellites, comets, nebulæ and stars are successively considered, and the broad practical experience of the author is made available to the reader in a most useful way. The results of the various observers are subjected to a careful and judicial criticism. A special value of this part lies in the suggestiveness of the treatment, for the lines in which future work is needed are no less plainly brought out than are the records of past observations. Especially prominent is the need of more accurate photometry of the Sun, and, as the author states, "here a rich and promising field of activity remains open for the astrophysicist." For the ratio of brightness of the Sun and full Moon the author does not adopt the value of Zöllner usually employed in text-books (618,000), but prefers the mean of Zöllner's two results and Bond's (470,980), which gives 569,500. The wide range of these determinations sufficiently illustrates the need of more work in this direction. As the values of the albedo of the planets differ considerably from those current in many text-books, it may be worth while to give the author's determinations here, according to the definitions of Lambert and Seeliger :

	Lambert	Seeliger		Lambert	Seeliger
Moon	0.129	0.172	Uranus	0.604	0.805
Mercury	0.140	0.187	Neptune	0.521	0.694
Venus	0.758	1.010	Jup.-Sat. I	0.412	0.550
Mars	0.220	0.293	Jup.-Sat. II	0.489	0.652
Jupiter	0.616	0.821	Jup.-Sat. III	0.259	0.346
Saturn	0.721	0.961	Jup.-Sat. IV	0.118	0.157

Especially valuable sections are those dealing with stellar photometry, entitled photometric catalogues, variable stars, spectral-photometric observations of stars, and the photographic brightness of stars. In view of the great number of variable stars, the discussion in that section is necessarily somewhat limited. Tables are given for reduction of phase and of atmospheric extinction, followed by an extensive bibliography. As pecuniary reasons will presumably forbid an English translation of this work, we must most strongly recommend it for the libraries of teachers as well as of astronomers.

E. B. F.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

It is intended to publish from time to time a bibliography of astrophysics, in which will be found the titles of recently published astrophysical and spectroscopic papers. In order that this list may be as complete as possible, and that current work in astrophysics may receive appropriate notice in other departments of the JOURNAL, authors are requested to send copies of all papers on these and closely allied subjects to both Editors

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

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THEORETICAL CONSIDERATIONS RESPECTING THE DEPENDENCE OF WAVE-LENGTH ON PRESSURE WHICH MESSRS. HUMPHREYS AND MOHLER HAVE OBSERVED IN THE ARC-SPECTRA OF CERTAIN ELEMENTS.

By J. Wilsing.

The object of the following article is to show that the interesting results, which have been obtained by Messrs. W. J. Humphreys and J. F. Mohler[1] from their study of the influence of pressure on the wave-lengths of lines in the spectra of the metals, can be explained as an effect of damping of the vibrations to which the emission of light is due, and can be brought into connection with the views which have been developed by Herr v. Lommel[2] in his memoir "Theorie der Absorption und Fluorescenz," and independently by Herr G. Jaumann[3] in a memoir entitled "Zur Kentniss des Ablaufes der Lichtemission."

By raising the pressure in the vessel containing the arc to 15 atmospheres, Messrs. Humphreys and Mohler succeeded in producing displacements of the metallic lines which amounted in exceptional cases to 0.2 tenth-meter, and which were of the same

[1] "Effect of Pressure on the Wave-lengths of Lines in the Arc-Spectra of Certain Elements." This Journal 3, 114, 1896; 4, 175, 249, 1896; 6, 169, 1897.

[2] *Wied. Ann.*, 3, 251, 1878.

[3] *Wied. Ann.*, 53, 832, 1894; this Journal 2, 215, 1895.

order of magnitude as certain systematic differences between the wave-lengths of corresponding solar and metallic lines found by Mr. L. E. Jewell[1] in carrying out his measurements for Rowland's "New Table of Standard Wave-lengths." On increasing the pressure the lines were always displaced toward the red end of the spectrum, that is, their wave-length was increased; while diminution of the pressure below one atmosphere produced a displacement toward the violet end of the spectrum. The amount of the displacement was unequal for lines of different elements. In the case of the different series of lines in the spectrum of any element, it had a definite relation to the number expressing the order of the series. For any line of the same series the change of wave-length was moreover proportional to the change of pressure and to the wave-length itself. Relations were also found between the displacements of lines of elements belonging to the same half of a Mendelejeff's group. Of special significance, however, is the author's remark, that in general displacement was not accompanied by any widening of the lines, such as is usually produced when the density of the vapor is alone increased. On the contrary, the metallic lines have essentially the same width and the same appearance with and without pressure. An explanation of these displacements as the result of an unsymmetrical broadening of the lines is therefore not permissible, although, according to the observations of Ebert,[2] an apparent displacement toward the red end of the spectrum is in general likewise produced, as a consequence of the great broadening which accompanies an increase in the quantity of the radiating vapor. According to Kundt the displacement of the absorption bands of substances dissolved in transparent media is also in the same direction; the greater the dispersion of the non-absorbing medium, the greater is the displacement of the absorption bands. In a later article Mr. Humphreys calls attention to these phenomena, which have a certain analogy to the changes of wave-length caused by pressure, in that the direc-

[1] "The Coincidence of Solar and Metallic Lines." This JOURNAL **3**, 89, 1896.

[2] *Wied. Ann.*, **34**, 39, 1888.

tion of the displacement is the same in both. He also remarks, in the same place, with reference to v. Lommel's theory of absorption, that the theorem, "An increase of the density or the pressure of a radiating gas causes a broadening of the bright lines in its spectrum and at the same time displaces them toward the red,"[1] necessarily leads to the conclusion that a displacement would be produced by increasing the density of a gas without changing its pressure, which is contrary to experience. This conclusion rests, however, upon a misapprehension which has been corrected by v. Lommel himself in a note on the articles by Jaumann and Prince Galitzin; for the conclusion holds, of course, only when the logarithmic decrement k of the vibration has a value other than zero, or when in general there is damping. In the case of an ideal gas which obeys Mariotte's law, k is, however, according to v. Lommel, always vanishingly small. On the other hand, Humphreys' remark, that displacement *and* broadening must take place simultaneously, according to the theory of v. Lommel, and that this theory is consequently incapable of explaining the phenomena observed in the arc-spectrum when the pressure is increased, since the broadening of the lines must be very considerable relatively to their displacement, is perfectly pertinent in all cases. In fact, the formulæ of v. Lommel require modification or extension, if they are to include the possibility of displacement of lines in consequence of damping, *without* perceptible broadening of the lines.

Von Lommel, like Jaumann, proceeds from the assumption that homogeneous light consists of simple, pendulum-like vibrations. If the vibrations are damped, the corresponding disturbance of the ether is to be regarded as compounded of an infinite number of simple partial vibrations, and therefore capable of being analyzed by Fourier's theorem. But since, in accordance with the fundamental assumption, the resultant obtained by the summation of all these vibrations no longer yields homogeneous light, the broadening of the spectral lines is seen to be a necessary consequence of damping; without, however, implying

[1] *Wied. Ann.*, 3, 267, 1878.

the necessary truth of the inverse theorem, that damping is always present when the spectral lines are broadened.

Von Lommel takes, for the equation of a damped simple harmonic vibration which is also subject to the periodic impulse $-F \sin qt$,

$$m \frac{d^2 x}{d t^2} = -K \frac{d x}{d t} - A x - B x^2 - C x^3 - \ldots\ldots - F \sin qt,$$

where x denotes the distance of the atom m at the time t from its position of equilibrium. Division by m reduces the equation to the form

$$(1) \quad \frac{d^2 x}{d t^2} + 2 k \frac{d x}{d t} + p^2 x + b \epsilon x^2 + c \epsilon^2 x^3 + \ldots\ldots + f \sin q t = 0$$

where

$$\frac{K}{m} = 2 k, \ \frac{F}{m} = f, \ \frac{A}{m} = p^2, \ \frac{B}{m} = b \epsilon, \text{ and } \frac{C}{m} = c \epsilon^2.$$

If we neglect the terms containing the small quantity ϵ, which is only of significance in the theory of fluorescence, the integral of the equation is

$$x = M \sin (q t - \alpha) + N e^{-kt} \sin (r t + \psi),$$
$$r = \sqrt{p^2 - k^2},$$

where k is the coefficient of damping, and $\frac{\pi}{r}$ is the period of the damped main vibration of the atom. The amount of the molecular absorption Δ of the vibration $f \sin qt$ is determined by the expression

$$\frac{m}{2}\left(\int_0^t \left(\frac{d x}{d t}\right)^2 d t - \left[\int_0^t \left(\frac{d x}{d t}\right)^2 d t\right]_{f=0}\right),$$

which gives on integration

$$\Delta = \frac{m f^2 \sin^2 \alpha}{16 k^2}; \quad \tan \alpha = \frac{2 k q}{p^2 - q^2}.$$

The absorption of the vibration is therefore greatest when $q = p$, that is, when the period of the excitant radiation coincides with that of the atom vibrating *without* damping. According to the theory of v. Lommel and Jaumann, therefore, damping of

the vibrations can produce only a widening of the dark absorption lines and not a displacement, while according to the observations of Humphreys and Mohler bright and dark lines are equally displaced.

The position in the emission-spectrum of the damped fundamental vibration is determined by the equation $r = \sqrt{p^2 - k^2}$, where $\frac{\pi}{r}$ and $\frac{\pi}{p}$ denote the periods of the damped and undamped vibrations respectively. But the maximum of intensity in the widened bright line does not exactly coincide with the place of the main vibration. In order to find the distribution of intensity in the bright line widened by damping, we have to seek the amplitude of the partial vibrations of which the damped vibration is compounded. From the equation

$$e^{-kt} \sin rt = \frac{4kr}{\pi} \int_0^\infty \frac{\mu}{(k^2 + r^2 + \mu^2)^2 - 4r^2\mu^2} \sin \mu t \, d\mu$$

we obtain for the amplitude f of the vibration whose period is $\frac{\pi}{\mu}$ the value

$$f = \frac{4kr}{\pi} \frac{\mu}{(k^2 + r^2 + \mu^2)^2 - 4r^2\mu^2}.$$

The period $\frac{\pi}{\mu_o}$ which corresponds to the maximum of intensity in the widened bright line is determined by the condition $\left(\frac{df}{d\mu}\right)_{\mu = \mu_o} = 0$, therefore by

$$\mu_o^2 = \tfrac{1}{3}(p^2 - 2k^2) + \tfrac{2}{3}\sqrt{p^4 - p^2k^2 + k^4},$$

or,

$$\mu_o^2 = r^2 + \tfrac{1}{4}\frac{k^4}{p^2} + \ldots$$

When the damping of the vibration is slight, that is, when $\frac{k^4}{p^2 r^2}$ is very small, the maximum of intensity practically coincides with the place of the main vibration r.

Since the period $\frac{\pi}{r}$ of the damped vibration is connected

with the period $\frac{\pi}{p}$ of the undamped vibration by the equation $r = \sqrt{p^2 - k^2}$, it follows, according to v. Lommel's theory, and in agreement with the observations of Humphreys and Mohler,* that damping must always produce a displacement of the bright line toward the red end of the spectrum. If, however, we regard $\frac{k}{p}$ as a small quantity of the first order, the amount of the displacement is of only the second order, while the breadth of the line increases very rapidly as k increases. Therefore a perceptible displacement of the bright line cannot take place unless it is accompanied by a very great increase in the width of the line.

The result of the foregoing discussion of v. Lommel's theory of absorption is as follows: the theory is so far in agreement with observation, that it requires a displacement toward the less refrangible end of the spectrum to follow an increased damping of the vibrations. On the other hand it is contradictory to observation in that the dark absorption lines actually suffer the same displacement as the bright lines when the pressure is changed, and in that observation shows no perceptible accompanying increase in the breadth of the lines. These contradictions may, however, be easily removed by an extension of the theory, without impairing the validity of its conclusions when applied to the explanation of the phenomena of fluorescence.

The theory of v. Lommel starts from the assumption of a simple sine vibration and finds its chief support in the analogy with the vibrations of a pendulum in a resisting medium. Bessel's investigations of this subject have, however, shown that the consideration of the actual resistance, the effect of which is chiefly to decrease the amplitude of the vibrations, is not alone sufficient to determine the motion of the pendulum. On the contrary, observation showed such a considerable increase in the period of the pendulum in consequence of damping, that the period could no longer be represented by means of the observed logarithmic decrement k. An increase in the moment of inertia

of the pendulum became necessary, which was recognized by Stokes as an effect of the internal friction of the air particles. The increase of the period expresses, therefore, the reaction of the particles of air which are set in motion by the pendulum and are carried with it. Herr Ketteler has introduced Bessel's extension of the theory of damped vibrations into optics, in his work, *Theoretische Optik gegründet auf das Bessel-Sellmeier'sche Princip.* If the equation of the damped vibration is made fundamental for the explanation of optical phenomena, Bessel's extension cannot be left out of consideration in questions concerning vibration-numbers. So long, however, as this view is used merely as a working hypothesis, it may be left an open question as to what physical conceptions are to be formed of the damping of the light-vibrations; whether, according to the older view, it is to be regarded as a decrease of the intra-molecular motion, or, according to the electro-magnetic theory, as a loss of energy which the vibrations of the valence-charges of the atoms suffer in consequence of radiation.

Taking into account this principle of Bessel, the following is to be substituted for equation (1);

$$(2) \qquad \frac{d^2 x}{d t^2} + 2 k_1 \frac{d x}{d t} + p_1^2 x \dots\dots + f_1 \sin qt = 0,$$

in which the new are connected with the former coefficients by means of the equations

$$k_1 = \frac{k}{1+\gamma}, \; f_1 = \frac{f}{1+\gamma}, \; p_1^2 = \frac{p^2}{1+\gamma},$$

where γ is a small positive quantity.

The consequences which follow from the introduction of γ are readily seen. First, the expression now obtained for the amount of absorption, corresponding to the period $\frac{\pi}{q}$ of the incident wave, is:

$$\Delta = \frac{m f_1^2}{16 k_1^2} \sin^2 \alpha \,;\; \tan \alpha = \frac{2 k_1 q}{p_1^2 - q^2}$$

so that for $q^2 = p_1^2 = \dfrac{p^2}{1+\gamma}$ the absorption becomes a maximum,

which corresponds to the darkest part of the line. In consequence of this a displacement of the absorption line toward the less refrangible part of the spectrum takes place, which is measured by the difference of the periods $\frac{\pi}{p_1}$ and $\frac{\pi}{p}$. $\left(\frac{\pi}{p^1} - \frac{\pi}{p} = \frac{\pi\gamma}{2p}\right)$. The expression Δ for the intensity of the absorption corresponding to q,

$$\Delta = \frac{m f_1^2}{16 k_1^2} \frac{1}{1 + \frac{(p_1^2 - q^2)^2}{4 k_1^2 q^2}}$$

shows that the line remains narrow and sharp as long as $\left(\frac{k_1}{q}\right)^2$ is small. If, therefore, γ and $\frac{k_1}{q}$ are quantities whose squares may be neglected, the damping of the vibrations essentially produces merely a displacement of the absorption lines toward the red end of the spectrum without giving rise to a noticeable broadening of the lines.

A similar consideration applies to the bright lines of the emission spectrum. For the period $\frac{t}{r_1}$ of the principal vibration emitted by the luminous gas we obtain from equation (2)

$$r_1 = \sqrt{p_1^2 - k_1^2} = \sqrt{\frac{p^2}{1+\gamma} - \left(\frac{k}{1+\gamma}\right)^2}$$

and if $\frac{k_1}{p}$ and γ are, in accordance with the previous assumption, quantities whose squares may be neglected: $\frac{\pi}{r_1} - \frac{\pi}{p} = \frac{\pi\gamma}{2p}$. The bright lines will, therefore, be displaced toward the red end of the spectrum by the same amount as the dark absorption lines. From the general expression f for the amplitude of the vibration μ:

$$f_\mu = \frac{4 k r_1}{\pi} \frac{\mu}{(k^2 + r_1^2 + \mu^2)^2 - 4 r_1^2 \mu^2}$$

or, if we place $\left(\frac{k}{p}\right)^2 = 0$

$$f_\mu = \frac{4 k p_1}{\pi} \frac{\mu}{(p_1^2 - \mu^2)^2},$$

it further follows that the ratio of the intensity of the vibration μ, lying indefinitely near $\frac{\pi}{p_1}$, to that of the principal vibration $\frac{\pi}{p_1}$,

$$\frac{I_\mu}{I_p} = \left(\frac{f_\mu}{f_p}\right)^2$$

vanishes, and that, therefore, the lines experience no perceptible broadening.

The observations of Messrs. Humphreys and Mohler are, therefore, fully explained on the assumption of a damping of the vibrations, if the quantity γ which expresses the damping may be regarded as a function of the pressure which has a definite value for any series of vibrations of the same gas, but which varies for different series and different gases. An analogy to this hypothesis is found in the theory of damped electrical vibrations, in the respect that the damping of the latter vibrations is dependent upon the form of the resonator and the dielectric constants of the medium. Taking as a first approximation for the displacement,

$$\lambda_r - \lambda_p = \frac{\gamma_o P}{2} \lambda_r,$$

where λ_r and λ_p are the wave-lengths corresponding to the periods $\frac{\pi}{r}$ and $\frac{\pi}{p}$, and P is the pressure, we get, for $\lambda_r - \lambda_p = 2 \cdot 10^{-8}$ mm., and $P = 12$ atmospheres, $\lambda_r = 4 \cdot 10^{-4}$ mm., $\gamma_o = 8 \cdot 10^{-6}$, while $\left(\frac{k}{p}\right)^2$ is to be regarded as vanishingly small, since no broadening of the lines takes place.

The slightly damped vibrations which are emitted by the vapors of the volatile metals in the electric arc obey Kirchhoff's law, and only in the case of actual fluorescence of dissolved or solid substances, for which $\left(\frac{k}{p}\right)^2$ becomes appreciable, does a displacement of the bright bands relatively to the absorption bands take place. While metallic vapors under ordinary pressure give out only slightly damped vibrations, bright lines may be expected in their fluorescence spectrum, in addition to the broad

and diffuse bands which usually appear. Thus Messrs. E. Wiedemann and G. C. Schmidt[1] succeeded in detecting the yellow sodium lines in the fluorescence spectrum of sodium vapor, as well as fluted bands. In the case of other readily volatilized elements it appears, according to E. Wiedemann, that the presence of bright lines in the fluorescence spectrum is also possible. Of special interest would be the investigation whether these bright lines in the spectrum undergo a displacement with respect to the absorption lines, since this determination would make it possible to decide on the correctness of the assumption that $\left(\frac{k}{p}\right)^2 = 0$.

The investigations of Messrs. Humphreys and Mohler, the significance of which, in astrophysics, has already been pointed out by Mr. Hale,[2] deserve special interest, because, like the separation of lines observed in a magnetic field by Herr Zeemann, they demonstrate the possibility of a change in the period of vibration of the luminous molecules, while in the case of refraction and magnetic rotation of the plane of polarization there is merely a change in the velocity of propagation of the light waves. In this connection, the displacements effected by pressure have to be considered with reference to the changes in the periods of vibration, which are explained, according to Doppler's principle, by the mechanical motion of the source of light or of the observer in the line of sight. On the validity of this principle is based the method of determining the motions of the stars by measuring the displacement of lines in their spectra. If, however, displacement of the lines may also be brought about by physical causes, through definite influences to which the luminous atoms are subjected, and which may be characterized as damping of their vibrations, there arises the necessity for an investigation as to how we can separate that part of these

[1] *Sitz. d. Phys.-med. Soc. Erlangen*, November 12, 1895. This JOURNAL, 3, 207, 1896.

[2] "Note on the Application of Messrs. Jewell, Humphreys, and Mohler's Results to Certain Problems of Astrophysics." This JOURNAL, 3, 156, 1896.

changes in the periods of vibration which is due to mechanical motion from that which is due to physical causes.

The amount of the displacement of the lines which is due to the motion of the source of light, is proportional to the wave-length. The proportional factor is constant for all the lines in the spectrum of a star, and is identical with the ratio of the velocity of the star in the line of sight to the velocity of light; while the change of wave-length due to pressure is different for lines which belong to different elements and different series, although the lines may be in the same part of the spectrum. If, therefore, the metallic vapors in the absorbing layers of a star are subject to a noticeable pressure, *relative* displacements of the lines of different metals must occur, the amount of which, if the same mean pressure is assumed for all the vapors, can be made to furnish an estimate of the pressure itself. This supposition need not be regarded as contradictory to the measurements of Mr. Jewell[1] relative to the solar spectrum, since the differences in pressure, which he finds for the different metallic vapors present in the solar atmosphere, are so small that for the greater part they may be attributed to the uncertainty of the measurements. These measurements of the amount by which approximately the metallic lines in the arc-spectrum, at atmospheric pressure, are displaced with reference to the corresponding lines in the solar spectrum (0.01 to 0.02 tenth-meters), lead to the conclusion that there is a comparatively small pressure (about five atmospheres) in the Sun's reversing layer. The same conclusion very probably holds good for stars of the second type, whose physical condition evidently closely resembles that of the Sun. For these stars the displacement due to pressure, amounting to about 0.02 tenth-meters (which, according to Doppler's principle, corresponds to a velocity of 1.5 kilometers at λ 4000) will therefore hardly reach the limit of accuracy of Herr Vogel's[2] measurements of the motion of the stars in the line of sight.

[1] "Note on the Pressure of the Reversing Layer of the Solar Atmosphere." This JOURNAL, 3, 138, 1896.

[2] *Pub. Obs. Potsdam*, 7, 25.

In the case of stars of the first type, whose spectra contain but few and weak metallic lines, the displacements of the artificially produced hydrogen lines, with reference to the hydrogen lines in the star spectra, will in most cases have to be measured directly. The investigations of Messrs. Humphreys and Mohler do not include hydrogen; nor can it be assumed that the pressure of the gas has alone any considerable influence on the period of vibration, since, under increased pressure, its lines broaden so greatly that even at a pressure of less than one atmosphere the spectrum becomes almost continuous. The significance of the displacement of the hydrogen lines in a stellar spectrum, in accordance with Doppler's principle is, therefore, not in the least doubtful. Of special significance for investigations of stars of Type I are Herr Vogel's[1] measurements in the spectrum of Sirius, from which no appreciable displacement of the absorption lines, due to pressure, can be deduced. The measurements of the displacements of the $H\gamma$ line in the stellar spectrum, with respect to the corresponding line produced by means of Geissler tubes, gave the following values:

1891, March 21 — 14.9 kilometers (first plate),
March 21 — 15.9 kilometers (second plate),
March 22 — 15.0 kilometers,

15.2 kilometers,

while the determinations of the displacement of the fine iron lines in the star spectrum, with respect to the corresponding lines of the iron spectrum, photographed on the same plate, furnished the values:

1891, March 21 — 13.4 kilometers,
March 21 — 14.9 kilometers,
March 22 — 16.3 kilometers,

14.9 kilometers.

The slight difference between these numbers is, considering the peculiar difficulty of such measurements, to be ascribed

[1] *Pub. Obs. Potsdam*, 7, 75 and 82.

solely to physiological causes, so that there is no displacement of the metallic lines with respect to the hydrogen lines.

According to these views, an effect of the differences in pressure on the position of the dark lines in the spectrum is to be expected only at the present limit of precision for the most accurate determinations of wave-lengths in stellar spectra. Therefore, in the determination of the motion of the stars by Doppler's principle, the wave-length may still be regarded as constant.

This statement cannot be made with equal certainty in the case of the emission spectrum. Since the number of stars whose spectra contain bright lines is comparatively very small, it is evident, on this ground alone, that the physical conditions under which the vapors on the surface of these stars are luminous, is to be regarded as anomalous.

This is especially true with reference to new stars. The occurrence of pairs of lines seems to be characteristic of their spectrum, which arises from the superposition and relative displacement of the emission spectrum and the absorption spectrum of the gases. Here the bright lines lie on the less refrangible side of the dark lines, precisely as in the case of the bright bands of fluorescing substances. The agreement in the direction of these displacements for different stars, and their surprising amount, oppose considerable difficulties to the purely mechanical explanation based on the motion of one or more luminous bodies, and appear to favor more directly the assumption of an essentially physical cause.

ASTROPHYSICAL OBSERVATORY, POTSDAM.
February 1898.

CORRESPONDENCE OF THE PHOTOGRAPHIC DURCHMUSTERUNG WITH THE VISUAL.

By R. H. TUCKER.

THE first volume of the *Cape Photographic Durchmusterung* has been published, and it forms the most extensive contribution to the needs of the practical observer that the photographic process has furnished.

In considering the results as a basis for present use, and for further development, the true aim of the *Durchmusterung* must be kept plainly in view. It is, briefly, to give a census of the stars, which shall be complete down to a certain definite limit of brightness. To furnish positions, within such limits of error as shall make it possible to easily identify the stars, and to set for them for purposes of accurate determination. And, finally, to rank them on a scale of magnitudes, which shall be in accord with certain standards, and shall be in true proportion throughout. If these several purposes be set down according to their importance, the order would be: completeness, uniformity of scale, and exactness of position; the last ranking considerably below the other two. Each purpose is liable to fall short of perfection in attainment, from a variety of causes; no work is free from the evidence of our own fallibility. There may be expected a certain number of mistakes, as long as the human observer has aught to do with investigation, even if his part should, sometime, be reduced to the transcription only, of results. In this class of work, there will be also systematic and accidental errors, affecting both the scale of magnitudes and the exactness of the places. Whatever limits of error be adopted, there are more to be feared the chance combinations and mistakes, that will occasionally produce abnormal results. The scope of the *Cordoba Visual Durchmusterung* was designed to be the tenth magnitude, to which limit the enumeration was intended to be complete. With respect to the scope of the *Photographic*,

Dr. Gill states, on page xxiii of the Introduction, that "probably every catalogued plate shows every star which Argelander would have considered it essential to include in a *Durchmusterung* similar to his own, with the exception, perhaps, of a very few red stars of the fainter class."

The ninth magnitude of the visual scale would thus be the limit of the complete census contemplated; while, necessarily, in order to be sure of including that grade, fainter stars will be gathered in, to a considerable extent.

Internal evidence, of some weight, may always be derived from a work of this character, as to its successful fulfillment of purpose; besides the authority, which is given by the ability and repute of the designers and constructors. Some such evidence is furnished by the count of the grades of the scale, to be given, later, in the tables.

The discussion of the scale itself has been made in the second part of the Introduction, by Professor Kapteyn, with comprehensive thoroughness. The magnitudes of the *Cordoba Zone Catalogue* have been adopted as the standard, the reduction is differential, and the aim has been to produce, from the photographic plates, a scale of magnitudes which shall be the same as the visual scale.

The results of an earlier investigation, by the same author, upon the distribution and brightness of the stars, as shown by photography, are quoted. In discussing the star density, while the comparison with Schönfelds *DM.* gives a legitimate conclusion, I do not think that the same validity attaches to that made with Gould's Zones.

The aim of the former was a complete enumeration of the stars. The aim of the zone work, on the contrary, was to obtain a large number of stars, generally distributed. The following concise statement of Dr. Gould's, which I quote from a letter written in 1893, makes this quite plain: "The object in view was to get as many stars as possible — not to make an exhaustive catalogue down to any limit of magnitude."

With respect to the light of the stars, Kapteyn's conclusions

are, in brief, that the effect of stars of the same visual magnitude, upon the photographic plate, varies in different parts of the sky. And that "the stars of the Milky Way are in general bluer than the stars in other regions of the sky." Thus far, from his earlier investigation. Resulting from the detailed comparisons, in the Introduction, the conclusion is drawn, also, that "the uniformity of color for the fainter stars is apparently much greater than that for the brighter stars."

In his comparison of the *Photographic DM.* and the *Visual*, a marked difference is found between the respective scales, depending apparently upon the galactic latitude. The photographic scale is intended to reproduce that of the *Zone Catalogue.* Accordingly, if the magnitudes of the visual *DM.* be assumed to be correct, "then the fainter stars of the *Z. C.* have been estimated half a magnitude *brighter* in the Milky Way than at the poles of that belt."

It is possible that the scale of either may be subject to systematic variation. The advantage would seem to lie with the zone work, for conditions favorable to consistency of scale, were it not for the fact that the consistent estimation of magnitudes was recognized as so important a part of the other work; while in the zones it was subordinate to the exact and rapid determinations of position. The effect of a bright background, due to the accumulated mass of stars in the Milky Way, would naturally reduce the apparent brilliancy of individual stars, observed in an unilluminated field, for the *Durchmusterung.* For the *Zone Catalogue*, the artificial illumination, required for meridian circle observations, would be so much in excess of the sky illumination, that the effect of the latter would be imperceptible.

Nevertheless, I am inclined to believe that it was possible for the *DM.* observers, with the state of the sky visibly changing, as the Milky Way was entered, to carry the same scale into and through those dense regions, without sensible variation. The general conditions were good, and were, as far as possible, uniform. Certainly a great amount of practice was obtained, under constant exercise of the same faculties. My own experi-

ence included three-fifths of the observations, between — 22° and —42°, and exactly half the revision, devoted to the settlement of doubtful cases.

The scale of the visual *DM.* was recognized, during the progress of reduction, to have a fairly consistent difference from that of the *Zone Catalogue*. I derived all the constants of reduction for the twenty degrees, comparing every hour of observation with all the stars of the catalogue. This was, ordinarily, accomplished within the month following the observations; though a few stars, for the determination of the exact limits of the zone observed, were compared on the day immediately following.

This difference may be stated, in general, to have increased gradually, up to one quarter of a magnitude at about 9; so that our 9 corresponded best with a type between 9 and 9½ of the *Zone Catalogue*, the individual estimates of which were made originally to half magnitudes. The 10 of the *Zone Catalogue* was usually found to be brighter than the 9½, and it appeared probable that the few observations made of that estimate were of brighter stars, under unfavorable conditions.

On the other hand, considering the photographic work by itself, the effect of a bright background, upon the plates, should be taken into account. This might be similar to the effect of moonlight, or of previous exposure to faint light, referred to by Dr. Gill, p. xxiii of the Introduction. There seems to be need of much further investigation of such features of the photographic process. And it is suggested by the many interesting points, developed in the treatment of the results, in part second.

That the scale of the photographic *DM.* bears quite a different relation to that of the visual, for the Milky Way and for regions outside, will be clearly evident in the comparisons which will follow.

The photographic scale has been compared by Professor Kapteyn, for the part common to Schönfeld's *DM.*; and with the *Cordoba DM.* for portions of the belt —22° to —32°, about 1300 stars having been used in the last. The Cordoba volume,

for the ten degrees next south, was not available at that time for further comparison. This has suggested an extension to the field covered by the second volume of the visual *Durchmusterung*.

The first point that attracts attention is the evident break in the continuity of the photographic enumeration, that occurs at -34°. This corresponds with the employment of the repolished object-glass for the photographs; and, apparently, with the use of plates of a different manufacture. The number of stars per degree rises from an average of 6700 on one side, to 9500 on the other. If these figures be translated into the equivalent of a zone one degree wide at the equator, the average rises from about 8000 to nearly 12,000. If there were any difference between the adjoining degrees, with relation to the Milky Way, an explanation might thus be given for the change. But the zones, one degree in width, cross the branches of the Milky Way quite symmetrically, with respect to each other.

Turning now to the visual *DM.* there is no break in the uniformity of the enumeration at this point. The average equivalent, for an equatorial zone of one degree, is 20,200 for the whole extent from -22° to -42°; and the average deviation is 4 per cent., while it does not exceed 14 per cent. for any degree.

In other terms, the photographic enumeration increases from an average of 42 per cent. of the visual for the first twelve degrees common to both, up to 59 per cent. for the four degrees next south of -34°.

For these comparisons a belt from $20'$ to $30'$ of declination has accordingly been chosen from each of the two degrees, -33 and -34. Each belt extends through the complete circuit of twenty-four hours, and above 2500 stars are included, nearly double the number used by Professor Kapteyn for the northern portion. Each belt has been divided into parts, in order to represent the Milky Way separately. It is, perhaps, needless to say that in identifying the stars, a scrutiny is made of the minutes of declination, adjoining the exact limits of the zone chosen. The stars of the Uranometry have not been included.

The magnitudes of the visual *DM.* for these stars were taken from the Uranometry, although the brightness was always estimated at the time of observation. In the second part of the photographic Introduction, Professor Kapteyn calls attention to a possible difference in the relation between telescopic and naked eye estimates, as affecting the photographic adjustment required for the plates. This is one of the many interesting points that will repay investigation by those who are following the development of astronomical photography. As illustrating one aim of these comparisons, the testing of the general consistency of the scale, there is given here in Table I the results of a count of the magnitudes of the visual scale, by tenths, which I made while in Cordoba, and which has not before been published in its entirety. It embraces the first ten degrees, from 0^h to 5^h inclusive; and the first five degrees for 7^h, this last representing the dense part of the Milky Way. Nearly 37,000 stars were counted. A small portion of these results (2000 stars in the first five degrees of 0^h), forms a part of the *DM.* Introduction, and had been previously printed in the *Astronomical Journal*, No. 230.

The individual tenths are fairly represented, as shown by the third column of percentages. The summation is given in the fourth column. By inspection of the third column it will be seen that the 10 includes a percentage equivalent to a total enumeration to 10¼ inclusive, on the same scale as that of the stars above it.

On the hypothesis of distribution by magnitudes, which has been found to represent the northern hemisphere, down to 9 mag. there would be 323,000 stars in the area —22° to —42°, down to 10¾ inclusive. The visual *DM.* actually has 340,000 in this area. There are two hypotheses to be adopted, however, in making this comparison; the same distribution for the two hemispheres, and the validity of the law beyond 9 mag.

As illustrating the possible range of the visual observations, for the telescope used, it served for the identification of an asteroid noted as twelfth magnitude, upon one occasion. It had

TABLE I.

CORDOBA VISUAL DM.

Mag.	Stars	Per 1000	Sum
To 6.9	202	6	6
7.0	66	2	8
7.1	26	1	9
7.2	53	1	10
7.3	54	1	11
7.4	28	1	12
7.5	97	3	15
7.6	54	1	16
7.7	104	3	19
7.8	78	2	21
7.9	57	2	23
8.0	184	5	28
8.1	137	4	32
8.2	183	5	37
8.3	217	6	43
8.4	159	4	47
8.5	366	10	57
8.6	300	8	65
8.7	421	11	76
8.8	370	10	86
8.9	400	11	97
9.0	871	24	121
9.1	852	23	144
9.2	959	26	170
9.3	978	27	197
9.4	1220	33	230
9.5	2000	54	284
9.6	2305	63	347
9.7	3216	87	434
9.8	3704	100	534
9.9	3171	86	620
10.0	13966	380	1000
Total	36798	1000	

been the custom to observe some of the asteroids that came to opposition far south, and of which determinations were desired. For this particular one, the ephemeris place was so uncertain that repeated trials failed to identify it in the 13-inch telescope by reason of the large number of faint stars within the area. I proposed trying for it with the *DM.* telescope. Its

aperture, five inches, should have rendered a 12 mag. just barely visible. I swept over the region in the usual manner of observing for the *DM.*, taking only the faintest stars, and a few brighter ones for identification. The following night a similar sweep was made and the asteroid was at once picked out from the reduction sheets with a difference in place corresponding to the daily motion. I have always regretted not having again turned to the region to see how the asteroid compared in light with the faintest stars visible.

In Table I, the constant $b = 3.9$, in the expression $a\, b^m$ for the sum of the stars down to any magnitude is clearly confirmed for the grades 7, 8 and 9. If, however, we look for the place of 10.0 by extension of the same hypothesis, it would be found at 9¾ of the scale, and there would thus be an indication that the stars estimated fainter than that really belong to 10 mag. From the manner in which the catalogue magnitudes are derived, the direct arithmetical mean of the separate estimates, the 9.8 and 9.9 were all observed once at least as 10, and it is not improbable that these stars may be truly tenth magnitude. The scale would then include a number of stars equivalent to a complete enumeration to 10½ inclusive. This is confirmed again by finding the percentage which is one-fourth of the total, at 9½ of the scale.

It must be borne in mind here and in what follows that the 10 of the visual scale is a flexible grade, including all stars estimated as tenth magnitude and fainter. It was in no sense our object to record the faintest stars visible, but to include all of which there was any doubt with respect to being approximately near the limit. In taking the means, it is always reckoned as 10.0, although the actual estimation may have been, and undoubtedly often was, consciously, beyond.

Referring again to the number of stars included in the photographic *DM.*, the total for 19° would indicate an equivalent of the complete enumeration down to 10¼ nearly, according to the law of distribution for the northern hemisphere. This limit would be different for the two parts above and below

—34°; the law giving 10.1 and 10.5, respectively, for the areas each side.

In terms of the visual scale, 42 per cent. would fall at 9.7, and 59 per cent. would reach nearly to 9.9, or, in round numbers, the limit has been extended one-fourth of a magnitude, after passing —34°. The actual scope of the work, the limit of brightness to which it is complete, will be much above. The count of scale in Table IV, and the number of stars missing from the fainter grades, Table V, must be referred to, for evidence from within and without, as to the completeness.

With respect to the law of distribution, results greatly at variance with the accepted constants have been found in the course of charting faint stars for standards of magnitude with the 36-inch telescope of this Observatory. A series of twelve small charts were completed, in extension of the scheme, originally of Professor Pickering (*M. N.*, **45**, 124), to furnish standards for comparison. The stars were grouped here in five classes for convenience in charting: the first including all to 9 mag. and each class below extending two magnitudes. The extent of area covered is too small—two square degrees—to base any definite conclusions upon, but the count indicates an entire change in the constants used in summing up magnitudes brighter than 9. The results are given here in condensed form:

Mag.	Stars	Sum	Per cent.
To 9	16	16	.011
10 and 11	91	107	.076
12 and 13	192	299	.214
14 and 15	327	626	.447
16 and 17	770	1396	1.000

The *b* of the formula, computed by extending the summation of 9 mag. to 17 mag., would be 1.75; in place of 3.91, adopted up to 9 inclusive.

Taking up in order the results of the *DM.* comparison for —33°, Table II has been formed by adopting, first, the visual magnitudes as the standard and grouping the individual stars as follows:

The magnitudes 7 to 8.
The magnitude 8.0 alone.
The magnitudes 8.1, 8.2, 8.3 together.
The magnitudes 8.4, 8.5, 8.6 together.
The magnitudes 8.7, 8.8, 8.9 together.

And so on up to 10. This distribution by groups is somewhat an arbitrary one, since perhaps the 8.9 belongs more precisely to 9 as the nearest quarter. But as there is an acknowledged tendency to gravitate towards the even magnitudes in direct observation, the whole units have been given a separate class. The table shows that a fair distribution of both scales has been effected, but Table IV should be consulted for final illustration of this point.

There are tendencies, also, in the photographic scale to exclude certain tenths; and there is great divergence both as to the limits and the tenths chosen in different portions. A few illustrations may be given, In $8^h - 34°$, for the zone compared, there are omitted entirely 9.5, 9.7, 9.9, 10.1, 10.3 and 10.5, while the scale includes 10.7 and 10.8; and the tenths omitted here are recorded commonly for other portions. The recorded limits vary from 9.6 to 11.4 over stretches of considerable length. In the zone through $5^h - 34°$, 60 per cent. of the stars are 10 mag. or fainter, almost to the exclusion of faint estimates between 9 and 10. While in 6^h, next following, but 20 per cent. are as faint as 10, and those mainly confined to the first half hour. These variations are, however, to be clearly anticipated, since by means of the photographic process the scale, founded upon only the brightest stars, is to be extended down. Thus the exposure, the sensitiveness of particular plates, and other conditions, must be taken into account. These difficulties are referred to by Dr. Gill, and they are most extensively treated in Scheiner's recent book upon Celestial Photography. The same class of uncertainty arises whenever differential work has to be extended beyond the base upon which it rests.

In making up the means of Table II, the visual 10 has been reckoned at 10.0. The number of stars in each group is not a

test of the distribution by quarters, since there are not included the stars of either catalogue, omitted by the other. For the photographic *DM.*, these begin to mount up perceptibly after passing 9 mag.

TABLE II.

COMPARISON OF VISUAL AND PHOTOGRAPHIC MAGNITUDES.—33°.

Visual standard				Photographic standard				Mean difference		
Vis.	Phot.	Stars	V.-P.	Phot.	Vis.	Stars	V.-P.	Mag.	V.-P.	Wt.
7.52	7.88	47	—.36	7.62	7.52	30	— .10	7½	—.26	4
8.0	8.34	21	—.34	8.0	8.04	14	+ .04	8	—.19	2
8.23	8.40	23	—.17	8.22	8.10	36	— .12	8¼	—.14	3
8.50	8.74	68	—.24	8.51	8.56	68	+ .05	8½	—.09	7
8.80	9.01	74	—.21	8.81	8.92	61	+ .11	8¾	—.06	7
9.0	9.15	77	—.15	9.0	9.05	59	+ .05	9	—.06	7
9.25	9.48	132	—.23	9.19	9.20	97	+ .01	9¼	—.13	11
9.52	9.70	241	—.18	9.51	9.42	216	— .09	9½	—.14	23
9.79	9.90	226	—.11	9.80	9.64	162	— .16	9¾	—.13	19
10.	10.08	111	—.08	10.0	9.70	91	— .30	10	—.18	10
				10.20	9.77	110	(— .43)			
		1020		10.45	9.73	58	(— .72)	Mean	—.14	
				10.80	9.75	12	(—1.05)			
				11.0	9.90	6	(—1.10)			

Seventeen stars from the *Uranometria Argentina* brighter than 7 have been omitted. Also eight differences of magnitude:

Vis.	Phot.
8.7	10.2
8.9	10.2
dup.	(2)
8.7	10.2
8.3	9.8
8.7	10.2
8.7	10.0
9.5	11.4

The same process is repeated for the second part of the table, except that the stars are combined with the photographic estimates as the standard. Thus each star is represented by a double comparison. The differences are taken out in all cases in the sense *Visual—Photographic*, and they thus conform to the

comparisons made by Professor Kapteyn. In the final tabulation of the Mean Difference, the results of the two preceding sections have been combined strictly according to the number of stars. The magnitudes may, then, be considered as representing types midway between those of the two scales; but the mean results are but slightly affected, by leaving out of account the systematic differences, in the combination. The scale of one section should be moved along upon that of the other in order to allow for systematic difference, and the magnitudes could thus be tabulated for the scale of either standard. The 10.0 of the photographic scale has been combined directly with the 10 of the visual for the final section, without carrying forward the fainter mean types of the photographic. The individual estimates have, however, been included in the first section with the visual standard. The last column of weights is made up with a basis of ten double comparisons as the unit.

The comparisons by the two standards show mutual progression of the two scales up to 10.0 of the photographic. From 10 to 11, however, there is apparently represented nearly the same type of the visual. But there is no way of distinguishing a star visually fainter than 10; and it is evident that if a group be taken, including any number of tens, the mean of the group will be brighter than 10 if one or more stars were estimated brighter. The 10 of the visual scale is probably best defined by the result in the first section alone.

The mean value for this table is, $V.-P.=-0.14$ mag., whether taken directly or with weights for the separate classes, and there is not much deviation from uniformity. The photographic estimates are fainter than the visual.

The explanation which has been given of Table II will apply, wholly, or in part, to the tables that follow. The method of combination, for the discussion of differences of magnitude, is the same that I used in all of the earlier comparisons of the scale with various catalogues, the results of which will be found in the Introduction to the *Cordoba DM.*

In order to represent the Milky Way separately it will not

be necessary to give the detailed results, in the full form of the table preceding. Accordingly there is condensed, in Table III, the final or mean differences, derived precisely in the same manner as those of the last section of Table II for the complete belt. The third and fifth columns include the Milky Way, and in each column is given the number, half the sum of the comparisons by both standards. The total for the column will be less than the whole number of stars compared, by one half the photographic estimates fainter than the 10.0.

The scale of the photographic *DM.* is seen to be in general one quarter fainter than the visual, outside of the Milky Way: while within those regions occur the only plus signs, indicating that the photographic is brighter than the visual. The change is more noticeable in the first crossing of the Milky Way.

TABLE III.

V.–P. GROUPS OF R. A. $-33°$

Mag.	1^h–5^h	No.	6^h–8^h	No.	9^h–15^h	No.	16^h–18^h	No.	19^h–23^h	No.
7½	—.10	7	—.40	4	—.37	12	—.28	9	—.10	6
8	—.15	3	—.26	2	—.29	4	—.20	2	—.09	5
8¼	—.20	7	—.11	4	—.17	10	+.04	3	—.10	4
8½	—.06	16	—.01	12	—.22	21	+.12	7	—.14	11
8¾	—.04	11	+.13	21	—.32	19	+.03	7	—.02	8
9	—.08	12	+.22	17	—.32	14	—.07	11	—.15	13
9¼	—.07	22	+.13	21	—.28	37	—.01	15	—.29	18
9½	—.28	40	+.16	57	—.30	52	—.08	44	—.28	34
9¾	—.31	24	+.15	54	—.20	29	—.17	64	—.43	22
10	—.51	9	+.03	46	—.33	5	—.23	31	—.38	14
Stars	173		246		219		229		153	

The dense parts of the Milky Way are: 7^h–8^h; and 17^h–18^h.

In Table IV, following, there is given the count of both catalogues for the zone compared, including in each, such stars as have been omitted from the other. The number of stars is given for the separate quarters, and the summation by quarters, with the percentages represented by these numbers.

The count of the visual scale closely resembles that of Table I, although the total includes but 7 per cent. of the number of stars. In comparing, the limiting tenths of the quarters must be reckoned: 8.3, 8.6, and 8.9 for 8¼, 8½, and 8¾. The agreement of the scale at 6.9, 7.9, and 8.9 with the constant *b* of the law of distribution, is excellent; but the tenth magnitude would evidently fall at 9.8 of the scale, by extending with the same constant.

The photographic scale, in the brighter part, does not conform to the theory, the stars from 7 to 8 falling behind. But this is possibly due to fainter estimates for the *U. A.* stars, which estimates have not been counted. The sums at 8¼ and 9¼ are in accord, and if all the faintest stars be grouped under magnitude ten, the 9 and 10 would agree with theory. In fact, the faint estimates are in such a small proportion to the stars above them, that one would be inclined to classify them farther up the scale, if uniformity were assumed. Considering the small variation of color, accepted for the fainter stars, there should be no reason for stars of 10¾ being photographed, when those of 9¾ do not produce an impression. And there is a noticeable falling off at 9½ of the scale, considered by itself entirely, which is ample internal evidence of the approach to the limit. It would appear that the expressions, by means of which the curves for magnitudes were extended below the standards, are not entirely in accord with the generally accepted increase in the distribution of the fainter stars. Of course a number of different plates are represented in this zone, but the characteristics of the whole belt are noticeable, as well, for smaller positions. In the dense parts of the Milky Way, the lower grades are, proportionally, much better represented.

As regards the scope of the photographic *DM.* the sums at 9 and 9½ would indicate, by the law of progression, that enough stars had been included up to this last grade. But by comparing the two catalogues the number at 9½ of the photographic is found to be not up to the scale of the visual, even allowing the proportional number of stars for the systematic

difference in scale. If, however, we take the sum at 9 mag. and make the proper allowance for the systematic difference (Table II) we find it covers the number at 9 visual, and the photographic *DM.* would appear to be complete to ninth magnitude of its own scale. Since the visual includes every star, estimated by the photographic as bright as 9.0, the completeness of the former at that point is pretty well assured.

TABLE IV.

COUNT AND PERCENTAGE. $-33^{\circ}\ 20'$ TO $30'$.

Visual DM.					Photographic DM.				
Mag.	Stars	Sum.	Per 1000	Sum.	Mag.	Stars	Sum.	Per 1000	Sum.
U. A.	17	17	6	6	U. A.	17	17	16	16
7–8	47	64	17	23	7–8	30	47	28	44
8	21	185	7	30	8	14	61	13	57
8¼	24	109	8	38	8¼	36	97	34	91
8½	68	277	25	63	8½	68	165	63	154
8¾	81	58	29	92	8¾	61	226	57	211
9	80	338	29	121	9	59	285	55	266
9¼	161	499	58	179	9¼	98	383	91	357
9½	395	894	143	322	9½	218	601	202	559
9¾	714	1608	260	582	9¾	169	770	157	716
10	1149	2757	418	1000	10	101	871	94	810
					10¼	121	992	112	922
					10½	64	1056	60	982
					10¾	12	1068	11	993
					11	7	1075	7	1000

The Uranometry stars include all to 6.9 mag.

There follows in Table V a list of the number of stars missing from either catalogue. It seems evident that the tenth magnitude has been generally included in the visual work. The missing stars occur entirely in the Milky Way, where the difficulty of observing all the stars in the densest parts would account for an occasional omission. For more than eighteen hours of the twenty-four the visual includes every star given by the photographic in this zone. Many of the stars missing

belong to pairs, or triple stars, for the separate components of which it would always be difficult to make distinct records. The photographic method presents here a decided advantage, in that no greater speed is required in measuring the most closely packed regions, and pairs can be picked apart at leisure. The rapidity of observation was not necessarily productive of bad estimates of magnitude, however; one's faculties are trained to alertness and to decision of judgment in such work, and it does not deteriorate with speed.

The stars missing from the photographic *DM.* are mainly fainter than 9 of the visual scale. The few exceptions may, very probably, be of stars estimated too bright in the visual work. All brighter than 9.2 lie outside of the Milky Way; and in general the proportion of missing stars is decidedly smaller in the Milky Way than in regions outside, corresponding with the evident difference in the relation of the two scales.

Of the visual 10 that have been included in the photographic work three-quarters for this zone are within the Milky Way.

TABLE V.

STARS MISSING. —33° 20′ TO 30′.

From Vis. DM.[1]			From Phot. DM.		
No.	Phot. Mag.		No.	Vis. Mag.	Sum.
1	9.1	This star is close to 4 mag.	1	8.7	1
2	9.4	One of these is close to 7 mag.	1	8.9	2
3	9.7	One belongs to very close pair.	3	9.0	5
2	9.8	Both belong to wide pairs.	7	9.1	12
1	9.9	Close to 7 mag.	2	9.2	14
21	10.0 to 10.5	Some of these belong to pairs.	20	9.3	34
—			22	9.4	56
30	Total	All of these stars are in the Milky Way, 5^h to 8^h; and 17^h to 19^h.	40	9.5	96
			90	9.6	186
			118	9.7	304
			98	9.8	402
			272	9.9	674
			1038	10.	1712

[1] I have looked up two cases of the brighter stars missing from the visual *DM.* with the meridian circle here. They were each of tenth magnitude.

The accidental error of the magnitudes deserves some consideration. The average difference between the respective estimates for the same star, and without taking account of the systematic difference between the two scales, is ± 0.30, including all between 7 and 10 of the photographic scale. The stars fainter than 10.0 of this last have not been included, in the mean; they appear to belong to the grade of faint ten.

The 10 of the visual has been rated at 10.0, and since but 10 per cent. of the visual tens are included in the photographic in this zone it seems probable that those included belong to the brightest class, near an actual 10 of the visual scale. The average difference does not show much variation between different parts of the scale, but is sensibly smaller for the faint stars from 9½ to 9¾.

The probable error of a magnitude, for stars of 10 or brighter, taken from either catalogue, would be ± 0.2 assigning the same weight to both authorities.

At the same time that these comparisons of magnitude were made, the difference between the respective places has been taken out. The average difference in right ascension is $\pm 0^s.90$. There is a considerable increase within the Milky Way, where it slightly exceeds 1^s. No account has been made of systematic difference, but there is a decided tendency towards *smaller* right ascensions in the faint stars of the visual. This is unquestionably due to the fact that the stars were generally recorded at the time of extinction behind the opaque bar in the field of view. The faint stars disappeared at contact, nearly, while the brighter ones required a perceptible interval to pass from sight. The brighter stars, from which the reduction constants were necessarily derived, would thus be recorded too late with respect to the others. This difficulty was recognized at the time, and it was intended to record the *bisection* of the bright stars by the edge of the bar. In the most hurried portions there was often not time to watch individual stars steadily, to obtain real bisections, and it is in these portions that the systematic difference is most marked. It amounts to one second,

evidently, for considerable stretches of the Milky Way, the faint stars requiring a *plus* correction to the right ascension.

There are often a number of stars coming to transit within one second. In observing such combinations two or more were recorded for the same tap of the chronograph key; but confusion between observer and recorder, or in the observer's recollection of the group, was apt to result from combining more than two. It was accordingly necessary, at times, to tap consciously too early for some stars, on the approach of a group, in order not to have the taps too close on the chronograph, and in order to get in the necessary calls for the recorder without confusion. A star had often to be recorded when it had fairly passed contact, by including it in the record for the last preceding star. Thus there enter exceptional sources of occasional large errors in the visual right ascensions, especially in dense groups of stars. Less than 1 per cent. of the differences have been excluded in making this comparison, which may be taken to fairly represent the limit of accidental error. The *probable errors* to be derived from the material, adjusting the respective errors in accord with the weight derived from direct comparison with catalogue places, would be

p. e. Phot. $\pm 0^s.4$
p. e. Visual $\pm 0^s.7$

With respect to declinations the speed required in the denser regions had, in general, much less injurious influence; it was simply a matter of accustoming oneself to rapid and decisive estimates of tenths of the divided scale. The average difference of declination is $\pm 0'.34$, slightly larger in the Milky Way than outside, no systematic error being evident, and without rejecting any discordant differences. As *probable error* this would be distributed between the two catalogues:

p. e. Phot. $\pm 0'.06$
p. e. Visual $\pm 0'.28$

It must be borne in mind that the unit of measure in the visual work is one minute of arc. The scale was divided into $10'$ spaces, and estimated to tenths only. It is evident that if

no errors were made in the observation, and the nearest tenth *invariably* recorded, the average error of a single determination of declination would be one-quarter of a minute, and one in five would have an error of half a minute.

It is not likely that any observer records the nearest tenth invariably. There is often a distinct personality, by which certain tenths are more often chosen. Thus between 2 and 3 and between 7 and 8 the uncertainty can well become real. I made tests, by counts of the tenths estimated by myself, at various times. The only noticeable tendency was to avoid the 5. As the point exactly half way is certainly 5, I apparently estimated the slightest shade either side of that, as 4 or 6. The declinations of the photographic work have been measured with the tenth of a minute as the unit. In considering the results of a comparison between the two systems, this should not be lost sight of; the absolute uncertainty of a determination, aside from the errors of observation, or of measurement, is ten times as large in one case as in the other.

For the declinations of the *Bonn Southern DM.*, Schönfeld used a scale divided into spaces $4'.5$ long. His unit of measure, consequently, comes between the other two. This point has nowhere been in evidence, in the comparisons which I have seen.

The probable errors, derived from comparisons of the brighter stars with catalogue places, are given in the Introduction to the Cordoba *DM.* All of those comparisons were made by me, and I see no need of further extending or of modifying them. But the accidental error of observation is best derived from the comparison of the individual determinations, and it is no larger for faint stars than for the brighter ones.

From a large number of zones, discussed at various stages of the work, and from the compilation of the separate determinations, the following values represent the average probable error of one observation:

p. e. R. A. $\pm\ 0^s.7$.

p. e. Dec. $\pm\ 0'.3$.

In the catalogue, the comparison with the photographic shows that differences between the right ascensions of both, amounting to 3^s, are not uncommon, including the systematic difference. In declination the maximum difference is about $2'$, and it rarely exceeds $1'$. These errors, and their limits are matters of importance to those using the catalogues ; there is often an evident lack of judgment with respect to the positions. The uncertainty may be safely stated as not exceeding, on the average, the respective units : one quarter of a magnitude in brightness, one second of time in right ascension, and one minute of arc in declination. These may be fairly said to conform to the purpose of the work, and further refinement is not essential.

The results for the $10'$ zone $-34°$ can be briefly summed

TABLE VI.

COMPARISON OF VISUAL AND PHOTOGRAPHIC MAGNITUDES. $-34°$.

Visual standard				Photographic standard				Mean difference		
Vis.	Phot.	Stars	V.−P.	Phot.	Vis.	Stars	V.−P.	Mag.	V.−P.	Wt.
7.51	7.88	47	−.37	7.56	7.58	32	+.02	7½	−.21	4
8.0	8.27	21	−.27	8.0	8.04	9	+.04	8	−.17	1
8.27	8.54	35	−.27	8.20	8.01	30	−.19	8¼	−.23	3
8.51	8.79	65	−.28	8.51	8.52	72	+.01	8½	−.13	7
8.78	9.06	77	−.28	8.81	8.93	75	+.12	8¾	−.08	8
9.0	9.24	77	−.24	9.0	9.01	59	+.01	9	−.13	7
9.24	9.41	145	−.17	9.20	9.31	145	+.11	9¼	−.03	14
9.53	9.69	323	−.16	9.51	9.46	275	−.05	9½	−.11	30
9.80	9.90	411	−.10	9.79	9.64	248	−.15	9¾	−.12	33
10	10.15	295	−.15	10.0	9.74	202	−.26	10	−.19	25
		———		10.19	9.84	176	(−.35)		———	
		1496		10.47	9.87	126	(−.60)			
				10.78	9.91	47	(−.87)	Mean	−.14	

17 stars brighter than 7, from the *Uranometria Argentina*, have been omitted. Also 6 differences of magnitude.

Vis.	Phot.	Vis.	Phot.
9.3	10.8	dup.	(2)
9.4	10.8	8.2	9.4
8.8	10.2	8.0	9.4

up, they have been derived in the same manner precisely as for the preceding degree. Table VI includes the comparison of magnitudes for the whole belt. The mean difference is —0.14, or by weight —0.125, the photographic scale being brighter throughout. The differences, with the visual estimates as standard, indicate a more nearly uniform progression in the types. The final mean difference should have largely eliminated variations due to individual estimates.

The average difference of magnitude for the two scales from 7 to 10 of the photographic, without taking account of systematic difference, is ±0.29. This varies, however, from ±0.18 in one branch of the Milky Way, to ±0.40 in the other; the values for stars outside being in accord with the mean. The average difference in right ascension is $\pm 0^s.99$, ranging from ±0.93 outside the Milky Way to $\pm 1^s.11$ within, where the systematic error for the faint stars is most marked. The mean difference would be much decreased by application of the systematic correction. The average difference in declination is $\pm 0'.33$. These differ but slightly from the preceding degree, and no further discussion appears necessary.

TABLE VII.

V.–P. GROUPS OF R. A. —34°.

Mag.[1]	0^h–6^h	No.	7^h–8^h	No.	9^h–11^h	No.
7½	—.01	8	.....	..	—.32	9
8	+.04	5	.00	1	+.10	1
8¼	+.01	8	—.11	3	—.41	3
8½	—.07	17	+.15	4	—.32	9
8¾	—.06	15	—.11	7	—.45	8
9	—.38	11	+.03	10	—.24	10
9¼	—.20	22	—.01	17	—.15	14
9½	—.27	39	+.02	36	—.16	51
9¾	—.28	38	—.03	66	—.27	62
10	—.44	14	—.19	90	—.30	57
Stars	200	..	290	..	288	..

[1] The Milky Way is 7^h to 8^h; and 17^h to 18^h.

TABLE VII — *Continued.*

V.–P. GROUPS OF R. A. — 34°.

Mag.[1]	12^h–16^h	No.	17^h–18^h	No.	19^h–23^h	No.
7½	—.45	9	—.06	6	—.13	6
8	—.43	3	+.10	1	—.31	2
8¼	—.41	7	—.26	2	—.27	8
8½	—.26	15	+.35	6	—.22	16
8¾	—.33	17	+.32	16	—.07	11
9	—.24	13	+.24	10	—.19	8
9¼	—.30	28	+.44	37	—.23	26
9½	—.21	67	+.17	60	—.22	45
9¾	—.21	61	+.19	60	—.22	41
10	—.18	48	+.31	24	—.37	14
Stars	291	..	233	..	194	..

[1] The Milky Way is 7^h to 8^h; and 17^h to 18^h.

In Table VII the material is divided to represent the Milky Way separately. The difference between the scales is evidently about one-quarter of a magnitude outside the Milky Way. In the first branch, the difference is extremely small, up to 10 mag. In the second, the photographic estimates are one quarter brighter than the visual, for the greater part. There is thus represented a change, in the relation, of a full half magnitude.

The zone through 8^h of right ascension, in this degree, probably exhibits the photographic *DM.* at its best, and deserves some special consideration. There are but few bright stars, comparatively; the scales differ systematically less than one tenth, and the visual 10 is estimated 9.9 of the photographic scale (8^h only included); while the average difference is but $\pm$ 0.18 per star.

The actual number of stars in each list is important, and is a good criterion for each, in this case. The photographic has 208 stars, of these the visual has omitted 2 of 9.8 and 33 of magnitudes below 10.0. The visual has 205 stars, of which the photographic has omitted 1 of 9.7, 1 of 9.9, and 30 of 10. In other words, there are missing, from either list, barely more than a few stars of the faintest grades ever included. There may be made objection, that there is here no actual standard of

comparison. But this is a case for the exercise of practical judgment, rather than for the logic of figures. With two extensive lists, the portion in each representing some of the densest regions observed, each list being subject to systematic and accidental errors, and to mistakes of record and of observation, the mutual agreement indicates a degree of efficiency nearly approaching the complete purpose of the work. The photographic has to contend with the errors introduced by color, and with differences in the plates, the exposure, and in conditions of the sky. The visual has mainly to contend with the difficulties arising from the necessary rapidity of observation and estimate. The number of stars rises above 30 per minute, in this hour, for a degree in width. This distribution is, however, much exceeded in the other branch of the Milky Way.

This zone, through 8^h, is the only case in which the photographic *DM.* includes more stars in one hour than the visual, for the regions compared. From 100 per cent. in this hour, the photographic falls to 20 per cent. of the visual, in other hours of the same degree. This is a large variation; it implies a difference of more than a whole magnitude in the scope of one list, or the other, for different portions.

Table VIII gives the number of stars, and the corresponding percentage, in each list. The percentages of the visual will be found to be in accord with Table I.

The count of stars, in the photographic evidently begins to fall off at 9½. If we compare the count for the two at this point, it will be seen that by adding a sufficient number of stars to correspond with the systematic difference in the scales of magnitude, the photographic summation will be brought up to the visual. The photographic total is 59 per cent. of the visual, precisely the same as for the mean of the four degrees south of — 34.

For the contents of Table IX, in general, no explanation is required. But the zone through 17^h and 18^h deserves treatment apart. The number of faint stars, missing from the visual, exclusive of 17^h and 18^h is three times as large as for the

TABLE VIII.

COUNT AND PERCENTAGE. —34° 20′ TO 30′.

Visual DM.					Photographic DM.				
Mag.	Stars	Sum.	Per 1000	Sum.	Mag.	Stars	Sum.	Per 1000	Sum.
U. A.	17	17	6	6	U. A.	17	17	10	10
7–8	47	64	17	23	7–8	32	49	19	29
8	21	85	8	31	8	9	58	5	34
8¼	37	122	13	44	8¼	30	88	18	52
8½	66	188	24	68	8½	72	160	44	96
8¾	77	265	28	96	8¾	75	235	46	142
9	78	343	28	124	9	59	294	36	178
9¼	149	492	54	178	9¼	145	439	89	267
9½	381	873	137	315	9½	300	739	182	449
9¾	740	1613	266	581	9¾	264	1003	161	610
10	1163	2776	419	1000	10	209	1212	127	737
					10¼	206	1418	125	862
					10½	146	1564	89	951
					10¾	81	1645	49	1000

The Uranometry stars include all to 6.9 mag.

degree preceding. There are none missing from 21^h to 6^h, inclusive.

The photographic now includes 25 per cent. of the visual 10; and it omits but one-fourth as many of the stars to visual 9¼, as in the preceding degree, and but one-third as many to 9½.

The photographic scale is sensibly different, in 17^h and 18^h, from that in any other region compared. The limit of the estimates for the full degree, from $17^h\ 29^m$ to $18^h\ 45^m$, is 9.8, and for the last half of 17^h it is 9.6. At the point of division, midway of 17^h, the photographic scale changes one-half mag. with respect to the visual, at the same time that the limit drops from 10.2 to 9.6.

This corresponds with the omission of a comparatively large number of stars from the visual *DM.* They lie, mainly, within the limits of the cluster, 7 Messier in Scorpius, which is included in Gould's photographic clusters. But one star of those missing in this zone from the visual *DM.* is in the *Cordoba General Cata-*

logue clusters, recorded as 10 mag., and this star is not in Gould's photograph. The estimates of the photographic *DM.* are a full half magnitude brighter than the estimates of the *G. C.* cluster in this section of the zone.

TABLE IX.

STARS MISSING. —34° 20′ TO 30′.

From Vis. DM.			From Phot. DM.		
No.	Phot. mag		No.	Vis. mag.	Sum.
1	9.6	These 5 stars are in the first branch of the	1	8.5	1
4	9.8	Milky Way, 7^h to 9^h.			
			1	9.0	2
7	10.0	Of 91 stars, 10 mag. and fainter, 68 are in the	1	9.2	3
30	10¼	first branch of the Milky Way, 7^h to 9^h.	5	9.3	8
20	10½		7	9.4	15
34	10¾	Many missing stars belong to pairs.	15	9.5	30
			36	9.6	66
96		Exclusive of 17^h and 18^h.	91	9.7	157
			58	9.8	215
24	9½	In 17^h. Phot. limit 9.6.	180	9.9	395
12	9¾	In 18^h. Phot. limit 9.8.			
			868	10	1263
36		See reference to these stars in text following.			
132		Total.			

In observing these clusters with the meridian circle, estimates were made as faint as 11. I was unable to reconcile the extremely faint estimates, made by the other observers, with my own scale based upon previous practice with the larger telescope of the Albany meridian circle; but the observation of so many faint stars, at this time, probably established the uniformity of the scale down to magnitude 10.

The evidence seems to be pretty conclusive, that for this special section of the photographic *DM.* the faintest stars are recorded much too bright; and that the visual has omitted only such stars as ordinarily were below its limit.

The number of stars per degree reaches 60 for one minute of right ascension in 17^h. Such thick clusters were occasion-

ally observed by narrowing down to a zone of 20′ in the *Durchmusterung* work, making it entirely feasible to obtain all the stars that were recognized as being within its scope.

There remains the direct exhibition of the two adjoining degrees, and this is effected in the columns of Table X. The summation of the count to each quarter magnitude is given, and the difference in the scales for each type.

The two degrees of the visual are in noticeable accord with respect to the summation. The same may be said of the photographic up to 9. At 9¼ is indicated the commencement of the increase in the extension of the photographic record, which becomes more strongly marked as the faint end is approached. The scales are, however, still in accord for the same method. The systematic difference between the visual and photographic scales has virtually the same character in both degrees.

The summation of the photographic will be close to that of

TABLE X.

COMPARISON OF −33° AND −34°.

Mag.	Visual DM.		Phot. DM.		Vis.—Phot.	
	Sum. −33	Sum. −34	Sum. −33	Sum. −34	Δ Mag. −33	Δ Mag. −34
U. A.	17	17	17	17	——	——
7–8	64	64	47	49	−.26	—.21
8	85	85	61	58	−.19	—.17
8¼	109	122	97	88	−.14	—.23
8½	177	188	165	160	−.09	—.13
8¾	258	265	226	235	−.06	—.08
9	338	343	285	294	−.06	—.13
9¼	499	492	383	439	−.13	—.03
9½	894	873	601	739	−.14	—.11
9¾	1608	1613	770	1003	−.13	—.12
10	2757	2776	871	1212	−.18	—.19
10¼			992	1418	——	——
10½			1056	1564	−.14	—.14
10¾			1068	1645		
11			1075			

the visual, at 9 in —33°, and at 9½ in —34°, if the number of stars, proportional to the systematic difference in magnitude, be added.

A short list of errors in print, for the two volumes that have been under comparison, is given here. No mistakes have been assumed in making the identification of the stars, though occasionally the adoption of a 10′ error would have brought the places of a faint star from each list into agreement.

The two cases of 10′ discrepancies noted below may very probably be real mistakes, they are of the nature most likely to occur in the direct observation as a result of misunderstanding by the recorder, or of actual error in count by the observer. If there are two mistakes, and both in one catalogue, it is a small number to be detected in the rigorous comparison of 2500 stars.

CORRIGENDA.

CORDOBA VOL. XVII.

Page	Star	Mag.	
105			Head of first column, read 15^m, in place of 14.
124	1920		read *U. A.* in place of *G. C.*
137	5903	9.9	The figures have slipped down to the following star.
	5904	9.5	
	5905	7.6	
	5906	9.9	

PHOT. DM. VOL. I.

Omission of catalogue reference.

Page	Star	
477	3867	*Z. C.* 9
485	5830	*Z. C.* 9
514	6433	*Z. C.* 9½ but the Δ Dec. is 2′ Vis. *DM.* has two stars.
525	8763	*G. C.* 9½ but the Δ R. A. is 3^s. Precedes 7¾ closely.

10′ DISCREPANCIES.

Visual.

		Mag.	Dec.
157	11826	9.8	35′.4
167	14821	9.4	21 .3

Photographic.

		Mag.	Dec.
517	6986	10.2	26′.2
525	8851	9.4	11 .7

As general conclusions the two scales may be stated to have a systematic difference of one quarter of a magnitude outside the Milky Way, the photographic estimates being the fainter.

This difference nearly disappears within the Milky Way and is for the following branch of $-34°$, actually reversed in sign, the photographic being one quarter brighter than the visual. The two portions of the photographic *DM.* north and south of $-34°$, have in general a consistent scale of magnitudes, but the full scope of the latter is from ¼ to ½ a magnitude greater. Completeness to the ninth magnitude is pretty well assured.

The extremely faint estimates of the scale belong, probably, to stars near the tenth magnitude, and the real extent of the photographic reproduction of the sky has much smaller variation than the difference of nearly two magnitudes (9.6 to 11.4), in the limit of estimates, would imply.

The influence of color is not greatly in evidence, unless the variation at the faint end, where it is least to be anticipated, is taken as indication.

The measurement of the plates is less likely to be influenced by systematic error in right ascension than was the direct observation. In either case the positions meet the requirements of the work with respect to accidental error and mistakes appear to be rare.

The evidence of the standing of the visual *Durchumsterung* is additional recompense for the employment of seven years in earnest and willing aid to its accomplishment.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
March 15, 1898.

THE CAUSTIC OF THE RIGHT PARABOLIC CYLINDER.

By C. W. Crockett.

Let OP be the parabolic director, O the principal vertex, F the focus, and P any point on the arc. Let SP be the direction of the rays of light, parallel to the plane of the parabolic director and making the angle a with the axis OF; then PD will be the reflected ray corresponding to SP.

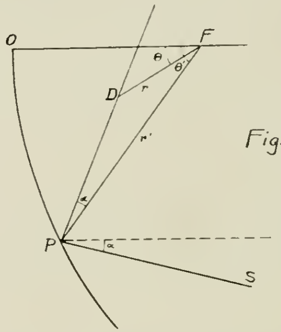

Fig. 1

With the pole at F and the initial line extending from F towards O, let

$r', \theta' =$ coördinates of P,

$r, \theta =$ coördinates of any point D on PD,

and $\frac{1}{2} p = OF$.

Then from the equation of the parabola,

$$r' = \frac{p}{1 + \cos \theta'} = \tfrac{1}{2} p \sec^2 \tfrac{1}{2} \theta' \quad - \; - \; - \; - \quad (1)$$

From the triangle PDF,

$$r \sin (a + \theta' - \theta) = r' \sin a ;$$

$$\therefore r \sin (a + \theta' - \theta) - \tfrac{1}{2} p \sin a \sec^2 \tfrac{1}{2} \theta' = 0 \quad - \quad (2)$$

Equation (2) is the equation of the ray reflected from the point whose vectorial angle is θ'. To find the equation of the caustic, θ' must be eliminated between (2) and its first derivative with respect to θ'. This derivative is

$$r \cos (a + \theta' - \theta) - \tfrac{1}{2} p \sin a \sec^2 \tfrac{1}{2} \theta' \tan \tfrac{1}{2} \theta' = 0 \quad - \quad (3)$$

Substituting the value of $\frac{1}{2} p \sin a \sec^2 \frac{1}{2} \theta'$ from (2) and reducing,

$$\cot (a + \theta' - \theta) = \tan \tfrac{1}{2} \theta';$$

$$\therefore \quad a + \theta' - \theta = 90^\circ - \tfrac{1}{2} \theta';$$

$$\therefore \quad \theta' = \tfrac{2}{3} (90^\circ + \theta - a) \quad - \quad - \quad - \quad - \quad (4)$$

or

$$\theta = \tfrac{3}{2} \theta' + a - 90^\circ \quad - \quad - \quad - \quad - \quad - \quad - \quad (4')$$

Substituting (4) in (2) and reducing, the equation of the caustic becomes

$$r \sin^3 (60^\circ + \tfrac{1}{3} a - \tfrac{1}{3} \theta) = \tfrac{1}{2} p \sin a \quad - \quad - \quad - \quad (5)$$

The coördinates of the point nearest the focus are

$$r_m = \tfrac{1}{2} p \sin a, \ \theta_m = a - 90^\circ \quad - \quad - \quad - \quad - \quad - \quad - \quad (6)$$

If the vectorial angles are measured from this minimum radius vector instead of from the axis, and if θ_1 represents the new vectorial angle,

$$\theta_1 = 90^\circ + \theta - a \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (7)$$

and (5) becomes

$$r = \tfrac{1}{2} \frac{p \sin a}{\cos^3 \frac{1}{3} \theta_1} \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (8)$$

showing that the caustic is symmetrical with respect to the minimum radius vector.

From (4′) and (7)

$$\theta_1 = \tfrac{3}{2} \theta' \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad (9)$$

The length of the arc of the caustic corresponding to a mirror with the angular aperture $2\theta'$ is

$$s = 3 p \sin a (\tan \tfrac{1}{2} \theta' + \tfrac{1}{3} \tan^3 \tfrac{1}{2} \theta') \quad - \quad - \quad (10)$$

Fig. 2 shows the portion of the caustic that falls within the mirror when $a = 30^\circ$ and the aperture is 240°, the degrees marked along the caustic being the vectorial angles of the corresponding points on the parabola.

In Fig. 3, the angular aperture $2\theta'$ is 40°, and $a\,o\,b$ is the caustic formed when $a = 30^\circ$, a corresponding to A, o to O, and b to B. If a is less than 30° the points a, o and b will fall

nearer the focus, the arc $a\,o\,b$ will be shorter, and the points a, o and b will describe the circular arcs $a\,F$, $o\,F$, and $b\,F$ respectively as α approaches zero.

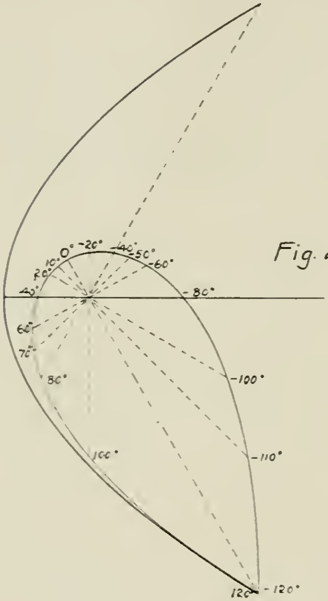

Fig. 2

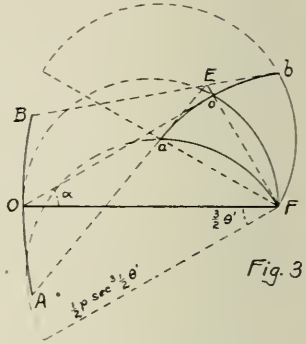

Fig. 3

The extreme rays $A\,a$ and $B\,b$ (Fig. 3) meet the line $F\,o$ at E and the cross section of the system of rays is $o\,E$. The coördinates of the point o are $(\frac{1}{2}\,p \sin \alpha, \alpha - 90^\circ)$. The direction angle of E is also $\alpha - 90^\circ$; substituting this for θ in (2) gives

$$F\,E = \tfrac{1}{2}\,\frac{p \sin \alpha}{\cos^2 \tfrac{1}{2}\,\theta' \cos \theta'}.$$

$$\therefore\; o\,E = F\,E - F\,o = \tfrac{1}{2}\,p \sin \alpha \left(\frac{1}{\cos^2 \tfrac{1}{2}\,\theta' \cos \theta'} - 1\right)$$

which reduces to

$$o\,E = \tfrac{1}{2}\,p \sin \alpha \tan^2 \tfrac{1}{2}\,\theta'\,\frac{2 + \cos \theta'}{\cos \theta'}.$$

If the angle at the vertex of the mirror subtended by $o\,E$ (*i. e.*, $o\,O\,E$) is represented by γ,

$$\tan\gamma = \frac{oE}{Oo} = \tan\alpha \tan^2 \tfrac{1}{2}\theta' \frac{2+\cos\theta'}{\cos\theta'} \qquad (11)$$

equivalent to the final expression for tan γ derived by Dr. Poor.[1] This applies, however, to the parabolic cylinder and not to the paraboloid of revolution.

Rensselaer Polytechnic Institute,
March 24, 1898.

[1] This Journal, February 1898, p. 119 and *Ast. Jour.*, **420**, p. 90.

THE PARABOLIC MIRROR.

By C. W. Crockett.

Let O-APB be a paraboloid of revolution, O the principal vertex, F the focus, P any point on the circle APB whose plane is perpendicular to the axis, and PN the normal at P.

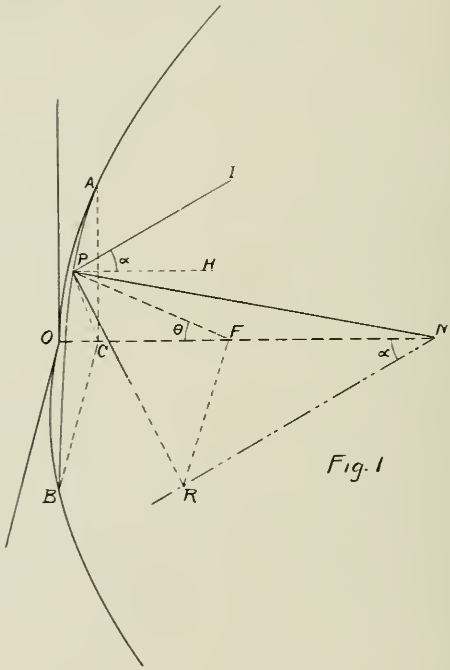

Fig. 1

Let the incident rays, such as IP, be parallel to the vertical plane AOF and make the angle a with the horizontal plane BOF, and let PR be the reflected ray corresponding to IP. Then the ray reflected from any point P on the circle APB must pierce the vertical plane AOF somewhere in the line NR drawn through N parallel to IP; for if a plane be passed through IP parallel to the vertical plane AOF, the plane IPR,

containing IP, PN and PR, must intersect the parallel planes in the parallel lines IP and NR.

Let $v = \angle IPN = \angle NPR$,

$r = FP$,

$\theta = OFP$,

$p = CN$,

$\frac{1}{2} p = OF$,

$\psi =$ angle between the vertical plane AOF and the plane PCN through the axis ON and P,

$= \angle ACP$.

I. In the plane triangle PFN, $PF = FN$.

$$\therefore \angle FPN = \angle FNP = \tfrac{1}{2}\theta.$$

$$\therefore PN = p \sec \tfrac{1}{2}\theta.$$

II. Constructing a sphere about P as a center, the planes $IPNR$ and $HPNF$ cut two intersecting arcs of great circles, which with the arcs cut by the planes IPH and FPR form two congruent spherical triangles, from which we find that the plane angle $FPR = a$ and the dihedral angle $NPF\text{-}RPF = 180° - \psi$.

III. Constructing a sphere about N, the planes FNP, FNR, and PNR give a spherical triangle having two sides and the included angle equal to two sides and the included angle of the triangles formed when the sphere was passed about P.

$$\therefore \text{Plane angle } PNR = v.$$

Hence in the plane triangle PNR, we have

$$\angle PNR = \angle NPR.$$

$$\therefore RP = RN.$$

IV. In the plane triangles FRP and FRN, it has been shown that

$$FP = FN;\quad RP = RN;\quad \angle RPF = \angle RNF = a.$$

Hence the two triangles are congruent, and the triangle FPR may be brought into coincidence with FNR by revolution about FR.

V. Pass a plane DFN (Fig. 2) through the axis, making an angle ψ_1 with the vertical plane AOF, and represent the plane angle NFD by θ_1 and the radius vector FD by r_1. Then from the plane triangle PFD,

$$r \sin \alpha = r_1 \sin FDP = r_1 \sin (DFP + \alpha).$$

$$\therefore \; r_1 = r \sin \alpha \operatorname{cosec} (\gamma + \alpha) \quad - \; - \; - \; - \; - \quad (1)$$

where $\gamma = DFP$.

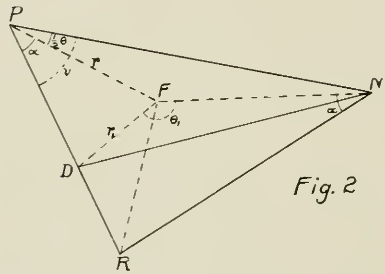

Fig. 2

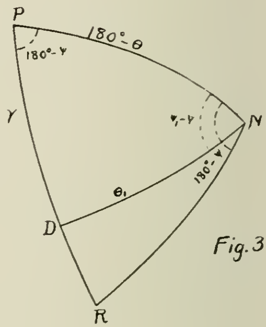

Fig. 3

VI. Constructing a sphere about F, the planes PFN, DFN, RFN and PFR will cut the surface in a figure, similar to that shown in Fig. 3, where

$$DP = \gamma,\; PN = 180° - \theta,\; DN = \theta_1,\; DNP = \psi_1 - \psi,$$
$$RNP = 180° - \psi = RPN$$

(shown in Section II). Hence

$$\sin \gamma = \frac{\sin (\psi_1 - \psi)}{\sin \psi} \sin \theta_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$\cos \gamma = - \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\psi_1 - \psi) \quad . \quad (3)$$

$$\sin \theta \cot \theta_1 = - \cot \psi \sin (\psi_1 - \psi) - \cos \theta \cos (\psi_1 - \psi) \quad (4)$$

VII. To find the point where the reflected ray pierces the plane through the focus perpendicular to the axis, make $\theta_1 = 90°$ in Eqs. (1), (2), (3) and (4).

$$\left.\begin{aligned} \therefore\ \tan(\psi_1-\psi) &= -\cos\theta\tan\psi \\ \sin\gamma &= \frac{\sin(\psi_1-\psi)}{\sin\psi} \\ \cos\gamma &= \sin\theta\cos(\psi_1-\psi) \\ r_1 &= \frac{r\sin\alpha}{\sin(\gamma+\alpha)} \end{aligned}\right\} \quad \cdots \quad (5)$$

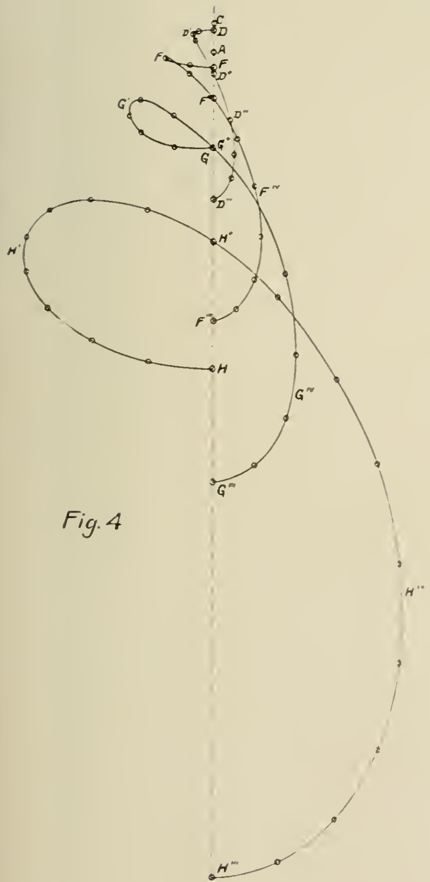

Fig. 4

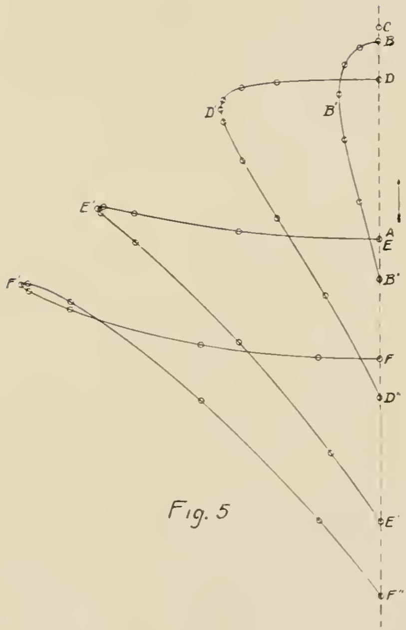

Fig. 5

VIII. When θ is less than $90^\circ-a$, the following conditions must be satisfied in Eqs. (5):

for $\psi<90^\circ$; $\psi_1>180^\circ$, $\psi_1-\psi$ between 90° and 270°;
for $\psi>90^\circ$; $\psi_1<180^\circ$, $\psi_1-\psi$ numerically less than 90°.

Conclusion.—Assuming that $a = 1°$ and considering the circles on the paraboloid such that θ has the values $0°$, $0°30'$, $0°40'$, $1°00'$, $1°20'$, $1°30'$, $2°$ and $3°$, Eqs. (5) lead to the results shown in Figs. 4 and 5, the latter figure showing the upper part of Fig. 4 on a larger scale. The plane of the paper is the focal plane, the vertical line FA passes through the focus which is above A at the distance $\frac{1}{2}p$ tan $1°$, and the vertex of the paraboloid is behind the paper, so that the observer is looking towards the vertex. In Fig. 4 the distance $AH = 0.0000199 \times \frac{1}{2}p$ and in Fig. 5 $AF'' = 0.0000029 \times \frac{1}{2}p$. The diagrams show the results corresponding to ψ from $0°$ to $180°$—*i. e.*, for half the mirror—and the curves should therefore be symmetrical with reference to the line FA.

The traces are as follows:

for $\theta = 0°$, the trace is the point A;
$\theta = 0°30'$, the trace is the curve $B^{II}\,B^{I}\,B$ (Fig. 5);
$\theta = 0°40'$, the highest point of the image is at C;
$\theta = 1°00'$, the trace is the curve $D^{III}\,D^{II}\,D^{I}\,D$;
$\theta = 1°20'$, the trace is the curve $E^{II}\,E^{I}\,E$ (Fig. 5);
$\theta = 1°30'$, the trace is the curve $F^{III}\,F^{IV}\,F^{II}\,F^{I}\,F$;
$\theta = 2°00'$, the trace is the curve $G^{III}\,G^{IV}\,G^{II}\,G^{I}\,G$;
$\theta = 3°00'$, the trace is the curve $H^{III}\,H^{IV}\,H^{II}\,H^{I}\,H$.

The curves were constructed from the points marked in the diagrams, as they indicate the forms of the curves with an accuracy sufficient for the purpose of this article.

It will be noticed that as θ increases from $0°$, the trace starting as the point A becomes a curve similar to B, the highest point moving upwards until it reaches C; then the trace becomes similar to D, the highest point that lies on the line FA moving downward, in the E curve coinciding with A. The loop first appears when $\theta = 1°25'$ approximately, increasing in size through the form of the F curve to that of the G, after which the trace is similar to H.

Rensselaer Polytechnic Institute,
March 24, 1898.

COMPARISON OF OXYGEN WITH THE EXTRA LINES IN THE SPECTRA OF THE HELIUM STARS, β CRUCIS, ETC., ALSO SUMMARY OF THE SPECTRA OF SOUTHERN STARS TO THE 3½ MAGNITUDE AND THEIR DISTRIBUTION.[1]

By FRANK McCLEAN.

IN a previous paper read before the Society on April 8, 1897, I suggested that the special lines present in spectra of the first division of helium stars (Type I, Division I*a*) might possibly be due to oxygen. These stars are associated by their position and distribution with the gaseous nebulæ, and some of the lines in their spectra correspond with bright lines observed by Campbell in nebulæ. The suggestion from this was that these stars are in the first stage of stellar development from gaseous nebulæ.

The special lines referred to are the extra lines which distinguished these spectra from those of the remaining helium stars of Division I*b*.

The indications in the spectra of the northern stars that these extra lines are due to oxygen are slight, as the lines at best are indistinct. Among the southern stars, however, there are several in the spectra of which these lines are better defined, and there is one, viz., β Crucis, in which they are very fairly defined.

The following stellar spectra are mounted on the plate presented to the Society, viz., κ Orionis, β Scorpii, β Canis Majoris, β Centauri, and β Crucis. These photographs are intended to show the gradual improvement in the definition of the extra lines, between κ Orionis and β Crucis, and to indicate their identity of origin throughout.

The extra lines in the spectrum of β Crucis are singled out by comparison with another helium star, viz., κ Argus, of Division I*b*, in which the extra lines do not appear. The lines are drawn out by themselves below the spectrum of β Crucis. They are then compared directly by juxtaposition with a drawing of the spectrum of oxygen as tabulated in the spectrum of air by Neovius (Stockholm, 1891, and Appendix E, 1894, of *Watts's Index*).

This comparison shows a close correspondence in the grouping of

[1] Read before the Royal Society.

the extra lines with the spectrum of oxygen. The most remarkable correspondence is in the case of the large group on either side of $H\delta$. A slight shift of about a tenth-meter is required to bring the groups into identical positions. However, the close similarity of the whole grouping of the two spectra as they appear on the plate admits of little doubt that the extra lines actually constitute the spectrum of oxygen. If this be established the spectrum of the first division of helium stars would be due to hydrogen, helium and oxygen.

The scale attached to the spectra is based on standard lines that can be identified with certainty in the stellar spectra. It is interpolated between the standard lines. Its position in relation to the spectra is determined by the hydrogen lines. The wave-lengths employed are in accordance with Ångström's scale.

On the original negatives the distance between (H) and (F) measures about 1 inch. The negatives are enlarged about eight and a half times. It is difficult to fix the position of the lines — and especially of the hydrogen lines — on these enlargements with sufficient accuracy. A further correction than this would account for is, however, required in order to reduce the two spectra to exact coincidence. I believe it should be sought to some extent in a reëxamination of the adopted wave-lengths of the hydrogen and of the oxygen spectra.

The spectrum of γ Argus is given on the plate in order to identify it as a helium star. It contains two crucial lines of helium. The Wolf-Rayet stars, of which it is the principal example, are thus classified as helium stars. There are also some coincidences between the bright lines of γ Argus and the spectrum of oxygen, which suggest a possible connection.

The spectrum of μ Centauri is also given as a bright line helium star. The bright lines in this case are due to hydrogen, and the spectrum resembles that of γ Cassiopeiæ. The spectrum of γ Centauri is similar.

I take this opportunity of presenting a summary of the spectra of 116 stars to the $3\frac{1}{2}$ magnitude in the Southern Hemisphere. They were photographed between May and October last by means of my own object-glass prism, mounted in front of the Cape astrographic telescope. This instrument, which is similar to my own telescope at Rusthall, with which the spectra of the northern stars were photographed, was kindly placed at my disposal by H. M. Astronomer, Dr. Gill. It may be a little time before the actual photographs of the stellar

spectra are ready for presentation, and meanwhile the results are of interest.

In my previous paper I divided the sphere into eight equal areas consisting of two galactic equatorial areas and two galactic polar areas, situated on either side of the galactic equator. The northern stars

TABLE I.

PHOTOGRAPHIC STELLAR SPECTRA—STARS TO MAGNITUDE 3½.
SUMMARY OF SOUTHERN STARS—REGIONS BB, CC, AND DD.

	Mag.	Div.	Area		Mag.	Div.	Area
Aquila				Centaurus			
λ	3.3	I (*b*)	CC	α	0.7	IV	CC
				β	1.2	I (*a*)	BB
Ara				γ	2.4	I (*b*)	BB
α	2.9	I (*b*)	CC	δ	2.8	II	BB
β	2.8	IV	CC	ε	2.6	I (*a*)	BB
γ	3.6	I (*a*)	CC	ζ	2.7	I (*b*)	BB
ζ	3.2	IV	CC	η	2.5	I (*b*)	BB
				θ	2.7	IV	BB
Argo				ι	3.0	III	BB
α	0.4	III	CC	κ	3.3	I (*b*)	BB
β	2.0	II	CC	λ	3.4	I (*b*)	CC
γ	3.0	I (*b*)	CC	μ	3.4	I (*a*)	BB
δ	2.2	II	CC				
ε	2.1	IV	CC	Circinus			
ζ	2.5	I (*b*)	CC	α	3.5	III	CC
θ	2.9	I (*a*)	CC				
ι	2.5	III	CC	Columba			
κ	2.7	I (*b*)	CC	α	2.7	I (*b*)	CC
λ	2.5	IV	BB	β	2.9	IV	CC
μ	2.9	IV	BB				
ν	3.5	I (*b*)	CC	Crux			
ξ	3.4	IV	BB	α	1.3	I (*a*)	BB
π	2.7	IV	CC	β	1.7	I (*a*)	BB
ρ	3.2	III	BB	γ	2.0	V	BB
σ	3.5	IV	CC	δ	3.4	I (*b*)	BB
τ	3.2	IV	CC				
υ	3.4	III	CC	Doradus			
				α	3.1	I (*b*)	DD
Canis Major							
α	−1.4	II	CC	Eridanus			
β	2.0	I (*a*)	CC	α	1.0	I (*b*)	DD
δ	1.9	IV	CC	θ	2.6	II	DD
ε	1.5	I (*a*)	CC	φ	3.5	I (*b*)	DD
ζ	3.0	I (*b*)	CC	χ	3.9	IV	DD
η	2.4	I (*b*)	CC	Grus			
o²	3.0	I (*b*)	CC	α	1.9	I (*b*)	DD
				β	2.2	V	DD
Capricornus				γ	3.0	I (*b*)	DD
β	3.4	IV	CC	ε	3.5	II	DD

TABLE I — *Continued.*

	Mag.	Div.	Area		Mag.	Div.	Area
Hydrus				Piscis Austr.			
α	2.9	III	DD	α	1.3	II	DD
β	2.7	IV	DD				
γ	3.2	V	DD	Recticulum			
				α	3.3	IV	DD
Indus							
α	3.1	IV	DD	Sagittarius			
				γ^2	3.0	IV	CC
Lepus				δ	2.8	IV	CC
α	2.7	III	CC	ϵ	2.1	I (*b*)	CC
β	3.0	IV	CC	ζ	2.9	II	CC
ϵ	3.3	IV	CC	η	3.0	V	CC
μ	3.3	I (*b*)	CC	λ	3.1	IV	CC
				π	3.1	III	CC
Libra				σ	2.3	I (*b*)	CC
σ (20)	3.2	V	BB	ϕ	3.3	I (*b*)	CC
Lupus				Scorpius			
α	2.6	I (*a*)	BB	α	1.1	V	BB
β	2.8	I (*a*)	BB	β'	2.9	I (*a*)	BB
γ	3.2	I (*a*)	BB	δ	2.5	I (*a*)	BB
δ	3.7	I (*a*)	BB	ϵ	2.2	IV	BB
ϵ	3.7	I (*b*)	BB	θ	2.1	III	CC
				ι'	3.3	III	CC
Musca				κ	2.6	I (*a*)	CC
α	2.9	I (*b*)	CC	λ	1.7	I (*a*)	CC
β	3.4	I (*b*)	CC	μ	3.6	I (*a*)	BB
				π	3.1	I (*a*)	BB
Ophiuchus				σ	3.0	I (*a*)	BB
β	2.9	IV	BB	τ	2.9	I (*a*)	BB
ζ	2.8	I (*a*)	BB	υ	2.8	I (*b*)	CC
η	2.6	II	BB				
θ	3.4	I (*b*)	BB	Serpens			
κ	3.4	IV	BB	η	3.4	IV	BB
Pavo				Telescopium			
α	2.1	I (*b*)	DD	α	3.5	I (*b*)	CC
β	3.3	III	DD				
δ	3.5	IV	DD	Toucan			
				α	2.8	IV	DD
Phœnix				Triangulum			
α	2.4	IV	DD	α	2.2	IV	CC
β	3.3	IV	DD	β	3.1	III	CC
γ	3.4	IV	DD	γ	3.1	II	CC

NOTE.—The magnitudes are taken from the *Nautical Almanac* (or from Gould).

already given occupy the upper or northerly lateral areas A, B, C and D, also the southerly area AA. The southern stars now given occupy the lower or southerly lateral areas BB, CC and DD. Their photo-

graphic spectra are distributed into these areas, and are classified on the same system as in the previous paper. The table of distribution for the whole sphere by areas and classes is given below.

TABLE II.

SUMMARY TABLES OF DISTRIBUTION OF GASEOUS NEBULÆ AND OF STELLAR TYPES. STARS TO THE 3½ MAGNITUDE.

No. 1.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Planetary nebulæ	2	3	8	2	(15)	2	7	3	0	(12)
Extended nebulæ.......	1	4	8	4	(17)	1	4	3	1	(9)
Total gaseous nebulæ.	3	7	16	6	(32)	3	11	6	1	(21)

NOTE.—Extracted from table in Frost's edition of *Scheiner's Astronomical Spectroscopy.*

No. 2.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Division I............	3	6	17	3	(29)	6	23	25	6	(60)
" II............	10	7	0	3	(20)	3	2	5	3	(13)
" III............	7	8	8	4	(27)	9	1	9	2	(21)
" IV............	14	8	9	13	(44)	9	9	16	9	(43)
" V...........	1	2	4	3	(10)	3	3	1	2	(9)
	35	31	38	26	(130)	30	38	56	22	(146)

No. 3.

Stellar types	A	B	C	D	Total	AA	BB	CC	DD	Total
Division I............	3	6	17	3	(29)	6	23	25	6	(60)
" II and III.....	17	15	8	7	(47)	12	3	14	5	(34)
" IV and V......	15	10	13	16	(54)	12	12	17	11	(52)
	35	31	38	26	(130)	30	38	56	22	(146)

No. 4.

Stellar types	A and AA	B and BB	C and CC	D and DD	Total
Division I..................	9	29	42	9	(89)
" II and III..................	29	18	22	12	(81)
" IV and V........	27	22	30	27	(106)
	65	69	94	48	(276)

There are in all 89 helium stars (Division I), distributed 71 in the galactic zones and 18 in the galactic polar areas, the areas being equal. There are 29 in the upper galactic zone (B and BB), and 42 in the lower galactic zone (C and CC). There are 9 in the upper polar areas (A and AA), and 9 in the lower polar areas (D and DD). There are 23 in the northerly halves of the two galactic zones (B and C) and 48 in the southerly halves (BB and CC).

The 81 stars in Division II, the Sirian stars, and Division III, the Procyon stars (which along with Division I constitute Secchi's Type I) are rather irregularly distributed throughout the sphere. There are 40 in the galactic zones and 41 in the galactic polar areas. There are 18 in the upper galactic zone (B and BB) and 22 in the lower (C and CC). There are 29 in the upper polar areas (A and AA) and 12 in the lower (D and DD). To the extent of the observations there is no condensation of stars of Divisions II and III in the galactic zones as there is in the case of stars of Division I.

The 106 stars in Divisions IV and V (II and III of Secchi's types) are fairly evenly distributed throughout the sphere. There are 52 in the galactic zones and 54 in the galactic polar areas. There are 22 in the upper galactic zone (B and BB) and 30 in the lower (C and CC). There are 27 in the upper polar areas (A and AA) and 27 in the lower (D and DD).

The general distribution of the types of spectra throughout the sphere to the extent of the observations bears out generally the conclusion that stars with spectra of the more advanced types, in order of development, are evenly distributed in space. Also that stars with spectra more recent in order of development are mostly congregated in the galactic zones. The helium stars of Division I are predominant in the Southern Hemisphere, being congregated in the lower or southerly halves of the galactic zones (BB and CC). They include 48 stars out of a total of 94 stars in those areas. They are also more closely congregated in the vicinity of the galaxy than is the case in the northerly halves of the galactic zones. In the contiguous constellations of Musca, Crux, Centaurus, Lupus and Scorpius, there are 27 helium stars out of a total of 36 stars included in the tables. (The distribution of the helium stars throughout the sphere was illustrated by two small hand charts, not reproduced, on which these stars are colored red.) Apparently the region in which the first stage of stellar development is now most active lies in the southerly half of the galaxy.

ARC-SPECTRA OF ZIRCONIUM AND LANTHANUM.[1]

By HENRY A. ROWLAND and CALEB N. HARRISON.

SOLAR LINES FOR STANDARDS.

PLATE 32.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3050.217	.212*	+.005	3218.399	.390‖	+.009
3053.180	.173‖	+.007	3224.382	.368‖	+.014
3055.823	.821*	+.002	3232.412	.404‖	+.008
3061.931	.930‖	+.001	3260.393	.384‖	+.009
3077.303	.303‡	.000	3267.840	.839‖	+.001
3078.145	.148‖	—.003	3274.096	.092†	+.004
3086.900	.891‖	+.009	3287.795	.791†	+.004
3094.740	.739‖	+.001	3292.174	.174†	.000
3095.013	.003‖	+.010	3295.954	.957‖	—.003
3115.166	.160‖	+.006	3302.505	.501†	+.004
3121.284	.275†	+.009	3306.475	.471†	+.004
3129.900	.882‖	+.008	3308.925	.928†	—.003
3137.456	.441‖	+.015	3318.158	.163†	—.005
3140.887	.869‖	+.018	3331.750	.741†	+.009
3153.882	.870‖	+.012	3351.871	.877‖	—.006
3167.299	.290‖	+.009	3356.215	.222‖	—.007
3172.187	.175†	+.012	3405.230	.272†	—.042
3176.118	.104‖	+.014	3406.549	.581‖	—.032
3188.173	.164‖	+.009	3406.916	.955‖	—.039
3200.033	.032‖	+.001			

PLATE 36.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3406.550	.581‖	—.031	3486.029	.036†	—.007
3406.920	.955‖	—.035	3490.704	.721*	—.017
3425.706	.721‖	—.015	3491.464	.464‖	.000
3440.730	.759*	—.029	3500.993	.993†	.000
3441.125	.135*	—.010	3510.990	.987‖	+.003
3456.368	.384‖	—.016	3518.487	.487†	.000
3465.972	.991†	—.019	3521.404	.404†	.000
3475.578	.594†	—.016	3540.263	.266†	—.003

[1] Attention is called to the fact that in the following tables the symbols *, †, ‡, ‖, represent the relative weights assigned to standards, and are in ascending scale of magnitude.

PLATE 36.—ZIRCONIUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3545.339	.333‡	+.006	3652.677	.692†	—.015
3549.149	.145	+.004	3658.686	.688†	—.002
3550.003	.006	—.003	3667.387	.397	—.010
3558.662	.670*	—.008	3680.047	.064*	—.017
3564.680	.680†	.000	3683.186	.202†	—.016
3570.228	.225*	+.003	3684.247	.259†	—.012
3581.335	.344*	—.009	3695.178	.194†	—.016
3583.478	.483‡	—.005	3707.170	.186†	—.016
3597.188	.192	—.004	3716.578	.585†	—.007
3609.009	.015*	—.006	3727.766	.763†	+.003
3612.214	.217	—.003	3732.528	.542†	—.014
3618.920	.924*	—.004	3747.075	.095*	—.020
3623.325	.332	—.007	3780.829	.846‡	—.017
3623.598	.603	—.005	3781.309	.330	—.021
3631.615	.619*	—.004	3783.656	.674†	—.018
3640.530	.536†	—.006	3788.058	.032*	+.026
3647.997	.995*	+.002			

PLATE 40.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3883.757	.773‡	—.016	4016.578	.578†	.000
3897.589	.599	—.010	4029.792	.796‖	—.004
3905.657	.666*	—.009	4034.638	.641†	—.003
3924.663	.669‡	—.006	4045.974	.975*	—.001
3925.335	.345‡	—.010	4048.888	.893†	—.005
3926.123	.123†	.000	4055.700	.701‡	—.001
3937.470	.474†	—.004	4062.601	.602‡	—.001
3941.020	.021‡	—.001	4063.750	.756*	—.006
3942.556	.559†	—.003	4073.918	.920‖	—.002
3950.097	.101‡	—.004	4083.755	.767†	—.012
3950.485	.497	—.012	4083.925	.928‖	—.003
3954.001	.001	.000	4103.095	.101†	—.006
3957.180	.180*	.000	4107.642	.646†	—.004
3960.430	.429	+.001	4114.602	.600‖	+.002
3971.478	.478	.000	4121.476	.481‖	—.005
3977.895	.891	+.004	4121.907		
3981.916	.914‡	+.002	4157.936	.948‖	—.012
3984.080	.078†	+.002	4185.045	.063‖	—.018
3986.906	.903*	+.003	4197.238	.251‡	—.013
4016.572	.578‖	—.006			

PLATE 44.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4283.148	.170†	—.022	4415.289	.299†	—.010
4289.504	.523†	—.019	4425.607	.609†	—.002
4293.216	.249*	—.033	4447.889	.899	—.010
4222.339	.381	—.042	4454.936	.950†	—.014
4226.848	.892†	—.044	4494.725	.735†	—.010†
4250.924	.956†	—.032	4497.026	.041†	—.015
4302.680	.689†	—.009	4499.063	.070†	—.007
4308.064	.034*	+.030	4501.438	.444‡	—.006
4343.382	.387†	—.005	4508.449	.456	—.007
4352.895	.903	—.008	4554.187	.213†	—.026
4359.770	.778†	—.008	4571.265	.277†	—.012
4369.930	.943	—.013	4572.142	.157†	—.015
4391.140	.149	—.009	4578.720	.731‡	—.011
4404.917	.927†	—.010	4588.369	.384	—.015
4407.838	.850†	—.012	4590.112	.129	—.017

PLATE 48.—ZIRCONIUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4611.455	.453*	+.002*	4805.263	.253†	+.010
4629.509	.515†	—.006†	4810.732	.723†	+.009
4637.683	.683	.000	4823.705	.697†	+.008
4638.189	.194‡	—.005	4824.334	.325	+.009
4668.300	.303*	—.003	4859.939	.934‡	+.005
4678.345	.353†	—.008	4883.876	.867†	+.009
4686.392	.395†	—.003	4893.045	.030†	+.015
4690.325	.324†	+.001	4917.410	.410†	.000
4691.574	.581*	—.007	4924.104	.109	—.005
4703.181	.180*	+.001*	4934 248	.247*	+.001
4722.355	.349‡	+.006	4936.005	.015†	—.010
4727.628	.628†	.000	4991.221	.247†	—.026
4754.230	.226†	+.004	4994.292	.316	—.024

THE ARC-SPECTRUM OF ZIRCONIUM.

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3054.929	—.002	10	3054.927	3205.028	—.012	3	3205.016
3060.220	—.002	5	3060.218	3208.444	—.012	3	3208.432
3061.451	—.002	3	3061.449	3212.135	—.012	5	3212.123
3064.748	—.002	4	3064.746	3212.974	—.012	3	3212.962
3065.315	—.002	3	3065.313	3214.318	—.012	7	3214.306
3072.240	—.002	1 n	3072.238	3228.933	—.011	7	3228.922
3085.465	—.004	2	3085.461	3234.250	—.010	7	3234.240
3094.914	—.006	2	3094.908	3235.884	—.010	1	3235.874
3095.180	—.006	8	3095.174	3241.180	—.009	7	3241.171
3095.441	—.006	1	3095.435	3244.116	—.008	1	3244.108
3099.331	—.007	10	3099.324	3247.680	—.008	1	3247.672
3109.453	—.009	1 n	3109.444	3250.570	—.007	7	3250.563
3106.682	—.008	10	3106.674	3260.245	—.005	3	3260.240
3110.654	—.009	1	3110.645	3264.949	—.004	3	3264.945
3110.980	—.009	10	3110.971	3269.791	—.003	7	3269.788
3119.330	—.010	1	3119.320	3272.335	—.002	7	3272.333
3120.861	—.010	8	3120.851	3273.170	—.002	7	3273.168
3125.319	—.011	4	3125.308	3279.400	—.001	10	3279.399
3126.021	—.011	10	3126.010	3280.601	—.001	1 n	3280.600
3129.286	—.011	10	3129.275	3282.969	—.000	2	3282.969
3129.865	—.011	10	3129.854	3284.827	—.000	10	3284.827
3132.177	—.011	7	3132.166	3286.025	—.000	2	3286.025
3133.335	—.011	2	3133.324	3288.934	+.001	5	3288.935
3133.595	—.011	8	3133.584	3306.409	+.002	7	3306.411
3137.083	—.011	4	3137.072	3310.022	+.002	3	3310.024
3138.775	—.011	10	3138.764	3311.480	+.002	2	3311.482
3149.937	—.012	3	3149.925	3313.830	+.002	5	3313.832
3155.792	—.012	7	3155.780	3314.613	+.002	7	3314.615
3157.108	—.012	7	3157.096	3316.323	+.002	1	3316.325
3157.944	—.012	7	3157.932	3318.640	+.002	4	3318.642
3164.423	—.012	10	3164.411	3319.146	+.002	7	3319.148
3165.570	—.012	7	3165.558	3323.115	—.001	7	3323.114
3166.076	—.012	7	3166.064	3326.545	—.001	1	3326.544
3166.387	—.012	6	3166.375	3334.381	+.001	7	3334.382
3166.749	—.012	1	3166.837	3334.743	+.001	7	3334.744
3178.205	—.012	7	3178.193	3338.543	+.002	7	3338.545
3185.183	—.012	8	3185.171	3340.611	+.003	7	3340.614
3182.050	—.012	7	3182.038	3344.913	+.004	7	3344.917
3182.965	—.012	10	3182.953	3353.775	+.008	2	3353.783
3191.340	—.012	7	3191.328	3376.375	+.020	5	3376.395
3192.024	—.012	7	3192.012	3377.565	+.020	5	3377.585
3194.548	—.012	1	3194.536	3380.038	+.022	1	3380.060
3196.844	—.012	1	3196.832	3387.976	+.026	7	3388.002
3204.485	—.012	3	3204.473	3388.345	+.026	7	3388.371

ZIRCONIUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3392.350	+.028	10	3392.378	3663.778	+.006	8	3663.784
3393.239	+.029	7	3393.268	3671.403	+.007	2	3671.410
3394.360	+.029	1	3394.389	3674.852	+.007	3	3674.859
3396.438	+.030	7	3396.468	3680.666	+.008	5	3680.674
3398.965	+.032	7	3398.997	3691.521	+.009	8	3691.530
3402.977	+.034	5	3403.011	3696.421	+.009	1 ?	3696.430
3403.806	+.034	1	3403.840	3697.593	+.010	2	3697.603
3404.955	+.033	5	3404.987†	3891.504	+.012	5	3891.516
3407.595	+.032	1 n	3407.627	3892.149	+.012	1, n	3892.161
3408.198	+.037	7	3408.235	3896.653	+.011	2	3896.664
3410.374	+.031	1	3410.405	3897.788	+.010	1	3897.798
3410.328	+.038	7	3410.366	3900.639	+.010	5	3900.649
3414.787	+.029	1	3414.816	3916.060	+.007	5	3916.067
3419.786	+.027	1	3419.813	3921.923	+.005	5	3921.928
3424.953	+.025	1	3424.978	3929.662	+.004	6	3929.666
3430.661	+.022	3	3430.683	3934.250	+.004	4	3934.254
3437.271	+.020	2	3437.291	3934.915	+.004	4	3934.919
3438.361	+.020	5	3438.381	3936.188	+.003	2	3936.191
3446.754	+.016	3	3446.770	3941.756	+.002	2	3941.758
3447.498	+.016	4	3447.514	3958.353	+.001	8	3958.354
3456.042	+.014	3	3456.056	3966.417	.000	1, n	3966.417
3457.327	+.013	2	3457.340	3973.549	.000	1	3973.549
3457.702	+.013	3	3457.715	3975.435	—.001	4	3975.434
3459.071	+.012	1	3459.083	3977.422	—.001	3	3977.421
3461.232	+.012	3	3461.244	3979.375	—.001	3	3979.374
3463.155	+.011	3	3463.166	3981.727	—.001	10	3981.726
3471.321	+.008	6	3471.329	3982.305	—.001	2	3982.304
3478.448	+.007	1	3478.455	3991.269	—.001	8	3991.268
3478.923	+.006	3	3478.929	4012.395	.000	4	4012.395
3481.295	+.005	5	3481.300	4018.520	.000	3	4018.520
3482.944	+.005	3	3482.949	4024.586	.000	3	4024.586
3483.674	+.005	3	3483.679	4025.060	.000	5	4025.060
3496.340	+.003	7	3496.343	4027.349	+.001	5	4027.350
3499.725	+.002	1	3499.727	4028.098	+.001	3	4028.099
3505.581	+.001	1	3505.582	4029.820	+.001	5	4029.821
3505.812	+.001	4	3505.813	4030.187	+.001	4	4030.188
3519.735	+.001	10	3519.736	4031.496	+.001	1	4031.497
3552.092	+.001	3	3552.093	4032.196	+.001	4	4032.197
3556.743	+.001	5	3556.744	4032.210	+.001	1	4032.211
3558.942	+.002	1	3558.944	4034.230	+.001	3	4034.231
3572.603	+.003	10	3572.606	4036.038	+.001	4	4036.039
3601.326	+.005	10	3601.331	4040.386	+.002	3	4040.388
3612.037	+.005	1	3612.042	4041.787	+.002	3	4041.789
3613.242	+.005	3	3613.247	4042.371	+.002	3	4042.373
3614.920	+.005	5	3614.925	4043.720	+.002	5	4043.722
3624.000	+.005	8	3624.005	4045.756	+.002	6	4045.758
3630.164	+.005	1	3630.169	4045.970	+.002	3	4045.972
3634.293	+.005	3	3634.298	4048.811	+.002	7	4048.813
3636.596	+.005	2	3636.601	4049.714	+.002	5	4049.716
3658.285	+.005	1, n	3658.290	4050.465	+.002	3	4050.467

ZIRCONIUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4054.579	+.002	3	4054.581	4183.459	+.013	4	4183.472
4055.171	+.002	5	4055.173	4186.821	+.013	2	4186.834
4055.849	+.002	4	4055.851	4191.635	+.015	2	4191.650
4056.653	+.002	1	4056.655	4191.931	+.016	1	4191.947
4057.982	+.002	2	4057.984	4194.147	+.016	2	4194.163
4058.769	+.002	2	4058.771	4194.909	+.016	4	4194.925
4060.231	+.002	1	4060.233	4196.280	+.017	3	4196.297
4060.728	+.002	2	4060.730	4227.839	+.041	8	4227.880
4061.676	—.002	4	4061.678	4231.644	+.039	4	4231.683
4064.301	+.002	8	4064.303	4231.716	+.039	3	4231.755
4068.870	+.002	2	4068.872	4234.717	+.038	3	4234.755
4071.240	+.002	2	4071.242	4236.153	+.037	5	4236.190
4072 840	+.002	10	4072.842	4237.517	+.037	2	4237.554
4075.070	+.002	3	4075.072	4239.392	+.036	8	4239.428
4076.676	+.002	3	4076.678	4240.420	+.034	8	4240.454
4077.199	+.002	2	4077.201	4241.286	+.033	8	4241.319
4078.455	+.002	4	4078.457	4241.770	+.033	8	4241.803
4081.359	+.002	10	4081.361	4253.660	+.030	2	4253.690
4082.439	+.002	2	4082.441	4256.546	+.029	2	4256.575
4083.239	+.002	2	4083.241	4258.142	+.029	7	4258.171
4084.450	+.002	3	4084.452	4261.304	+.027	2	4261.331
4085.838	+.002	5	4085.840	4261.526	+.027	2	4261.553
4087.836	+.002	2	4087.838	4264.115	+.026	1, n	4264.141
4090.662	+.002	6	4090.664	4265.015	+.026	1	4265.041
4090.943	+.002	3	4090.945	4266.828	+.025	1	4266.853
4093.311	+.002	2	4093.313	4268.116	+.025	5	4268.141
4094.416	+.002	2	4094.418	4273.620	+.023	4	4273.643
4096.781	+.002	3	4096.783	4274.861	+.023	2	4274.884
4099.458	+.002	2	4099 460	4277.465	+.022	2	4277.487
4107.690	+.002	3	4107.692	4282.306	+.020	5	4282.326
4108.544	+.002	3	4108.546	4285.354	+.019	1, n	4285.373
4110.195	+.002	1	4110.197	4286.615	+.019	1, n	4286.634
4110.801	+.002	1	4110.803	4289.266	+.018	1, n	4289.284
4113.115	+.002	1	4113.117	4290.314	+.018	1, n	4290.332
4120.309	+.002	1	4120.311	4291.454	+.016	2	4291.470
4121.601	+.002	5	4121.603	4294.897	+.017	7	4294.914
4128.125	+.002	1	4128.127	4296.311	+.016	2	4296.327
4135.828	+.002	4	4135.830	4300.681	+.015	1, n	4300.696
4140.158	+.003	2	4140.161	4302.990	+.015	5	4303.005
4146.034	+.004	1 n	4146.038	4304.803	+.014	3	4305.817
4149.339	+.004	10	4149.443	4306.034	+.014	1	4306.048
4151.118	+.005	4	4151.123	4309.931	+.013	1	4309.944
4152.788	+.005	4	4152.793	4312.342	+.012	1	4312.354
4156.377	+.006	8	4156.383	4317.424	+.011	5	4317.435
4161.345	+.007	7	4161.352	4319.164	+.011	2	4319.175
4166.501	+.008	4	4166.509	4321.287	+.011	2	4321.298
4169.494	+.009	1	4169.503	4324.145	+.011	2	4324.156
4171.616	+.009	3	4171.625	4325.554	+.010	4	4325.564
4179.953	+.011	3	4179.964	4329.685	+.010	2	4329.695
4182.710	+.012	5	4182.722	4333.380	+.010	2	4333.390

ZIRCONIUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4336.463	+.010	1, N	4336.473	4490.392	+.010	1, N	4490.402
4337.741	+.010	1	4337.751	4494.550	+.010	3	4494.560
4341.248	+.010	7	4341.258	4495.588	+.010	1	4495.598
4342.356	+.010	1, n	4342.366	4497.139	+.010	7	4497.149
4343.160	+.010	1	4343.170	4507.250	+.010	4	4507.260
4343.517	+.010	1, n	4343.527	4526.265	+.010	1, n	4526.275
4346.641	+.010	2	4346.651	4535.877	+.010	5	4535.887
4347.349	+.010	1	4347.359	4542.351	+.010	4	4542.361
4347.469	+.010	1	4347.479	4550.260	+.011	2	4550.271
4348.009	+.010	8	4348.019	4553.142	+.011	1	4553.153
4358.870	+.010	1	4358.880	4554.100	+.011	1	4554.111
4359.852	+.010	8	4359.862	4554.174	+.011	2	4554.185
4360.427	+.010	4	4360.437	4555.659	+.011	2	4555.670
4366.571	+.010	4	4366.581	4558.175	+.011	1, n	4558.186
4371.078	+.010	7	4371.088	4565.587	+.012	1, n	4565.599
4373.203	+.010	1	4373.213	4574.633	+.013	1, n	4574.646
4379.899	+.010	7	4379.909	4582.435	+.014	1, n	4582.449
4389.746	+.010	1, n	4389.756	4590.675	+.016	1, n	4590.691
4395.066	+.010	3	4395.076	4614.092	+.004	1	4614.096
4400.365	+.010	1	4400.375	4629.223	+.004	1	4629.227
4401.481	+.010	1	4401.491	4634.139	+.004	1	4634.143
4403.472	+.010	1, n	4403.482	4640.290	+.004	1, n	4640.294
4414.439	+.010	2	4414.449	4644.982	+.004	1, n	4644.986
4414.665	+.010	3	4414.675	4657.795	+.004	1	4657.799
4420.588	+.010	3	4420.598	4661.958	+.004	1, n	4661.962
4427.373	+.010	2	4427.383	4667.314	+.004	1, n	4667.318
4429.235	+.010	1	4429.245	4683.592	+.004	1	4683.596
4431.619	+.010	3	4431.629	4687.971	+.004	3	4687.975
4435.976	+.010	1, n	4435.986	4688.621	+.004	1	4688.625
4436.900	+.010	1, n	4436.910	4707.952	+.002	1, n	4707.954
4454.929	+.010	5	4454.939	4710.250	+.002	3	4710.252
4455.564	+.010	1	4455.574	4712.085	+.002	1	4712.087
4456.428	+.010	1	4456.438	4717.795	.000	1, n	4717.795
4457.552	+.010	4	4457.562	4719.291	.000	1, n	4719.291
4460.485	+.010	4	4460.495	4732.510	—.003	1	4732.507
4460.920	+.010	1	4460.930	4739.655	—.004	1	4739.651
4467.044	+.010	1	4467.054	4762.955	—.008	1	4762.947
4468.354	+.010	1	4468.364	4772.498	—.009	2	4772.489
4469.654	+.010	1, n	4469.664	4785.103	—.009	1	4785.094
4470.451	+.010	2	4470.461	4788.862	—.009	1	4788.853
4470.688	+.010	4	4470.698	4806.060	—.009	1	4806.047†
4482.170	+.010	1	4482.180	4934.245	—.009	1	4934.236
4485.577	+.010	1, n	4485.587				

SOLAR LINES FOR STANDARDS.

PLATE 32.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3094.998	95.003‡	—.005	3247.667	.680*	—.013
3106.659	.677*	—.018	3260.373	.384‡	—.011
3115.152	.160†	—.008	3267.832	.839‖	—.007
3129.875	.882‡	—.005	3274.070	.092†	—.022
3137.443	.441†	+.002	3287.771	.791‖	—.020
3140.875	.870†	+.005	3292.153	.174*	—.021
3153.872	.870†	+.002	3295.932	.957‖	—.025
3167.294	.290‡	+.004	3302.485	.501‡	—.016
3172.174	.175†	—.001	3303.624	.648*	—.024
3176.103	.104†	—.001	3306.446	.471†	—.025
3188.165	.164‡	+.001	3308.907	.928*	—.021
3200.031	.032†	—.001	3318.131	.163‖	—.032
3218.390	.390‖	.000	3331.719	.741‡	—.022
3224.370	.368†	+.002	3347.990	48.011*	—.021
3231.437	.421*	+.016	3356.190	.222‖	—.032
3232.403	.404‡	—.001	3377.620	.667*	—.047
3646.123	.124‖	—.001	3389.847	.887‖	—.040

PLATE 36.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3441.103	.135*	—.032	3570.416	.402*	+.014
3444.026	.032*	—.006	3581.340	.344‖	—.004
3455.353	.384‖	—.031	3583.476	.483‖	—.007
3464.585	.609‖	—.024	3600.880	.880*	.000
3477.987	78.001‖	—.014	3606.829	.831*	—.002
3486.022	.036‡	—.014	3612.210	.217‖	—.007
3491.448	.464‖	—.016	3617.918	.920*	—.002
3500.695	.721‡	—.026	3618.922	.924*	—.002
3500.985	.993‡	—.008	3622.149	.149*	.000
3510.981	.987‡	—.006	3623.330	.332‖	—.002
3513.941	.947†	—.006	3623.601	.603‖	—.002
3518.482	.487‖	—.005	3623.603	.603‖	.000
3521.402	.404*	—.002	3628.846	.853*	—.007
3540.260	.266‖	—.006	3628.858	.853*	+.005
3545.335	.333‖	+.002	3631.624	.619*	+.005
3549.995	50.006*	—.011	3638.430	.435*	—.005
3564.673	.680*	—.007	3647.995	.995†	.000
3565.530	.528*	+.002	3652.690	.692‡	—.002

PLATE 36 — LANTHANUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3653.645	.639†	+.006	3749.620	.623*	—.003
3658.694	.688‖	+.006	3754.661	.664*	—.003
3667.397	.397 |	.000	3756.203	.211‡	—.008
3680.056	.064†	—.008	3758.364	.379*	—.015
3683.198	.202‖	—.004	3763.924	.442*	—.018
3687.595	.607*	—.012	3767.331	.344*	—.013
3695.188	.194‖	—.006	3770.116	.130*	—.014
3705.711	.711*	.000	3774.460	.480*	—.020
3707.185	.186 |	—.001	3780.826	.846‡	—.020
3710.429	.438*	—.009	3781.315	.330‡	—.015
3716.583	.585 |	—.002	3783.654	.674‡	—.020
3720.081	.086†	—.005	3788.016	.032*	—.016
3722.693	.691*	+.002	3793.995	94.014†	—.019
3727.758	.763*	—.005	3795.127	.150*	—.023
3735.020	.014*	+.006			

PLATE 40′— LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3883.767	.773‡	—.006	4003.910	.916‡	—.006
3886.420	.427†	—.007	4016.578	.578 |	.000
3897.593	.599‡	—.006	4029.781	.796 |	—.015
3905.662	.666†	—.004	4030.900	.914*	—.014
3916.872	.875‡	—.003	4033.213	.225*	—.012
3924.664	.669‡	—.005	4034.623	.641*	—.018
3926.120	.123†	—.003	4035.864	.880*	—.016
3928.070	.071*	—.001	4045.960	.975*	—.015
3937.476	.474‡	+.002	4048.880	.893*	—.013
3941.019	.021‡	—.002	4055.688	.701‖	—.013
3944.145	.159*	—.014	4062.589	.602 |	—.013
3954.001	.001‡	.000	4063.741	.756*	—.015
3957.170	.180*	—.010	4071.882	.904*	—.022
3961.667	.676*	—.009	4073.901	.920 |	—.019
3971.465	.478‡	—.013	4077.860	.883*	—.023
3973.830	.835*	—.005	4083.739	.767*	—.028
3977.887	.891‖	—.004	4088.702	.716‖	—.014
3981.916	.914‡	+.002	4103.075	.101‖	—.026
3984.085	.078†	+.007	4107.626	.640*	—.014
3989.222	.216†	+.006	4114.578	.600 |	—.022

PLATE 40′′—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
3950.101	.101	.000	4063.749	.756*	—.007
3954.005	.001	+.004	4071.893	.904*	—.011
3957.182	.180†	+.002	4073.918	.920*	—.002
3960.430	.429	+.001	4107.639	.646†	—.007
3971.478	.478†	.000	4114.602	.600	+.002
3984.085	.078†	+.007	4121.475	.481‡	—.006
3987.226	.216*	+.010	4157.923	.948	—.025
4003.918	.916‡	+.002	4185.036	.063	—.027
4016.577	.578	—.001	4197.224	.251†	—.027
4029.786	.796	—.010	4077.874	.883*	—.009
4030.900	.914†	—.014	4083.905	.928	—.023
4033.223	.225†	—.002	4088.710	.716	—.006
4034.632	.641†	—.009	4114.591	.600	—.009
4044.280	.293*	—.013	4157.918	.948	—.030
4045.980	.975*	+.005	4185.026	.063	—.037
4048.881	.893†	—.012	4197.222	.251†	—.029

PLATE 44.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4222.347	.381	—.034	4435.140	.132†	+.008
4254.498	.502*	—.004	4435.865	.852‡	+.013
4260.630	.638*	—.008	4447.909	.899‡	+.010
4293.232	.249†	—.017	4454.953	.950*	+.003
4318.811	.818†	—.007	4494.742	.735‡	+.007
4352.907	.903	+.004	4508.461	.456	+.005
4369.956	.943†	+.013	4554.210	.213†	—.003
4376.108	.103†	+.005	4571.273	.277‡	—.004
4407.850	.850†	.000	4578.733	.731	+.002
4425.624	.609‡	+.015			

PLATE 44′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4250.258	.290*	—.032	4383.722	.721†	+.001
4250.945	.956†	—.011	4391.161	.149‡	+.012
4254.476	.502‡	—.026	4404.937	.927†	+.010
4260.623	.638*	—.015	4407.846	.850*	—.004
4267.928	.958*	—.030	4415.305	.299*	+.006
4271.914	.924†	—.010	4435.853	.852‡	+.001
4274.941	.958*	—.017	4435.133	.132*	+.001
4283.154	.170*	—.016	4447.912	.899‡	+.013
4289.510	.523*	—.013	4454.947	.950†	—.003
4289.872	.881*	—.009	4456.038	.047*	—.009
4293.231	.249*	—.018	4494.736	.735‖	+.001
4299.135	.152*	—.017	4497.031	.041‡	—.010
4302.681	.689*	—.008	4499.075	.070*	+.005
4307.889	.904*	—.015	4499.308	.315*	—.007
4308.057	.071*	—.014	4501.449	.444‖	+.005
4318.804	.818‖	—.014	4508.459	.456‖	+.003
4325.925	.940†	—.015	4554.206	.213‡	—.007
4343.400	.387*	+.013	4563.941	.939‖	+.002
4352.904	.903‖	+.001	4571.269	.277‖	—.008
4359.784	.778*	+.006	4572.146	.157‖	—.011
4369.946	.943‖	+.003	4578.724	.731‖	—.007
4376.117	.103‖	+.014	4588.368	.384‖	—.016

PLATE 44″.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4215.617	.616†	+.001	4307.890	.904†	—.014
4226.860	.892*	—.032	4308.060	.034†	+.026
4250.268	.290†	—.022	4318.804	.818‡	—.014
4254.477	.502*	—.025	4325.927	.940†	—.013
4260.625	.638†	—.013	4359.786	.778†	+.008
4267.923	.958‡	—.035	4369.930	.943‡	—.013
4271.910	.924*	—.014	4376.102	.103‡	—.001
4274.947	.958†	—.011	4383.720	.721*	—.001
4283.555	.523*	+.032	4407.844	.850†	—.006
4283.142	.170*	—.038	4415.290	.299†	—.009
4289.511	.523*	—.012	4425.607	.609†	—.002
4293.231	.249†	—.018	4435.128	.132†	—.004
4299.144	.152*	—.008	4435.856	.852‡	+.004
4302.683	.689*	—.006	4447.894	.899‡	—.005
4306.057	.071†	—.014	4454.958	.950*	+.008

PLATE 44‴.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4293.238	.249‡	—.011	4407.850	.850‡	.000
4318.814	.818†	—.004	4435.140	.132‡	+.008
4352.914	.903‖	+.011	4447.918	.899‡	+.019
4369.960	.943‡	+.017	4454.958	.950†	+.008
4376.115	.103‡	+.012	4494.750	.735‡	+.015
4425.623	.609‡	+.014	4508.468	.456‡	+.012
4435.871	.852‡	+.019	4554.218	.213†	+.005
4447.920	.899‡	+.021	4571.276	.277†	—.001

PLATE 48.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4629.505	.515‡	—.010	4824.335	.325‡	+.010
4637.669	.683‖	—.014	4810.723	.723*	.000
4643.631	.645‖	—.014	4903.483	.488†	—.005
4686.389	.395‖	—.006	4919.175	.183†	—.008
4703.999	.986‖	+.013	4924.955	.955‖	.000
4703.182	.180†	+.002	4934.244	.247†	—.003
4722.348	.349†	—.001	4973.262	.274‖	—.012
4727.638	.628‡	+.010	4978.760	.782†	—.022
4754.237	.226†	+.011	4994.301	.316‖	—.015
4859.930	.934‡	—.004			

PLATE 48′.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
4602.152	.183‖	—.031	4783.607	.601*	+.006
4607.470	.509‡	—.039	4805.260	.253†	+.007
4611.423	.453‡	—.030	4810.734	.723*	+.011
4629.501	.515‡	—.014	4823.690	.697†	—.007
4637.672	.683‖	—.011	4824.337	.325‡	+.012
4643.625	.645‖	—.020	4859.936	.934‖	+.002
4648.821	.835†	—.014	4890.923	.945†	—.022
4668.289	.303†	—.014	4805.253	.253†	.000
4678.345	.353†	—.008	4810.728	.723*	+.005
4683.738	.743‖	—.005	4783.614	.601†	+.013
4686.398	.395‖	+.003	4754.222	.226‡	—.004
4690.312	.324‖	—.012	4727.620	.628‡	—.008
4691.567	.581†	—.014	4722.340	.349†	—.009
4691.570	.581†	—.011	4810.732	.723*	+.009
4703.186	.180†	+.006	4823.687	.697‡	—.010
4714.600	.599*	+.001	4824.329	.325‡	+.004
4722.341	.349‡	—.008	4859.938	.934‡	+.004
4727.634	.628†	+.006	4859.933	.934‡	—.001
4754.232	.226†	+.006	4900.091	.098‡	—.007

PLATE 52.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5005.851	.904‡	—.053	5225.691	.690	+.001
5006.251	.303‡	—.052	5230.020	.014‡	+.006
5014.379	.422*	—.043	5233.111	.124‡	+.013
5020.173	.210‡	—.037	5242.662	.662‡	.000
5049.972	50.008	—.036	5250.374	.391‡	—.017
5060.226	.252‡	—.026	5253.630	.649	—.019
5064.810	.833‡	—.023	5261.868	.880‡	—.012
5068.921	.946	—.025	5266.714	.729‡	—.015
5083.501	.525‡	—.024	5269.716	.722*	—.006
5090.933	.959‡	—.026	5273.548	.554*	—.006
5109.807	.825‡	—.018	5281.950	.968‡	—.018
5115.554	.558‡	—.004	5283.789	.803‡	—.014
5127.520	.530‡	—.010	5288.688	.708‡	—.020
5146.651	.664‡	—.013	5296.858	.873‡	—.015
5155.931	.937‡	—.006	5300.902	.918‡	—.016
5165.575	.588‡	—.013	5307.519	.546‡	—.027
5171.779	.783‡	—.004	5324.345	.373‡	—.028
5183.786	.792†	—.006	5333.060	.092‡	—.032
5189.020	.020†	.000	5349.617	.623†	—.006
5188.843	.863*	—.020	5361.775	.813†	—.038
5193.141	.139‡	+.002	5367.617	.670‡	—.053
5198.875	.885‡	—.010	5370.120	.165‡	—.045
5202.475	.483†	—.008	5379.724	.776‡	—.052
5204.700	.708†	—.008	5383.518	.576‡	—.058
5215.344	.352‡	—.008	5393.318	.378‡	—.060
5217.561	.559‡	+.002	5397.284	.346‡	—.062

PLATE 56.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5455.829	.826*	+.003	5590.345	.342†	+.003
5463.491	.493	—.002	5594.685	.695†	—.010
5466.601	.608	—.007	5598.519	.555*	—.036
5477.126	.128	—.002	5598.702	.715*	—.013
5487.959	.968	—.009	5601.501	.501‡	.000
5497.730	.731	—.001	5603.096	.097*	—.001
5501.683	.685	—.002	5615.525	.526	—.001
5513.204	.207	—.003	5615.882	.879‡	+.003
5528.631	.636†	—.005	5624.245	.253	—.008
5535.059	.073	—.014	5624.776	.768	+.008
5544.160	.158	+.002	5634.171	.167	+.004
5555.109	.113	—.004	5641.663	.661	+.002
5569.844	.848‡	—.004	5645.831	.835	—.004
5576.317	.319	—.002	5655.714	.707	+.007
5588.975	.980†	—.005	5658.073	.096*	—.023

PLATE 56.—LANTHANUM—*continued.*

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5662.744	.745	—.001	5742.059	.066	—.007
5675.648	.648	.000	5752.246	.257	—.011
5679.245	.249	—.004	5754.869	.884‡	—.015
5682.859	.861	—.002	5763.205	.215†	—.010
5688.431	.434‡	—.003	5772.358	.360‡	—.002
5701.772	.769‡	+.003	5782.342	.346†	—.004
5708.614	.620†	—.006	5784.066	.081	—.015
5711.304	.318*	—.014	5788.123	.136‡	—.013
5715.300	.309‡	—.009	5791.192	.207*	—.015
5731.980	.973‡	+.007	5798.056	.087	—.031

PLATE 54.—LANTHANUM.

Micrometer reading	Standard	Difference	Micrometer reading	Standard	Difference
5784.074	.081	—.007	5884.062	.048*	+.014
5788.130	.136	—.006	5905.895	.895‡	.000
5798.071	.087	—.016	5914.379	.384†	—.005
5798.391	.400‡	—.009	5916.480	.475‡	+.005
5806.950	.954‡	—.004	5919.853	.855‡	—.002
5809.438	.437‡	+.001	5930.405	.410‡	—.005
5816.590	.594†	—.004	5934.885	.883‡	+.002
5831.820	.832	—.012	5948.761	.761‡	.000
5853.898	.903	—.005	5956.923	.925‡	—.002
5857.666	.672†	—.006	5975.570	.576	—.006
5862.585	.580†	+.005			

THE ARC-SPECTRUM OF LANTHANUM.

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
3104.694	+.008	1	3104.702	4031.846		7	4031.847†
3142.882	—.001	2	3142.881	4035.328	.000	1, n	4035.328
3176.105	—.002	2	3176.103	4035.878	.000	1, n	4035.878
3215.936	—.001	2	3215.935	4036.976		1, n	4036.974†
3236.670	+.001	1	3236.671	4043.071		8	4043.070†
3245.247	+.003	5	3245.250	4045.961	+.001	10	4045.962
3249.477	+.004	4	3249.481	4050.215		6	4050.217†
3265.786	+.009	5	3265.795	4059.631	+.002	1, n	4059.633
3303.219	+.022	5	3303.241	4060.460		4	4060.459†
3337.597	+.033	6	3337.630	4062.886	+.002	1, n	4062.888
3344.671	+.034	5	3344.705	4063.732	+.003	1	4063.735
3376.436	+.036	3	3376.472	4064.922		3	4064.922†
3381.010	+.036	8	3381.046	4065.713		3	4065.715†
3452.296	+.034	3	3452.330	4067.520		5	4067.519†
3453.279	+.033	3	3453.312	4076.841	+.004	2	4076.845
3461.300	+.027	3	3461.327	4077.499	+.004	10	4077.503
3510.116	+.005	3	3510.121	4077.852	+.004	1	4077.856
3513.045	+.005	3	3513.050	4078.886	+.004	1	4078.890
3514.187	+.004	4	3514.191	4086.843	+.005	10	4086.848
3574.560	+.005	5	3574.565	4089.746	+.006	1	4089.752
3641.677	—.002	5	3641.675	4090.541	+.006	1	4090.547
3645.549	—.002	5	3645.547	4090.161	+.006	1	4090.167
3649.664	—.003	5	3649.661	4090.914	+.006	6	4090.920
3650.316	—.003	5	3650.313	4095.271	+.007	3	4095.278
3672.150	—.003	3	3672.147	4099.671		8	4099.678†
3680.050	—.002	1	3680.048	4105.018		10	4105.026†
3734.995	+.006	3	3735.001	4123.382	+.012	3	4123.394
3759.205	+.012	7	3759.217	4181.856		10	4141.872†
3790.935	+.018	6	3790.953	4144.075	+.017	2	4144.092
3794.886	+.018	7	3793.904	4152.098	+.021	8	4152.119
3883.948	+.007	2	3883.955	4152.100	+.020	2	4154.120
3886.489		7	3886.495†	4192.465	+.036	8	4192.501
3916.050	+.003	4	3916.053	4194.332	+.037	1	4194.369
3936.351	.000	7	3936.351	4194.617		1	4194.654†
3949.256	.000	6	3949.256	4204.155	+.063	5	4204.218
3988.669	.000	5	3988.669	4215.637	+.046	2	4215.683
3995.903	.000	5	3995.903	4217.690	+.043	6	4217.733
4013.400	—.001	1	4013.399	4226.862	+.036	6	4226.898
4013.535	.000	1	4013.535	4231.074	+.033	4	4231.107
4015.535		2	4015.531†	4238.512		10	4238.543†
4023.720		3	4023.717†	4250.126		5	4250.144†
4023.999	.000	4	4023.999	4254.478	+.026	1	4254.504
4025.786	.000	3	4025.786	4263.734		7	4263.742†
4026.014		7	4026.013†	7275.782		4	4275.797†

† Mean of several values obtained from different plates.

LANTHANUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
4280.411		3	4280.418†	4692.678	+.002	6	4692.680
4282.537		2	4282.560†	4699.810	+.001	1, n	4699.811
4283.165	+.015	1	4283.180	4703.458	+.001	1	4703.459
4287.111		10	4287.128†	4703.429	.000	1	4703.429
4296.196		10	4296.210†	4716.624	—.001	3	4716.623
4300.765		2	4300.782†	4716.705	—.001	1, n	4716.704
4302.683		1	4302.699†	4728.598		5	4728.594†
4316.060		8	4316.076†	4743.277		7	4743.273†
4318.813	+.012	1	4318.825	4748.915		4	4748.911†
4333.924		15	4333.934†	4767.078	—.005	2	4767.073
4335.126		7	4335.134†	4796.868		1, n	4796.862†
4354.565		8	4354.565†	4804.221		4	4804.219†
4363.223		3	4363.221†	4809.188		5	4809.183†
4378.282		8	4378.274†	4809.695	—.005	5	4809.690
4383.628		7	4383.624†	4824.243		5	4824.239†
4385.360		4	4385.364†	4839.703		2	4839.697†
4411.380		2	4411.379†	4840.207		3	4840.203†
4419.332		3	4419.329†	4850.775		1, n	4850.772†
4423.350		1, n	4423.354†	4851.000	—.000	1, n	4851.000
4424.083		4	4424.082†	4861.082		4	4861.081†
4425.595	+.003	1, n	4425.598	4900.099	—.003	6	4900.096
4427.736		8	4427.741†	4921.148	+.001	6	4921.149
4430.079		10	4430.075†	4921.978	+.001	6	4921.979
4432.295	+.001	1, n	4432.296	4935.000	+.003	3	4935.003
4435.141		1	4435.139†	4949.941	+.006	3	4949.947
4435.389	+.003	1, n	4435.392	4952.232	+.006	1	4952.238
4452.331		5	4452.327†	4970.556	+.010	5	4970.566
4455.968		5	4455.965†	4987.044	+.015	6	4987.059
4499.223		2	4499.223†	4991.436	+.016	3	4991.452
4522.540		10	4522.544†	4999.625	+.017	0	4999.642
4525.468	—.002	7	4525.466	5001.929	+.050	11, n	5001.979
4526.279	+.014	8	4526.293	5002.255	+.050	1, n	5002.305
4549.677		7	4549.679†	5047.019	+.032	1, n	5047.051
4550.338		1	4550.337†	5050.704	+.030	1	5050.734
4550.952		2	4550.956†	5056.600	+.028	1	5056.628
4558.653		7	4558.660†	5063.071	+.026	1, n	5063.097
4568.085		5	4568.094†	5106.380	+.015	1	5106.395
4570.202		5	4570.215†	5114.709	+.014	4	5114.723
4575.048		8	4575.059†	5123.137	+.012	4	5123.149
4580.234		5	4580.245†	5145.573	+.009	1	5145.582
4605.922	+.029	4	4605.951	5156.881	+.007	1	5156.888
4613.532	+.025	5	4613.557	5157.575	+.007	1	5157.582
4620.022		3	4620.054†	5158.839	+.007	1	5185.846
4655.650		8	4655.667†	5163.762	+.006	1	5163.768
4662.661		5	4662.678†	5177.448	+.005	3	5177.453
4663.936	+.007	5	4663.943	5183.559	+.005	0, d	5183.664
4669.076		5	4669.080†	5188.366	+.005	11	5188.371
4671.985		4	4671.994†	5204.289	+.004	1	5204.293
4688.820		1, n	4688.824†	5212.010	+.004	1, n	5212.014
4691.349		4	4691.364†	5234.430	+.005	2	5234.435

LANTHANUM—*continued.*

Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected	Wave-length uncorrected	Correction	Intensity and Character	Wave-length corrected
5259.531	+.008	2	5259.539	5565.928	+.003	3	5565.931
5271.327	+.010	2	5271.337	5568.671	+.003	5	5568.674
5290.973	+.015	3	5290.988	5588.555	+.003	3	5588.558
5302.116	+.019	3	5302.135	5657.939	+.001	1	5657.940
5302.774	+.019	2	5302.793	5671.765	.000	2	5671.765
5303.686	+.022	3	5303.708	5703.530	.000	1	5703.530
5340.803	+.033	1	5340.836	5720.221	+.002	2, n	5720.223
5358.003	+.041	1, n	5358.044	5735.155	+.004	2, n	5735.159
5377.213	+.052	2	5377.265	5740.866	+.005	4	5740.871
5381.114	+.054	2	5381.168	5744.619	+.006	4	5744.625
5381.911	+.055	1, n	5381.966	5762.034	+.010	4	5762.044
5382.051	+.055	2	5382.106	5769.273	+.012	5	5769.285
5455.348	+.003	8	5455.351	5769.533	+.012	5	5769.545
5458.884	+.003	1	5458.887	5789.427	+.011	6	5789.438
5464.571	+.003	2	5464.574	5791.528	+.011	4	5791.539
5475.448	+.002	1, n	5475.450	5797.776	+.009	4	5795.785
5480.938	+.002	1, n	5480.940	5805.976	+.008	2	5805.984
5482.472	+.002	2	5482.474	5808.517	+.007	2	5808.524
5491.275	+.002	1	5491.277	5822.180	+.005	1	5822.185
5493.654	+.002	2	5493.656	5824.032	+.005	1	5824.037
5501.557	+.002	8	5501.559	5829.926	+.003	1, n	5829.929
5502.463	+.002	2	5502.465	5845.242	+.001	1, n	5845.243
5502.876	+.002	2	5502.878	5848.583	+.001	1, n	5848.584
5504.019	+.002	4	5504.021	5855.792	.000	1	5855.792
5515.500	+.002	1	5515.502	5863.903	+.001	2	5863.902
5517.560	+.002	2	5517.562	5874.202	+.002	1, n	5874.200
5535.886	+.003	3	5535.889	5874.943	+.002	1, n	5874.941
5545.129	+.003	2	5545.132	5930.330	.000	1	5830.330
5565.657	+.003	5	5565.660				

Minor Contributions and Notes.

WILLIAM A. ROGERS.

On March 1, of the present year, in the city of Waterville, Me., the seat of Colby University, William Augustus Rogers passed away. He had been engaged, even up to a few weeks before his death, in teaching duties, and in investigations connected with standards of length.

He was born in Waterford, Conn., November 13, 1832, prepared for college at Alfred Academy, New York, and graduated from Brown University in 1857. For the next thirteen years he was connected with Alfred Academy, first as tutor, later as professor of mathematics and astronomy. During this time, however, he was allowed to spend one year as student of theoretical and applied mechanics in the Sheffield Scientific School of Yale College, and one year as student of astronomy in the Harvard College Observatory. In 1870 he was appointed assistant in the Observatory, and from 1877 to 1886 he filled the position of assistant professor of astronomy.

During his stay at Harvard Observatory he was engaged in observing and mapping out all the stars down to the ninth magnitude in a belt of five degrees a little north of our zenith. This work formed part of the German survey of the northern heavens under the auspices of the Astronomische Gesellschaft. Five volumes containing the results of this work have been published, and one is in process of compilation.

About this time he became interested in the construction of comparators. In connection with George M. Bond, of Hartford, Conn., he designed the Rogers-Bond universal comparator. One of these instruments was used by the Pratt and Whitney Company in establishing their system of standard gauges, and another was used by Professor Rogers in Waterville in the comparison of standards of length.

In 1879 the American Academy of Arts and Sciences sent Professor Rogers to Europe to obtain authorized copies of the Imperial yard and the mètre des archives. The copies so obtained have been used in the comparisons of yard and meter bars made by him for the Depart-

ment of Standards of the British Board of Trade, the United States Signal Service, the Lick Observatory, and for most of the large universities of the United States and Canada. A paper by Professor Rogers presented to the A. A. A. S. in 1880, "On the Present State of the Question of the Standards of Length," was at its time an exceedingly important contribution.

His work on standards led him to the consideration of various dependent problems, such as the action of a diamond in ruling lines on glass the radiation of heat between metals, and the practical solution of the perfect screw problem. His contributions to the literature and data of these problems were important. As a result of his work we have the Rogers-Ballou process of cutting a screw. His very perfect dividing engines are the embodiment of his principles.

In 1886 Professor Rogers accepted the chair of Physics and Astronomy in Colby University. The Shannon Physical Laboratory, designed by him, was specially suited for constant temperature work. Here he carried on his work of the comparison of bars.

In 1890 Professor Morley became associated with Professor Rogers in the determination of the absolute expansion of metal bars in wave-lengths of sodium light. The investigations were carried on in the Shannon Laboratory. Though results were obtained for only a few bars, these went to show that the experimental difficulties had at last been gotten rid of and that the accuracy of the method was all that could be desired.

Professor Rogers was the recipient of many honors. In 1880 he was made a Fellow of the Royal Society of England, and five years later was elected an Honorary Fellow of that body. He was also an Honorary Fellow of the Royal Microscopical Society; a Fellow of the American Association for the Advancement of Science; twice vice president of Section A, and once of Section B, of the A. A. A. S.; was one of the hundred members of the American Academy of Arts and Sciences, and was also a member of the National Academy of Sciences.

He received the honorary degrees of A.M., Ph.D., LL.D., from Yale, Alfred, and Brown Universities respectively.

In disposition, Professor Rogers was kindly and generous. His broad sympathies led him to take an active and intelligent interest in civic and religious life. He was honored and loved by the student, and his co-workers on the faculty. In 1857, just after his graduation from Brown, he married Rebecca Jane Titsworth who, with two sons, Fred.

P. Rogers, M.D., of Providence, R. I., and Arthur K. Rogers, Fellow in Philosophy in the University of Chicago, survives him.

G. F. H.

PHOTOGRAPHIC SPECTRUM OF THE AURORA.[1]

VARIOUS attempts have been made at this Observatory to photograph the spectrum of the aurora. In 1886 on several occasions long exposures were given to plates during bright auroras, but no result was obtained. On April 1, 1897, Mr. Edward S. King succeeded in obtaining a photograph in which four bright lines were visible, but uncertainty existed regarding their wave-lengths. The exposure was 147 minutes. During the bright aurora of March 15, 1898, he obtained a photograph showing two bright lines. The exposure was 141 minutes. The brightest of these lines extends in wave-length from about 3892 to 3925, and the wave-length of the second is 4285. Assuming the two brighter lines photographed in 1897 to be identical with these, the four lines on that plate have the wave-lengths 3862, 3922, 4288 and 4694. The first of these lines is very faint.

The errors of measurement of these lines do not exceed one or two units, but much greater uncertainty exists in the reduction owing to difficulties in comparing them with the lines of the solar spectrum which was photographed upon the same plate. Probably the two auroras gave different spectra. That in 1897 was taken with a wide slit, but the images of the lines were well defined on the edges and of equal width, so that the line 3922 was probably really narrow and coincident with the edge of greater wave-length of the line 3892 to 3925. The spectroscope used was not especially designed for photographing faint surfaces and it is hoped that better results may be obtained with a new instrument now in course of construction. As is the case with all results announced in these *Circulars* it is expected that full details will be published later in the *Annals* of the Observatory.

EDWARD C. PICKERING.

March 23, 1898.

[1] *Harvard College Observatory, Circular* No. 28.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation,

It is intended to publish from time to time a bibliography of astrophysics, in which will be found the titles of recently published astrophysical and spectroscopic papers. In order that this list may be as complete as possible, and that current work in astrophysics may receive appropriate notice in other departments of the JOURNAL, authors are requested to send copies of all papers on these and closely allied subjects to both Editors

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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